Improving Amplify-and-Forward Relay Networks: Optimal Power Allocation versus Selection

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Abstract— We consider an Amplify-and-Forward (AF) cooperative diversity system where a source node communicates with a destination node with the help of multiple relay nodes. The conventional system assumes all relay nodes participate, with the available channel and power resources equally distributed over all nodes. This approach being clearly sub-optimal, we first present an optimal power allocation scheme to maximize the system throughput for an AF system. The main contribution in this paper is a new selection scheme where only one, the "best" relay node is chosen to participate in the transmission. We show that at reasonable power levels the selection AF scheme maintains full diversity order, and has significantly better outage behavior and average throughput than the conventional scheme or that with optimal power allocation.

I. INTRODUCTION

Prior work has shown that a cooperative diversity gain is available in distributed wireless networks with nodes that help each other by relaying transmissions [1]–[3]. The system under consideration comprises $m+2$ wireless nodes, one of which is a source and one the destination. The most popular cooperation protocols remain amplify-and-forward (AF – the relay simply amplifies the source transmission and retransmits) and decode-and-forward (DF – the relay decodes the source transmission, re-encodes and retransmits). AF was studied in [1], where, given the m potential relays, the available channel resources are split into $m + 1$ (relays + source) orthogonal (non-interfering) transmissions, e.g., time slots or frequency bins. All m relays then help the source, achieving order-$(m + 1)$ diversity. The average throughput of such an "all participate" AF (AP-AF) network is upper-bounded by the case of perfect knowledge of all channel gains, and using that information for optimal power allocation (OPA).

OPA in AF networks has been studied recently in [4]–[8]. Most of these (e.g., [4]–[6]) focus on the single-relay case, and solve for the optimal power division between the source and relay nodes. OPA in multi-hop systems was discussed in [6], [7], where the relay nodes are used to extend the coverage area, not for diversity. Employing multiple relay nodes with distributed beamforming for diversity gain was studied in [8].

This paper first revisits Laneman's framework in [1], deriving the OPA algorithm for an AP-AF network with multiple relay nodes, to maximize throughput. We present the OPA scheme as an extended water-filling process under both total and individual power constraints. However, we also realized that the performance of AP-AF is limited by the orthogonal partition of the system resources, especially when the number of relay nodes are large, even with OPA.

To solve this problem, a new cooperation structure is introduced in the second half of the paper. The new protocol is called selection AF (S-AF), in which only one "best" node is chosen as a relay. The selection algorithm is implemented at the destination, which is assumed to have knowledge of all channel gains, including those between the source and all relays.

As we will show, both S-AF and AP-AF achieve the maximum diversity order of $m+1$. But more importantly, we show that S-AF achieves a higher throughput than AP-AF whenever $m > 2$, where m is the number of relay nodes, except when SNR is unrealistically low. These analytical results can be justified intuitively: (a) in a *distributed network* instead of using only $1/(m+1)$ of the channel resources for transmitting information, S-AF uses $1/2$ of the resources and so achieves a higher throughput, and (b) since S-AF chooses the best of m relays, the relays still provide a diversity order of m.

The rest of the article is organized as follows. Section II-A introduces the system model for our discussion. Section II-B sets up the OPA problem for the AP-AF scheme and finds a closed-form solution for a special case of practical interest. Section III presents the S-AF scheme and compares it with AP-AF for both throughput and outage probability. Finally, Section IV concludes the paper.

II. ALL-PARTICIPATE AMPLIFY-AND-FORWARD

In this section, we consider a system in which a source node 's' transmits information to a destination node 'd' with the help of m relay nodes. Transmissions are orthogonal, either through time or frequency division. For convenience, we assume time division and so each node is assigned one of $m + 1$ time slots for each information packet.

