OPTIMAL STBC PRECODING WITH CHANNEL COVARIANCE FEEDBACK FOR MINIMUM ERROR PROBABILITY

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Abstract

This paper develops the optimal linear transformation (or precoding) of orthogonal space-time block codes (STBC) for minimizing probability of decoding error, when the channel covariance matrix is available at the transmitter. We build on recent work that stated the performance criterion without solving for the transformation. In this paper, we provide a waterfilling solution for multi-input single-output (MISO) systems, and present a numerical solution for multi-input multi-output (MIMO) systems. Our results confirm that eigen-beamforming is optimal at low SNR or highly correlated channels, and full diversity is optimal at high SNR or weakly correlated channels, in terms of error probability. This conclusion is similar to one reached recently from the capacity-achieving viewpoint.

Keywords: MIMO, Space-Time Block Coding, beamforming, Linear Precoding.

1 Introduction

In wireless communications, the adverse effects of channel fading can be mitigated by transmission over a diversity of independent channels. A large, and growing, body of results have firmly established the potential of space-time coding [1–3] in multi-input multi-output (MIMO) systems, which use antenna arrays at the transmitter and the receiver to provide spatial diversity at both ends of a communications link.

In [3], Tarokh et. al. introduced the well-known rank and determinant criteria for the design of space-time codes without channel knowledge at the transmitter. Furthermore, it was argued [2, Sect. II-C] that these criteria apply to both spatially independent and dependent fading channels. In other words, without channel state information (CSI) at the transmitter, space-time codes should be designed using the rank and determinant criteria, even when the spatial channels are correlated. This result was confirmed by El Gamal [4, Prop. 7], who proved that with spatial correlation and quasi-static flat fading, full-diversity space-time codes such as orthogonal Space-Time Block Codes (O-STBC) extract the maximum diversity gain achievable, without CSI at the transmitter.

While spatial correlation does not affect diversity gain, Shiu and Foschini showed that correlation between spatial channels leads to a loss in capacity [5]. It is also known that spatial correlation results in a smaller coding advantage [2, Sec. II-C]. This paper explores practical approaches to recover this performance loss. However, given that nothing can improve the performance of current state-of-the-art full-diversity space-time codes without CSI at the transmitter, it is natural to consider performance improvements when this assumption is relaxed.

In this paper, we study the design of a linear precoder for O-STBC in spatially correlated, quasti-static, flat fading channels with knowledge of the channel covariance at the transmitter. The objective is to minimize the probability of decoding error. The channel covariance information may be fed back from the receiver. Such a system may be considered more practical than the case when true CSI is available at the transmitter, because in that case the feed back channel may be too heavy an overhead on the communication system. Prior work done on this topic developed the optimality criterion [6] to be satisfied by the precoding matrix, but no closed-form or numerical solution was provided. In this paper, a numerical solution is provided for MIMO systems with an arbitrary number of transmit and receive antennae. Furthermore, we derive an exact waterfilling solution for MISO systems. Assuming uncorrelated fading at the receiver as in [7], this solution is shown to be exact in MIMO systems as well.

This problem setting ties in with recent work on determining the capacity-achieving signal correlation matrix when the channel covariance matrix is available at the transmitter [7–9]. In contrast, our research is focused on minimizing the error probability, given a linear precoding structure based on orthogonal STBC. Because of the orthogonal structure of the code matrices used, this transmitter has complexity only linear in the number of transmit antennas despite use of a maximum likelihood receiver [10].

The rest of the paper is organized as follows: Section 2 presents the background material needed in the rest of the paper. Section 3 discusses the optimal precoding under various scenarios while Section 4 introduces three simplified strategies that are shown to result in minimal performance loss. Simulation examples are presented in Section 5. Finally, Section 6 presents conclusions.

2 Background

Consider a MIMO system with M transmit and N receive antennae. Orthogonal STBC is used, and a linear transformation unit is applied prior to transmission to take account of the channel covariance information. The transformation matrix $\mathbf{W} \in \mathbb{C}^{M\times M}$ is to be determined to minimize the maximum pairwise error probability (PEP) between codewords in correlated fading. A maximum-likelihood (ML) receiver is used. Illustrations of the transmitter and receiver for such a system are shown in Fig. 1.

