# Spectral Analysis of Negative Refractive Index Metamaterials Utilizing Signal Processing Techniques and Time-Domain Simulations

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**Abstract**

*In this paper, signal processing techniques, mostly used for the Direction of Arrival (DoA) estimation in smart antenna systems, are combined with time-domain simulations in order to perform dispersion analysis of Negative Refractive Index (NRI) metamaterials. The time-domain response of infinite arrays of unit cells giving rise to NRI metamaterials is calculated using the Finite Difference Time Domain (FDTD) method together with Periodic Boundary Conditions (PBC). Subsequently, the correlation method, the MUSIC algorithm and the Matrix Pencil method are applied to the FDTD results and the resonances of those structures are estimated. A comparative study of the aforementioned algorithms is performed and the advantages of the Matrix Pencil algorithm are pointed out.*

## 1 Introduction

Recently, synthesized structures (artificial dielectrics) exhibiting simultaneously negative dielectric permittivity and magnetic permeability have been proposed. Such *metamaterial* structures are characterized by a negative refractive index (NRI). The electromagnetic properties of NRI media have been theoretically studied since the 1960s [1]. However, it was not until few years ago that NRI realizations were presented in the literature [2],[3].

Despite their experimental confirmation, the unusual properties of NRI media, including negative refraction, were theoretically disputed on grounds of causality, stemming from the inspection of their steady state response. On the other hand, time-domain analysis illuminates their transient response, significantly contributing to the clarification of apparent contradictions and the support of experimental observations in metamaterial structures [4]. Therefore, the time-domain modeling of NRI metamaterials, with methods such as the Finite Difference Time Domain (FDTD), is strongly motivated.

In this paper, the FDTD method, augmented with periodic boundary conditions, is combined with signal processing techniques in order perform spectral analysis of NRI metamaterials using time-domain data. The ultimate goal of this work is the development of a fast integrated CAD tool, capable of performing both time-domain and spectral analysis of NRI metamaterials.

## 2 Time-Domain Modeling of NRI Metamaterials

In recent years, several approaches to time-domain modeling of metamaterials have been reported [4]-[5]. The one presented in [6] explicitly models one unit cell of the loaded transmission line NRI metamaterial of [3]. In this approach, PBCs are enforced along the edges of each unit cell of the structure (Fig. 1). The PBCs are implemented in time-domain via the well known sine-cosine [7] method. Thus, a wavevector $\overline{k}$ within the irreducible Brillouin zone is enforced and the corresponding resonant frequencies of the modes, that are characterized by this wavevector and can be possibly supported, are extracted by spectral analysis of the time-series of sampled field components. This spectral analysis is further discussed in later parts of this paper.

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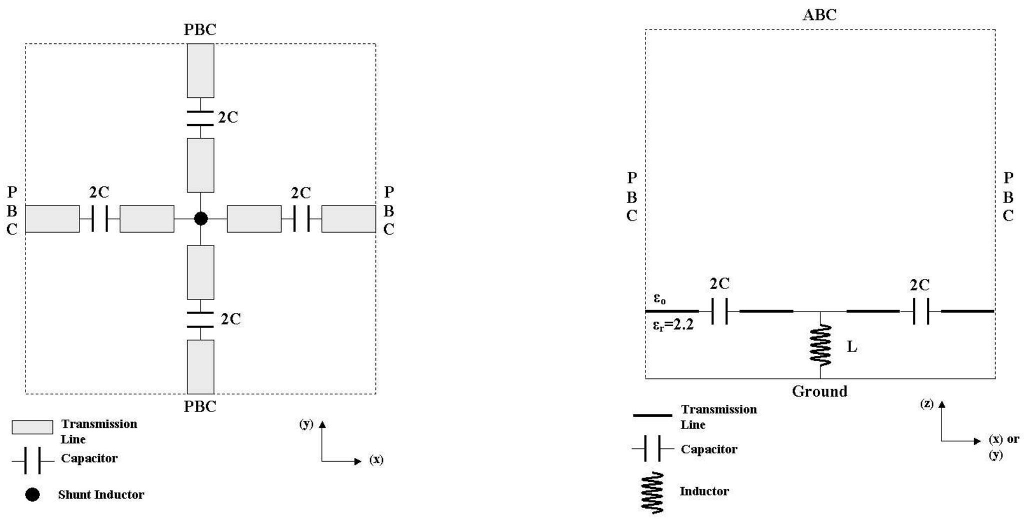


Figure 1: Unit cell of the Negative-Refractive-Index metamaterial under consideration.