A. System Model

In the first, data-sharing, time slot, the source node transmits to the destination as well as the relay nodes. In this phase, the

signals received at the destination and the relays are

$$y_{s,d} = \sqrt{E_s}h_{s,d}x + n_{s,d}, \tag{1}$$

$$y_{s,i} = \sqrt{E_s}h_{s,i}x + n_{s,i}, \quad i = 1, \cdots, m, \tag{2}$$

where x, $y_{s,d}$ and $y_{s,i}$ denote the (unit energy) transmitted signal and the signals received at the destination and the ith relay node, respectively. $h_{s,i}$ and $h_{s,d}$ are channel coefficients of the source-relay and source-destination channels, which include the effect of shadowing, channel loss and fading. E_s is the average energy transmitted in this time slot. Assuming all the time slots have unit duration, E_s can be seen as the transmission power. $n_{s,d}$ and $n_{s,i}$ are additive white Gaussian noise (AWGN) in the corresponding channels, modelled as having the same variance N_0, i.e., $n_{s,d}, n_{s,i} \sim \mathcal{CN}(0, N_0)$.

In subsequent time slots, the m relay nodes normalize their received signals and retransmit them to the destination in m time slots. For the ith relay, the normalization factor is $\sqrt{\mathrm{E}\{|y_{s,i}|^2\}}$ (where $\mathrm{E}\{\cdot\}$ denotes the expectation operator) and thus the signal transmitted from the ith relay is

$$x_i = \frac{y_{s,i}}{\sqrt{\mathrm{E}\{|y_{s,i}|^2\}}} = \frac{\sqrt{E_s}h_{s,i}x + n_{s,i}}{\sqrt{E_s|h_{s,i}|^2 + N_0}}. \tag{3}$$

Note that we assume that $|h_{s,i}|$ is known at the ith relay and that $E|x|^2 = 1$.

Based on (3), the signal received by the destination from the ith relay node is

$$\begin{aligned} y_{i,d} &= \sqrt{E_i}h_{i,d}x_i + n_{i,d} \\ &= \sqrt{\frac{E_sE_i}{E_s|h_{s,i}|^2 + N_0}}h_{i,d}h_{s,i}x + \tilde{n}_{i,d}, \end{aligned} \tag{4}$$

where $h_{i,d}$ is the channel gain from node i to the destination, and E_i is the power used by node i for transmission in its time slot. $n_{i,d} \sim \mathcal{CN}(0, N_0)$ denotes the AWGN of the relay-destination channel. $\tilde{n}_{i,d}$ is the equivalent noise term in $y_{i,d}$. It can be easily shown that $\tilde{n}_{i,d} \sim \mathcal{CN}(0, \omega_i^2 N_0)$ with

$$\omega_i^2 = 1 + \frac{E_i|h_{i,d}|^2}{E_s|h_{s,i}|^2 + N_0}. \tag{5}$$

The energies available at the source and relay nodes are constrained by a total energy and a per-node energy constraint,

$$E_s + \sum_{i=1}^{m} E_i = E_T, \quad E_s \leq E_s^{max}, \quad E_i \leq E_i^{max}. \tag{6}$$

B. Optimal Power Allocation

AP-AF assumes that complete channel state information (CSI) i.e. $h_{s,d}$, $h_{s,i}$ and $h_{i,d}$, is available at the destination node, so the destination can use this information to decode the signal as well as assign transmit powers to the relay nodes. The manner in which the destination obtains the CSI is beyond the scope of this paper.

We write the received signals from all the time slots in a block in vector form as [9]:

$$\mathbf{y}_d = \mathbf{h}x + \mathbf{n}, \tag{7}$$

where

$$\mathbf{y}_d = [y_{s,d} \ y_{1,d}/\omega_1 \ \cdots \ y_{i,d}/\omega_i \ \cdots \ y_{m,d}/\omega_m]^T \tag{8}$$

$$\mathbf{h} = \left[\sqrt{E_s}h_{s,d} \ \frac{1}{\omega_1}\sqrt{\frac{E_sE_1}{E_s|h_{s,1}|^2 + N_0}}h_{1,d}h_{s,1} \right.$$
$$\left. \cdots \ \frac{1}{\omega_m}\sqrt{\frac{E_sE_m}{E_s|h_{s,m}|^2 + N_0}}h_{m,d}h_{s,m} \right]^T \tag{9}$$