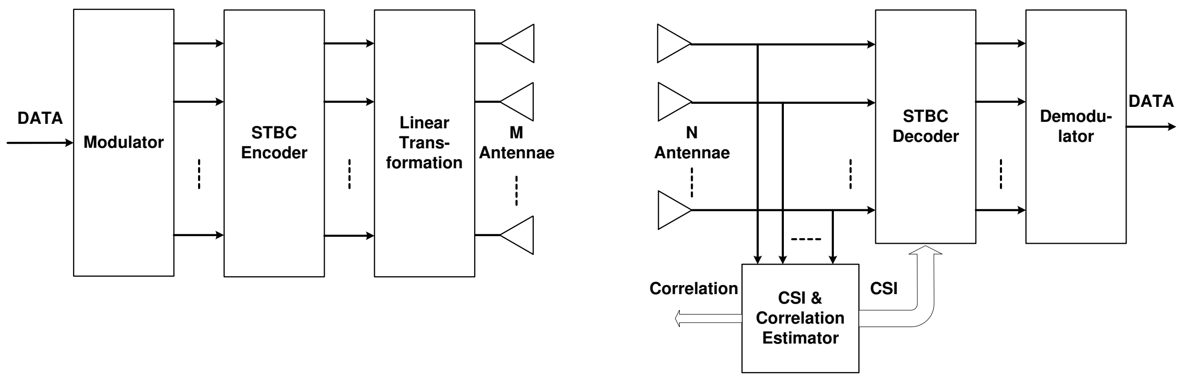

Figure 1: Precoded STBC transmitter and receiver block diagrams.

The MIMO channel between the transmitter and the receiver, assumed flat and Rayleigh, is described by the $N \times M$ matrix

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1M} \\ h_{21} & h_{22} & \dots & h_{1M} \\ \vdots & \vdots & & \vdots \\ h_{N1} & h_{N2} & \dots & h_{NM} \end{bmatrix}, \tag{1}$$

where the element h_{nm} is the fading coefficient between the mth transmit antenna and the nth receive antenna. The channel correlation matrix is

$$\mathbf{R} = E[\mathbf{h}\mathbf{h}^\dagger], \tag{2}$$

$$\mathbf{h} = vec(\mathbf{H}), \tag{3}$$

where $(\cdot)^\dagger$ denotes Hermitian transpose, and $vec(\cdot)$ denotes the vectorization operator which stacks the columns of $\mathbf{H}$. Note that this definition is identical to the one in [3].

The STBC encoder organizes data into an $M \times L$ matrix $\mathbf{C}$ and successive columns of this matrix are transmitted over L time indices. The corresponding $N \times L$ received signal matrix $\mathbf{X}$ can be written as

$$\mathbf{X} = \mathbf{HWC} + \mathbf{E}, \tag{4}$$

where $\mathbf{E}$ is an $N \times L$ matrix with i.i.d. complex Gaussian elements representing additive thermal noise. The receiver employs an ML decoder, thus the decoded codeword $\widehat{\mathbf{C}}$ can be expressed as

$$\widehat{\mathbf{C}} = \arg\min_{\mathbf{C}} \|\mathbf{X} - \mathbf{HWC}\|_F^2, \tag{5}$$

where $\|\cdot\|_F$ is the Frobenius norm [11]. Note that, because $\mathbf{HW}$ is equivalent to a modified channel matrix $\tilde{\mathbf{H}}$, maximum likelihood decoding of $\mathbf{C}$ requires only the simple linear operation described in [10].

It is known that the exact probability of error is hard to compute, so in much of the literature (see e.g. [6]) we work with the maximum PEP, which is the dominating term of the probability of error, and try to minimize a bound on it. This approach was taken in [6] and the result is that the tight upper bound on the Gaussian tail for the maximum PEP is minimized by a transformation matrix $\mathbf{W}$ that satisfies

$$\mathbf{Z}_{opt} = \mathbf{W}_{opt}\mathbf{W}_{opt}^{\dagger} = \arg \max_{\substack{\mathbf{Z} \\ \mathbf{Z}\succeq 0, tr(\mathbf{Z})=M}} \det\left[(\mathbf{I}_N \otimes \mathbf{Z})\eta + \mathbf{R}^{-1}\right], \tag{6}$$

where $\mathbf{Z}$ has to be positive semi-definite because $\mathbf{Z} = \mathbf{W}\mathbf{W}^{\dagger}$, and the trace constraint is necessary to avoid power amplification. $\otimes$ denotes the Kronecker product, while $\eta = \mu_{\min}/4\sigma^2$ with

$$\mu_{\min} = \arg\min_{\mu_{kl}} \left\{\mu_{kl}\mathbf{I} = (\mathbf{C}_k - \mathbf{C}_l)(\mathbf{C}_k - \mathbf{C}_l)^{\dagger}\right\}, \tag{7}$$

among all possible codewords. In this paper, we follow this approach as well and solve the optimization problem defined in (6).