## 3 Signal Processing Techniques for Spectral Analysis

Using the methodology briefly mentioned in the previous section (and extensively presented in [6]), the time-domain response $y(t)$ of NRI metamaterials, for a given wavevector $\overline{k}$, is obtained, sampled at time intervals $T_s$. The sampled response at a randomly chosen point in the substrate of the structure is in the form $\bar{y} = \begin{bmatrix} y_0 & y_1 & \ldots & y_{N-1} \end{bmatrix}$. A number of different spectral analysis techniques exist in the literature. In this paper, three different techniques will be employed and evaluated: the correlation method, the Multiple Signal Classification (MUSIC) algorithm and the Matrix Pencil method. The correlation method is a non-adaptive method and is based on the correlation of the sampled signal with exponential Fourier basis functions, while the MUSIC algorithm is an adaptive algorithm, which is based on the autocorrelation of the sampled signal. Further details for these algorithms can be found in [8]. The third method used in this paper, the Matrix Pencil algorithm, is also an adaptive algorithm, which is *not* based on any kind of correlation or autocorrelation. This algorithm is extensively presented in [9].

## 4 Correlation Method, MUSIC, Matrix Pencil: A Comparative Study

In this section, the three spectral analysis methods under study are applied for the extraction of field resonances. Field time-series are determined through the application of the sine-cosine version of the FDTD technique [7].

### 4.1 MUSIC algorithm compared to the Correlation method

Initially, the correlation and MUSIC algorithms are applied to determine the resonances of the structure. The frequency region of interest is that from 0-10 GHz, in which the structure is expected to support backward waves. The results are shown in Fig. 2. In this plot, it is proved that both methods manage to capture the dominant mode of the structure which corresponds to a backward wave. The resonant frequency is around 3.1 GHz. More specifically, after carefully detecting the peak of each curve, it is found that the correlation method calculates that mode to be at 3.12 GHz while the MUSIC algorithm pseudo-spectrum has its peak at 3.07 GHz. By running many similar simulations, it is proved that this divergence between the two methods is consistent and that the frequency extracted using MUSIC method turns to be more accurate and closer to the actual resonant frequency, which is 3.03 GHz. This is caused by the fact that the accuracy of the correlation method strongly depends on the number of the samples $N$ and the sampling interval $T_s$.

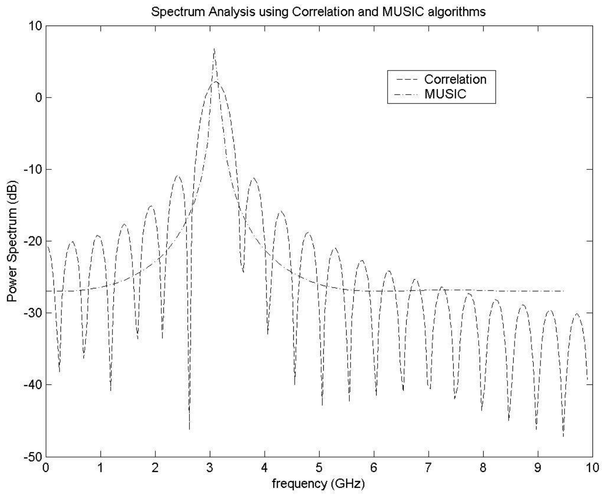


Figure 2: Unit cell of the Negative-Refractive-Index metamaterial under consideration.

The performance of the correlation method is improved as one decreases the sampling time $T_s$ and increases the numder of time steps $N$. This dependence is the major disadvantage of the method because these parameters mostly define the computational power and the execution time needed for the time domain simulation. The number of samples $N$ can be increased by increasing the number of time steps in the simulation, and therefore the execution time.

As far as the MUSIC algorithm is concerned, it is more accurate and not dramatically dependent on the parameters $N$ and $T_s$. Besides, it can be implemented in such way that all spatial modes can be captured. On the other hand, the drawback of this method is that it is computationally demanding as it performs both correlation and eigen-decomposition of $N \times N$ matrices, with $N$ being in the order of thousands.