with $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ and ω_i defined in (5). Note that normalizing the received signal $y_{i,d}$ with ω_i does not change the signal-to-noise ratio (SNR), but the normalized noise covariance matrix simplifies the computations needed later. Denote $|h_{s,d}|^2 = \alpha_0$, $|h_{s,i}|^2 = \alpha_i$ and $|h_{i,d}|^2 = \beta_i$. Then the source-destination channel capacity for a given $\mathbf{h}$ is

$$\begin{aligned} \mathcal{I}_{AP} &= \frac{1}{m+1}\log_2\left(1 + \mathbf{h}^H\mathbf{h}/N_0\right) \\ &= \frac{1}{m+1}\log_2\left[1 + \frac{1}{N_0}\left(E_s\alpha_0 + \sum_{i=1}^{m}\frac{E_sE_i\alpha_i\beta_i}{E_s\alpha_i + E_i\beta_i + N_0}\right)\right] \\ &= \frac{1}{m+1}\log_2\left[1 + \frac{1}{N_0}\left(E_s\sum_{i=0}^{m}\alpha_i \right.\right. \\ &\qquad\qquad \left.\left. - \sum_{i=1}^{m}\frac{E_s^2\alpha_i^2 + N_0E_s\alpha_i}{E_s\alpha_i + E_i\beta_i + N_0}\right)\right] \end{aligned} \tag{10}$$

in bits per time slot.

We can model our goal of allocating power among source and relay nodes to maximize $\mathcal{I}_{AP}$ as an optimization problem. Since $\log_2(1 + x)$ is a strictly increasing function of x, based on (6) and (10), we have:

$$\begin{aligned} [E_s \ E_1 \ \cdots \ E_m]_{opt} &= \arg \max_{\substack{E_s + \sum_{i=1}^{m}E_i = E_T \\ 0 \leq E_s \leq E_s^{max}, \ 0 \leq E_i \leq E_i^{max}}} \\ &E_s\left(\sum_{i=0}^{m}\alpha_i\right) - \sum_{i=1}^{m}\frac{E_s^2\alpha_i^2 + N_0E_s\alpha_i}{E_s\alpha_i + E_i\beta_i + N_0}. \end{aligned} \tag{11}$$

Solving optimization problem (11) in closed form appears to be difficult. But if we relax the problem to one with a fixed pre-determined E_s, then the new problem

$$[E_1 \ \cdots \ E_m]^{opt} = \arg \min_{\substack{\sum_{i=1}^{m}E_i = E_r \\ 0 \leq E_i \leq E_i^{max}}} \sum_{i=1}^{m}\frac{E_s^2\alpha_i^2 + N_0E_s\alpha_i}{E_s\alpha_i + E_i\beta_i + N_0}, \tag{12}$$

where $E_r = E_T - E_s$ is the total power constraint for the relay nodes, has a closed-form solution. The relaxed problem (12) is equivalent to having the source node transmit at some reasonable power, and then allocating the remaining power among the relay nodes. Without individual power constraints, the problem can be shown (using the Lagrange multiplier method) to have a water-filling solution [10] and the optimal allocation is