3 Optimal Transformation

3.1 General Solution

To solve the optimization problem (6), we begin by introducing a reasonable assumption of the channel correlation: the correlation between two subchannels is equal to the product of the correlation at the transmitter and that at the receiver [12]. In matrix form, letting $\mathbf{R}_T = \frac{1}{M}E\{\mathbf{H}^H\mathbf{H}\}$ denote the correlation between different transmit antennae, and $\mathbf{R}_R = \frac{1}{N}E\{\mathbf{H}\mathbf{H}^H\}$ the correlation between receive antennae, the channel correlation is

$$\mathbf{R} = \mathbf{R}_R \otimes \mathbf{R}_T. \tag{8}$$

It has been shown that the validity of this assumption is supported by measurement results for mobile links [12]. With this assumption, the optimal $\mathbf{Z}$ matrix is

$$\mathbf{Z}_{opt} = \arg \max_{\substack{\mathbf{Z} \\ \mathbf{Z}=\mathbf{Z}^*\succeq 0, tr(\mathbf{Z})=M}} \det[(\mathbf{I}_N \otimes \mathbf{Z})\eta + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}], \tag{9}$$

since $\mathbf{R}^{-1} = \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}$ [13].

The problem is to choose a positive semi-definite matrix $\mathbf{Z}$ to maximize $\det[(\mathbf{I}_N \otimes \mathbf{Z})\eta + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}]$ subject to the trace constraint $tr(\mathbf{Z}) = M$. Notice that the correlation matrix $\mathbf{R}_R$ and $\mathbf{R}_T$ are both positive semi-definite and we can decompose them into

$$\mathbf{R}_T = \mathbf{U}_T\mathbf{\Lambda}_T\mathbf{U}_T^{\dagger}, \text{ where } \mathbf{U}_T\mathbf{U}_T^{\dagger} = \mathbf{I}_M, \tag{10}$$

$$\mathbf{R}_R = \mathbf{U}_R\mathbf{\Lambda}_R\mathbf{U}_R^{\dagger}, \text{ where } \mathbf{U}_R\mathbf{U}_R^{\dagger} = \mathbf{I}_N, \tag{11}$$

then

$$
\begin{aligned}
&\det\left[(\mathbf{I}\otimes\mathbf{Z})\eta+\mathbf{R}_R^{-1}\otimes\mathbf{R}_T^{-1}\right] \\
&= \det\left[(\mathbf{I}\otimes\mathbf{Z})\eta+(\mathbf{U}_R\boldsymbol{\Lambda}_R^{-1}\mathbf{U}_R^\dagger)\otimes(\mathbf{U}_T\boldsymbol{\Lambda}_T^{-1}\mathbf{U}_T^\dagger)\right] \\
&= \det\left[(\mathbf{I}\otimes\mathbf{Z})\eta+(\mathbf{U}_R\otimes\mathbf{U}_T)(\boldsymbol{\Lambda}_R^{-1}\otimes\boldsymbol{\Lambda}_T^{-1})(\mathbf{U}_R\otimes\mathbf{U}_T)^\dagger\right] \\
&= \det\left[(\mathbf{U}_R\otimes\mathbf{U}_T)[(\mathbf{U}_R\otimes\mathbf{U}_T)^\dagger(\mathbf{I}\otimes\mathbf{Z}\eta)(\mathbf{U}_R\otimes\mathbf{U}_T)+\boldsymbol{\Lambda}_R^{-1}\otimes\boldsymbol{\Lambda}_T^{-1}](\mathbf{U}_R\otimes\mathbf{U}_T)^\dagger\right] \\
&= \det\left[\mathbf{U}_R\otimes\mathbf{U}_T\right]\det\left[(\mathbf{U}_R^\dagger\mathbf{I}\mathbf{U}_R)\otimes(\mathbf{U}_T^\dagger\mathbf{Z}\eta\mathbf{U}_T)+\boldsymbol{\Lambda}_R^{-1}\otimes\boldsymbol{\Lambda}_T^{-1}\right]\det\left[\mathbf{U}_R^{-1}\otimes\mathbf{U}_T^{-1}\right] \\
&= \det\left[\mathbf{I}_N\otimes\mathbf{B}+\boldsymbol{\Lambda}_R^{-1}\otimes\boldsymbol{\Lambda}_T^{-1}\right]
\end{aligned}
$$

where $\mathbf{B}=\mathbf{U}_T^\dagger\mathbf{Z}\eta\mathbf{U}_T$. The intermediate steps above come from the fact that [13]

$$\left(\prod_{i=1}^N \mathbf{A}_i\right)\otimes\left(\prod_{i=1}^N \mathbf{B}_i\right)=\prod_{i=1}^N \mathbf{A}_i\otimes\mathbf{B}_i,$$

and $\det\left[\mathbf{U}_R\otimes\mathbf{U}_T\right]=1$. The trace constraint becomes

$$
\begin{aligned}
tr(\mathbf{B}) &= tr(\mathbf{U}_T^\dagger\mathbf{Z}\eta\mathbf{U}_T) \\
&= tr(\mathbf{U}_T\mathbf{U}_T^\dagger\mathbf{Z}\eta)=tr(\mathbf{Z}\eta) \\
&= \eta M,
\end{aligned}
$$

since $tr(\mathbf{AB})=tr(\mathbf{BA})$.