### 4.2 Matrix Pencil compared to the Correlation method

In the following, the dependence of the Matrix Pencil and correlation methods on the number of time steps used and the spatial gridding of the time-domain simulation is studied. Initially, these two methods are applied in order to estimate the resonant frequency of the metamaterial, using a different number of time steps. The results are shown in Fig. 3. The estimated resonant using the correlation method strongly depends on the number of time steps $N$. Thus, for small numbers of time steps (at the order of one or two full periods of the wave function) the estimated resonance diverges dramatically from its actual value. On the other hand, the estimated resonant frequency by the Matrix Pencil method converges to is actual value even for the least possible number of time steps (only some more than a full period) while its performance seems to be independent of the increase of the number of time steps used. This is a conclusion of great importance as the use of Matrix Pencil method can result in the reduction of the number of time steps needed in order to perform an accurate frequency analysis.

Furthermore, the behavior of these methods with respect to the dispersive characteristics of FDTD is examined. It is known that the computational domain is effectively anisotropic and therefore is characterized by a specific numerical dispersion relation. The two different methods are used in order to define the limit at which the nonphysical dispersion of FDTD does not critically affect the simulated structure. This is achieved by examining the convergence of the resonant frequency of the structure in respect to the number of Yee cells (Nx,Ny) used to discretize the unit cell of Fig. 1. The results of this study are shown in Fig. 4.

The results show that both methods converge to the actual resonance frequency as the number of Yee cells per unit cell is increased. What is really important, though, is that the results of

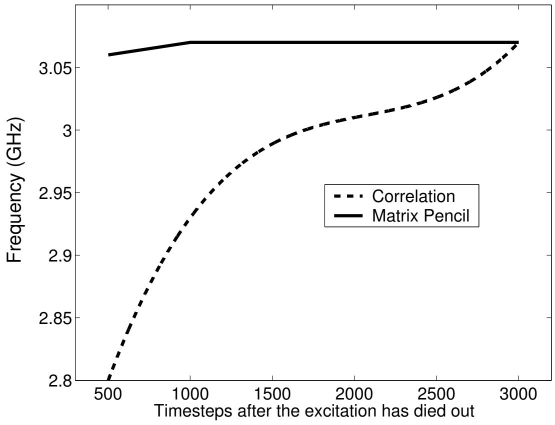


Figure 3: Convergence of Matrix Pencil and Correlation in terms of the number of time steps used.

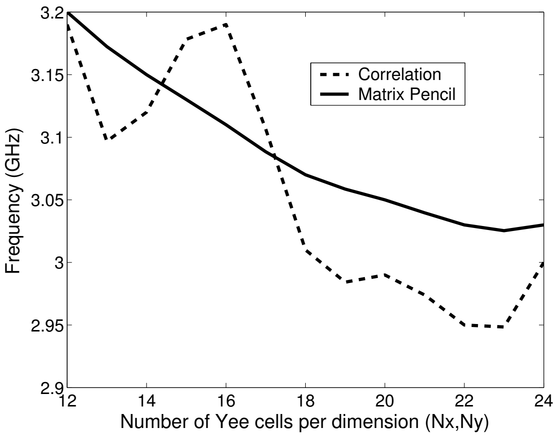


Figure 4: Convergence of Matrix Pencil and Correlation in terms of spatial griding.

Matrix Pencil method are converging smoothly to that actual resonance. That is not the case for the correlation method as it seems that other sources of error (such as the time resolution $T_s$ or the number of times steps $N$) are of greater importance to its performance. Therefore, the correlation method can not be used as criterion for the elimination of the numerical dispersion.

## 5 Conclusion

Three spectral analysis methods, namely the correlation method, the MUSIC algorithm and the Matrix Pencil, were applied for the purpose of accelerating the FDTD-based dispersion analysis of periodic metamaterial structures. The superiority of the Matrix Pencil was confirmed by a number of numerical experiments. This method allows the extraction of the frequencies corresponding to a wavevector in the Brillouin zone of the metamaterial structure, within seconds, since it converges rapidly with the number of FDTD time steps.

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