$$E_i = \left(\sqrt{\frac{E_s^2\alpha_i^2 + N_0E_s\alpha_i}{\beta_i}}\lambda - \frac{E_s\alpha_i + N_0}{\beta_i}\right)^+. \tag{13}$$
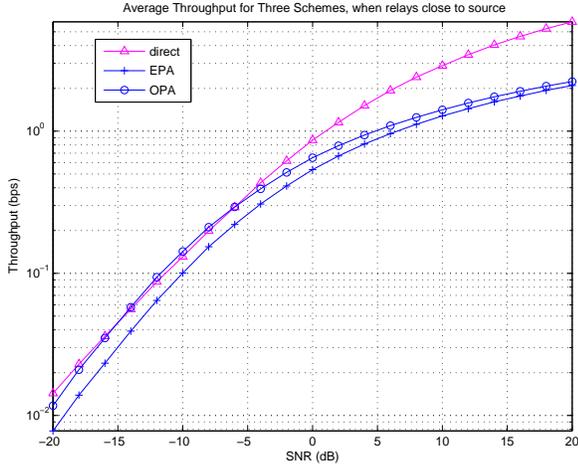
Fig. 1. Average Throughput of the Three Schemes.
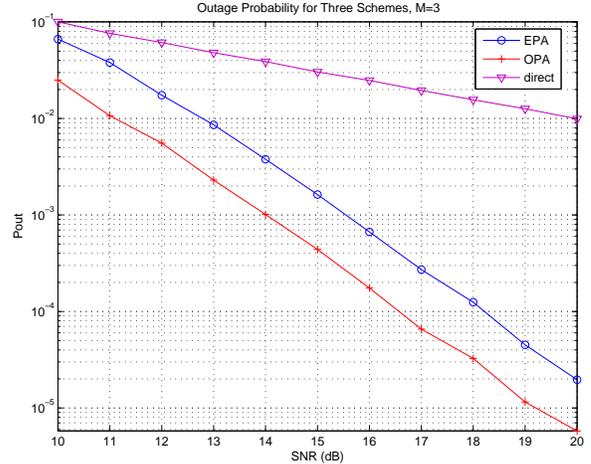
Fig. 2. Outage Probability of the Three Schemes.

Now consider the individual constraints. Since the objective function (12) is a monotonically decreasing convex function of E_i, the optimal point must be on the boundary. This can also be easily verified by the Kuhn-Tucker conditions [11]. Therefore, the solution for the power allocation problem is

$$E_i = \left(\sqrt{\frac{E_s^2 \alpha_i^2 + N_0 E_s \alpha_i}{\beta_i}} \lambda - \frac{E_s \alpha_i + N_0}{\beta_i} \right)_0^{E_i^{max}}, \quad (14)$$

where λ is a constant chosen to satisfy the total power constraint and $(x)_a^b = a\,(x < a), = x\,(a \leq x \leq b), = b\,(x > b)$. This solution can be considered as an extended water-filling process, with each vessel having both a bottom *and a lid*.

In the high SNR regime, we can simply ignore the noise term N_0, thus the optimal solution for high SNR is

$$E_i = \left(\frac{E_s \alpha_i}{\sqrt{\beta_i}} \lambda - \frac{E_s \alpha_i}{\beta_i} \right)_0^{E_i^{max}}. \quad (15)$$

C. Simulation Results

In this section we evaluate the impact of power allocation on AP-AF networks. We simulate an AP-AF network with three relay nodes ($m = 3$). Three transmission schemes are compared: OPA AP-AF is our proposed optimal scheme, equal power allocation (EPA) AP-AF is the conventional AF scheme where all relays use the same power ($E_i = E_s = E_T/(m+1)$), and Direct Transmission is when the source sends information to the destination directly without help from the relays. The power constraints are $E_s = 1$, $E_T = m$ and $E_i^{max} = 2$. We also assume Rayleigh fading channels with parameters $h_{s,d}, h_{i,d} \sim \mathcal{CN}(0,1)$, $h_{s,i} \sim \mathcal{CN}(0,10)$. This represents the case where the relays are close to the source node, thus the source-relay channels are much better than the relay-destination channels.

Figure 1 compares the average throughput of the three schemes, where results are obtained by averaging over $5,000$ channel realizations. From the figure we can see that the OPA

scheme improves the average throughput by about 2 dB at low SNR[1]. Figure 2 shows the outage probability of the three schemes. The OPA results in a gain of 1.5dB. Note that, from these figures, direct transmission has greater throughput, but far poorer outage probability (diversity order of 1, not $m+1$).

III. SELECTION AMPLIFY-AND-FORWARD SCHEME

A. Algorithm Description

In the previous section we showed that power allocation can improve system throughput for the AP-AF scheme. However, in order to realize orthogonal transmissions, every node can only transmit in a slot with length $1/(m + 1)$ of the entire block. Although this orthogonal transmission can achieve full diversity order, the TDMA factor $1/(m+1)$ in (10) has a large adverse effect on throughput when m is large.