The problem therefore reduces to finding a positive semi-definite matrix

$$\mathbf{B}_{opt}=\arg\max_{\substack{\mathbf{B}\\ \mathbf{B}\succeq 0, tr(\mathbf{B})=\eta M}}\det[(\mathbf{I}_N\otimes\mathbf{B})+\boldsymbol{\Lambda}_R^{-1}\otimes\boldsymbol{\Lambda}_T^{-1}]. \tag{12}$$

Since $\boldsymbol{\Lambda}_T^{-1}$ and $\boldsymbol{\Lambda}_R^{-1}$ are both diagonal, $\mathbf{B}$ must be also diagonal [14]. Let the ith diagonal element of $\boldsymbol{\Lambda}_T$ and $\mathbf{B}$, and the jth diagonal elements of $\boldsymbol{\Lambda}_R$ be λ_{ti}, b_i and λ_{rj}, respectively. The problem (9) becomes finding a set of non-negative b_i's to maximize

$$\prod_{i=1}^M\prod_{j=1}^N(b_i+\lambda_{rj}^{-1}\lambda_{ti}^{-1}) \tag{13}$$

under the trace constraint $tr(\mathbf{B})=\sum_i b_i=\eta M$. This problem is an extension of the waterfilling problem to two parameters (i and j), so we can view it as a generalized waterfilling problem. The closed form solution to this problem is unknown. However, we can find the solution by numerical methods such as Sequential Quadratic Programming (SQP) [15]. Results of the numerical scheme are provided in Section 5.2.

Since $\mathbf{Z}\eta=\mathbf{U}_T\mathbf{B}\mathbf{U}_T^\dagger$, the diagonal matrix $\mathbf{B}$ is actually the eigenvalue matrix of $\mathbf{Z}\eta$. Thus $\mathbf{Z}$ and $\mathbf{W}$ can be derived from $\mathbf{B}$ as follows:

$$
\begin{aligned}
\mathbf{Z} &= (1/\eta)\mathbf{U}_T\mathbf{B}\mathbf{U}_T^\dagger, && (14)\\
\mathbf{W} &= (1/\sqrt{\eta})\mathbf{U}_T\sqrt{\mathbf{B}}\boldsymbol{\Phi}, && (15)
\end{aligned}
$$

where $\mathbf{\Phi}$ can be any $M \times M$ unitary matrix, so $\mathbf{W}_{opt}$ is not unique. For simplicity we choose the identity matrix in this paper, i.e. $\mathbf{\Phi} = \mathbf{I}_M$.

3.2 Waterfilling Solution for MISO systems

We now consider the special case of a multi-input single-output (MISO) system, i.e. a system with only a single receive antenna (N=1). This is a reasonable model for the downlink of mobile communication systems since it may be impractical to employ more than one antenna at the mobile terminal. Under this assumption the Kronecker product in (12) disappears and we need to solve

$$\mathbf{B}_{opt} = \arg \max_{\substack{\mathbf{B}\succeq 0 \\ \mathrm{tr}(\mathbf{B})=\eta M}} \det\left[\mathbf{B} + \mathbf{\Lambda}_T^{-1}\right], \tag{16}$$

where $\mathbf{B}$ is still a positive semi-definite diagonal matrix. This is identical to the water-filling problem in information theory [14], which has the solution

$$b_i = \max(\nu - \lambda_{ti}^{-1},\ 0), \quad \text{for } i = 1, \dots, M, \tag{17}$$

where ν is a constant chosen to satisfy the trace constraint and $\mathbf{B} = \mathrm{diag}(b_1, \dots, b_M)$. The optimal transformation matrix is

$$\mathbf{W}_{opt} = \frac{1}{\sqrt{\eta}} \mathbf{U}_T \sqrt{\mathbf{B}}_{opt}. \tag{18}$$