To solve this problem, we introduce a new scheme called Selection Amplify-and-Forward (S-AF) where the transmission is divided into only two slots. The first slot implements the data-sharing phase of AP-AF. However, the relaying phase of S-AF contains only one slot, in which a relay node selected by the destination amplifies and forwards its received signal from the source. To focus on the idea of relay selection, we assume equal power allocation between the source and relay nodes.

Let the transmit SNR $\gamma = \frac{E_s}{N_0} = \frac{E_i}{N_0}$. Then the capacity of the source-destination channel when relay i is chosen for relaying is

$$\mathcal{I}_S(i) = \frac{1}{2} \log_2 \left(1 + \gamma \alpha_0 + \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} \right) \quad (16)$$

bits per time slot. The maximum capacity is therefore attained when the relay with the largest

$$P_i = \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} \quad (17)$$

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[1]SNR is defined as $E_s/N_0 = 1/N_0$.
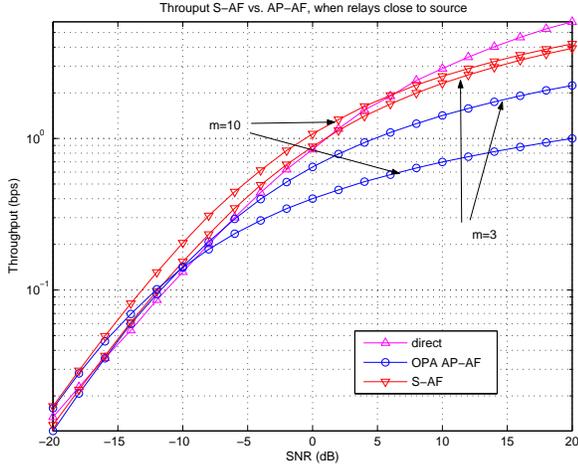
Fig. 3. Throughput S-AF vs. AP-AF, when relays close to source.

is selected, resulting in a capacity of

$$\mathcal{I}_S = \frac{1}{2}\log_2\left[1 + \gamma\alpha_0 + \max_i \frac{\gamma^2\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1}\right]. \quad (18)$$

The destination node needs only to make the selection and notify the selected relay node, instead of computing and feeding back the power allocated to every relay node, therefore the complexity of S-AF is lower than that of AP-AF. But because S-AF only repeats information once whereas AP-AF repeats m times, S-AF actually has a higher throughput.

Figure 3 plots the throughput of S-AF and OPA AP-AF schemes. The channel parameters are the same as in Fig. 1. The figure shows that S-AF achieves much larger average throughput than AP-AF. As expected, the gains of using S-AF over AP-AF increases with increasing m. Furthermore, unlike S-AF, as the number of relays increases, the throughput of AP-AF actually decreases due to the lower TDMA factor of $1/(m + 1)$ in (10).

B. Outage Analysis

In this section we compare the outage probability for the two AF schemes. We prove that both schemes can achieve full diversity order, which defines high SNR outage performance. Combined with the throughput analysis in the previous section, we can expect that S-AF has better outage probability than AP-AF in general since it has larger average throughput and the same diversity order. This is verified with simulation results.

An outage event occurs when the mutual information $\mathcal{I}$ falls under a required rate R. The outage probability is therefore $P_{out} = P[\mathcal{I} < R]$. The mutual information, $\mathcal{I}$ has the form of (10) and (18) for the AP-AF and S-AF schemes, respectively.

It is extremely difficult to compute the exact outage probabilities for the two AF schemes for arbitrary SNRs because the probability density functions (PDF) of the mutual information are hard to obtain. However, in the following two theorems, we present the high SNR approximations of the outage probabilities for the two schemes, from which the diversity orders

for both schemes can be easily obtained.

Theorem 1: At high SNR, the outage probability of the S-AF scheme can be approximated as

$$P_{out}^S = P[\mathcal{I}_S < R] \approx \frac{\lambda_0\Pi_{i=1}^m(\lambda_i + \xi_i)}{m + 1}\left(\frac{2^{2R} - 1}{\gamma}\right)^{m+1}, \quad (19)$$

with λ_0, λ_i and ξ_i the parameters of the exponential distribution of $\alpha_0 = |h_{s,d}|^2$, $\alpha_i = |h_{s,i}|^2$ and $\beta_i = |h_{i,d}|^2$ in (18), respectively.

Proof: See Appendix A.

Theorem 1 shows that full diversity order of $m + 1$ can be achieved by the S-AF scheme since P_{out}^S is proportional to $(1/\gamma)^{m+1}$. To compare this with AP-AF, next we consider the high SNR approximation of the outage probability of AP-AF. To simplify the problem, we focus on the AP-AF scheme with equal power allocation, a good approximation of the performance of OPA AP-AF at high SNR.

It is difficult to directly obtain the high SNR approximation for outage probability of EPA AP-AF scheme. However, we can find a pair of upper and lower bounds.

Theorem 2: In the high SNR regime, the outage probability of AP-AF scheme can be bounded as

$$\frac{\lambda_0\Pi_{i=1}^m(\lambda_i + \xi_i)}{(m + 1)m^m}\left(\frac{2^{(m+1)R} - 1}{\gamma}\right)^{m+1} \leq P_{out}^{AP}$$
$$\leq \frac{\lambda_0\Pi_{i=1}^m(\lambda_i + \xi_i)}{m + 1}\left(\frac{2^{(m+1)R} - 1}{\gamma}\right)^{m+1}. \quad (20)$$

Proof: See Appendix B.

Since both the upper and the lower bounds in Theorem 2 are proportional to $(1/\gamma)^{m+1}$, the AP-AF scheme must have full diversity order of $m + 1$. Finally, by comparing the high SNR outage probability of S-AF with the lower bound of AP-AF we arrive at the next corollary.

Corollary 1: $P_{out}^S < P_{out}^{AP}$ when the target rate R satisfies $R > (\log_2 m)/(m - 1)$.

Note that the condition $R > (\log_2 m)/(m - 1)$ is obtained by using the lower bound of AP-AF, and therefore is sufficient but not necessary i.e., even when the condition is not satisfied, S-AF may still have smaller outage probability than AP-AF. The threshold $(\log_2 m)/(m - 1)$ is easily reached in practice. For instance, when $m = 8$, the required target rate is only $R > 3/7$ bits/time slot. Therefore we can safely say that in practice S-AF provides better outage performance than AP-AF.

C. Simulation Results

The following figures simulate the outage probabilities for the three schemes, S-AF, AP-AF and direct transmission. In Fig. 4 we consider a network with three relay nodes with equal-gain channels i.e., $h_{s,d}, h_{s,i}, h_{i,d} \sim \mathcal{CN}(0, 1)$. The required outage is set to be $R = 1$. The figure shows that S-AF achieves a huge improvement in outage probability of about 5 dB or 2 orders of magnitude over AP-AF, while both achieve full diversity order. It also verifies that Direct Transmission has relatively poor performance at high SNR because it does not benefit from cooperative diversity.
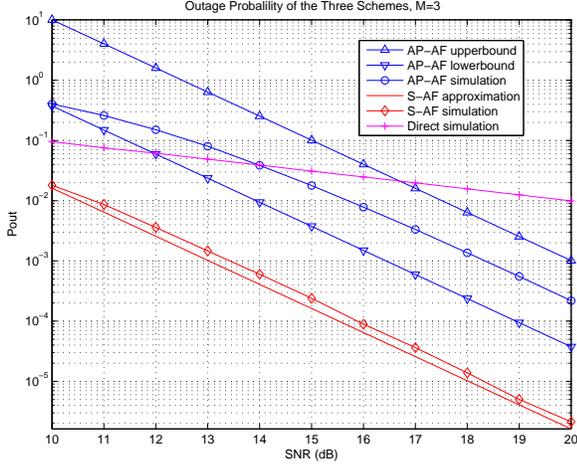
Fig. 4. Outage Probabilities for Three Schemes, M=3.
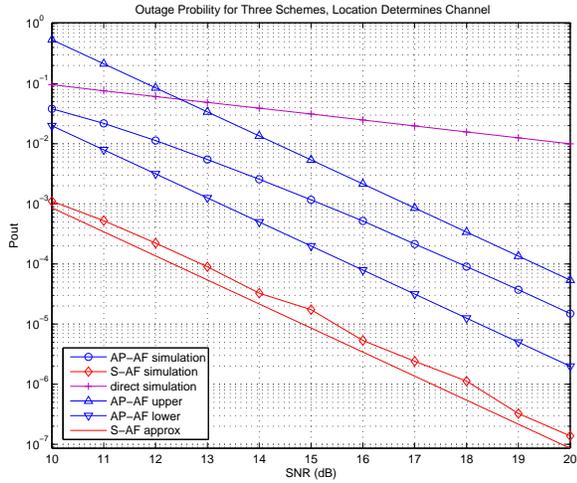
Fig. 5. Outage Probabilities for Three Schemes, Location Determines Channel, M=3.

Figure 5 shows a more practical situation where channels are determined by the locations of nodes. Consider a circle centered at the origin of the x-y plane with radius $r = 1$. The source and destination nodes are located at $(-0.5, 0)$ and $(0.5, 0)$, respectively. Four relay nodes are uniformly distributed in the circle. The channel between two nodes is $h_{i,j} \sim \mathcal{CN}(0, 1/d^\nu)$, where d is the distance between the two nodes, and $\nu = 2.5$ is the distance attenuation factor. The superiority of the S-AF scheme is again confirmed.

IV. CONCLUSION

Cooperative diversity is a powerful idea to achieve spatial diversity even when multiple antennas are unavailable at each node. Previous works have developed several schemes to realize this cooperative diversity gain, among which Amplify-and-Forward is attractive for its low complexity. The conventional AF (or AP-AF) scheme assumes that all the relay nodes participate in packet forwarding, and that the same power is used at all the nodes. In this paper we first consider optimal power allocation among the relay nodes for maximum system throughput with total and individual power constraints. We showed that the optimal power allocation can be obtained by an extended water-filling process. The main contribution of the paper is a selection scheme, called S-AF, where only one relay node is chosen to relay the source signal. We showed that S-AF maintains full diversity order while greatly increasing the throughput, and therefore also achieves better outage behavior than AP-AF.

APPENDIX

A. Proof of Theorem 1

Based on the mutual information formula (18), the outage probability of S-AF scheme can be written as

$$
\begin{aligned}
P_{out}^S &= P\left[\frac{1}{2}\log_2\left(1 + \gamma\alpha_0 + \max_i \frac{\gamma^2\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1}\right) < R\right], \\
&= P\left[\alpha_0 + \max_i \frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1} < \frac{2^{2R} - 1}{\gamma}\right], \\
&= P\left[\max_i \frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1} < \delta - \alpha_0\right], \quad (21)
\end{aligned}
$$

where $\delta = \frac{2^{2R}-1}{\gamma}$. Since α_0 is exponentially distributed variable with parameter λ_0, we have

$$
\begin{aligned}
P_{out}^S &= \int_0^\delta P\left[\max_i \frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1} < \delta - x\right]\lambda_0 e^{-\lambda_0 x}dx \\
&= \int_0^1 P\left[\max_i \frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1} < \delta x'\right]\delta\lambda_0 e^{-\lambda_0\delta(1-x')}dx' \\
&= \int_0^1 \left(\prod_{i=1}^m P\left[\frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i + \gamma\beta_i + 1} < \delta x'\right]\right)\delta\lambda_0 e^{-\lambda_0\delta(1-x')}dx' \\
&= \delta^{m+1}\lambda_0 \int_0^1 \left(\prod_{i=1}^m \frac{P\left[\frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i+\gamma\beta_i+1} < \delta x'\right]}{\delta x'}\right)(x')^m e^{-\lambda_0\delta(1-x')}dx',
\end{aligned}
$$
(22)

using $(x' = 1 - x/\delta)$ Note that δ is a function of transmit SNR γ, and $\delta \to 0$ when $\gamma \to \infty$. Thus

$$
\lim_{\gamma\to\infty} e^{-\lambda_0\delta(1-x')} = 1. \quad (23)
$$

It has been proved in [2] that

$$
\lim_{\gamma\to\infty} \frac{P\left[\frac{\gamma\alpha_i\beta_i}{\gamma\alpha_i+\gamma\beta_i+1} < \delta x'\right]}{\delta x'} = \lambda_i + \xi_i, \quad (24)
$$

Substitute (23) and (24) into (22) we have

$$
\begin{aligned}
\lim_{\gamma\to\infty} \frac{P_{out}^S}{\delta^{m+1}} &= \lambda_0 \int_0^1 \left(\prod_{i=1}^m (\lambda_i + \xi_i)\right)(x')^m dx' \\
&= \frac{\lambda_0\Pi_{i=1}^m(\lambda_i + \xi_i)}{m + 1} \quad (25)
\end{aligned}
$$

Therefore, at high SNR, P_{out}^S can be approximated as

$$P_{out}^S \approx \frac{\lambda_0 \Pi_{i=1}^m (\lambda_i + \xi_i)}{m+1} \delta^{m+1}. \tag{26}$$

Theorem 1 is proved. ∎

B. Proof of Theorem 2

The mutual information formula of the AP-AF with equal power allocation is

$$\mathcal{I}_{EAP} = \frac{1}{m+1} \log_2 \left[1 + \gamma \alpha_0 + \sum_{i=1}^m \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} \right]. \tag{27}$$

Thus the outage probability can be written as

$$\begin{aligned} P_{out}^{EAP} &= P[\mathcal{I}_{EAP} < R] \\ &= P\left[\alpha_0 + \sum_{i=1}^m \frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{2^{(m+1)R} - 1}{\gamma} \right] \end{aligned} \tag{28}$$

Since

$$\sum_{i=1}^m \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} > \max_i \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1}, \tag{29}$$

an upper bound for P_{out}^{EAP} can be introduced as

$$\begin{aligned} P_{out}^{EAP} &< P\left[\alpha_0 + \max_i \frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{2^{(m+1)R} - 1}{\gamma} \right] \\ &\overset{highSNR}{\approx} \frac{\lambda_0 \Pi_{i=1}^m (\lambda_i + \xi_i)}{m+1} \left(\frac{2^{(m+1)R} - 1}{\gamma} \right)^{m+1}, \end{aligned} \tag{30}$$

where the high SNR approximation can be derived using (26) in Appendix A, but with a new $\delta = \left(2^{(m+1)R} - 1 \right)/\gamma$.

For the lower bound, we use the inequality

$$\sum_{i=1}^m \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < m \max_i \frac{\gamma^2 \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1}, \tag{31}$$

therefore

$$\begin{aligned} P_{out}^{EAP} &> P\left[\alpha_0 + m \max_i \frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{2^{(m+1)R} - 1}{\gamma} \right] \\ &= P\left[\max_i \frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{\frac{2^{(m+1)R} - 1}{\gamma} - \alpha_0}{m} \right] \\ &= \int_0^\delta P\left[\max_i \frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{\frac{2^{(m+1)R} - 1}{\gamma} - x}{m} \right] \lambda_0 e^{-\lambda_0 x} dx \\ &= \left(\frac{2^{(m+1)R} - 1}{\gamma} \right)^{m+1} \frac{\lambda_0}{m^m} \int_0^1 \left(\prod_{i=1}^m \frac{P\left[\frac{\gamma \alpha_i \beta_i}{\gamma \alpha_i + \gamma \beta_i + 1} < \frac{2^{(m+1)R} - 1}{m\gamma} x' \right]}{\frac{2^{(m+1)R} - 1}{m\gamma} x'} \right) (x')^m e^{-\lambda_0 \frac{2^{(m+1)R} - 1}{\gamma}(1-x')} dx' \\ &\overset{highSNR}{\approx} \frac{\lambda_0 \Pi_{i=1}^m (\lambda_i + \xi_i)}{(m+1)m^m} \left(\frac{2^{(m+1)R} - 1}{\gamma} \right)^{m+1}, \end{aligned} \tag{32}$$

where the high SNR approximation can be easily derived using (23) and (24) in Appendix A.

Theorem 2 is therefore proved. ∎.

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