With $\mathbf{W}_{opt}$ given by (18), the transmitted signal is

$$\begin{aligned} \mathbf{Wx} &= (\mathbf{u}_{t1}, \cdots, \mathbf{u}_{tM}) \begin{bmatrix} \sqrt{\frac{b_1}{\eta}} & & \\ & \ddots & \\ & & \sqrt{\frac{b_M}{\eta}} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} \\ &= \sum_{i=1}^{M} \mathbf{u}_{ti} \sqrt{\frac{b_i}{\eta}} x_i, \end{aligned} \tag{19}$$

and thus occupies the subspace spanned by the subset of eigenvectors of $\mathbf{R}_T$ corresponding to non-zero b_i in (17). Notice that

$$rank(\mathbf{WC}_k - \mathbf{WC}_l) = rank[\mathbf{U}_T \mathbf{B}(\mathbf{C}_k - \mathbf{C}_l)] = rank(\mathbf{B}), \tag{20}$$

since both $\mathbf{U}_T$ and $(\mathbf{C}_k - \mathbf{C}_l)$ are full rank. Therefore, the dimension of this subspace is equal to the transmit diversity order, as defined in [2].

In the case of very high correlation, only one b_i – the one corresponding to the principal eigenvector – is non-zero, and we have eigen-beamforming. On the other hand, all the eigenvectors are used when the correlation is low, and we have full diversity. In the uncorrelated channel where $\mathbf{R} = \mathbf{I}$, it can be easily shown that $\mathbf{W} = \mathbf{I}$, meaning that O-STBC is already optimal, as expected.

In between beamforming and full diversity, the waterfilling scheme determines the number of active eigen-channels, and distributes the power over them with more power devoted to the stronger ones. In this transition region, the optimal scheme may be considered to have a partial diversity order. In all cases, the diversity order is equal to the number of non-zero b_i's.

3.3 Relation to Capacity Analyses

There has been much interest in the information theory community in MIMO channels with covariance feedback [7–9]. In those works the goal is to find the input covariance matrix $\mathbf{S}_{x,opt}$ necessary to achieve ergodic channel capacity, while in contrast our goal is to find the optimal linear transformation to achieve minimum error probability. Interestingly, the conclusions reached are strikingly similar for both approaches, and warrant some comment.

1. *Transmitting over the eigenvectors of the transmit correlation matrix is optimal* assuming only the channel correlation is available at the transmitter. The two schemes both result in allocating transmission power over the eigenvectors of the transmit correlation matrix. The strategy is similar: the stronger eigen-channel gets more power. However the exact amount allocated to each eigen-channel may differ for the two schemes since different optimization criteria are applied.

2. *Beamforming is optimal at high correlation/low SNR.* When the channels are highly correlated, both minimizing error probability and maximizing capacity require transmission over the strongest eigen-channel only. This statement is also true for the low SNR region where the errors are caused mainly by Gaussian noise. Thus focusing all the energy into one particular direction results in maximizing the received SNR. Diversity is not helpful as it is noise, and not fading, that limits performance.

3. *Optimal diversity order increases with SNR.* At low SNR, only the strongest eigen-channel is used. As the SNR increases, more eigen-channels come into use, so the diversity order increases until full diversity order is achieved. However, the SNR points where the diversity order changes may not be the same for the two schemes.

4. *Full diversity is optimal in uncorrelated channels.* For the extreme case of an uncorrelated channel, no transformation of STBC is required to minimize error rate, while uncorrelated transmit signals maximize bit rate. Similarly, in the high SNR region, the optimal scheme should use all the eigen-channels, because in this case diversity can be taken advantage of.

Besides these similarities, the transmitter structures of the two schemes are very similar. The channel signals (STBC codewords in our scheme or randomly coded Gaussian signals in capacity-achieving scheme) are first modulated on the eigenvectors of the transmit correlation matrix. Then these vectors are transmitted with different powers, determined by the eigenvalues of the channel correlation matrix. These two steps can be implemented with a linear transformation unit. Therefore, if we replace the STBC encoder with a random encoder and Gaussian signal modulator, the linear transformation structure becomes a capacity-achieving one.

4 Simplified Schemes

From Section 3 we know that the optimal transformation scheme is not simple to determine. For the general MIMO systems, the computation of the transformation matrix involves complex numerical algorithms. Even for the simpler case of MISO systems, the waterfilling solution still requires an iterative process. In this section, we introduce several simplified schemes to reduce the complexity. Simulation results in Section 5 will show that these schemes can achieve very similar performance to the optimal one with much lower complexity.

4.1 Ignoring the Receive Correlation

Due to differences in their physical surroundings, the transmitter and receiver on the downlink of a mobile network have different correlation properties. The extended "one ring" model introduced in [5] is a well known scattering model for channel correlation. If we use this model to simulate the downlink of a mobile connection, the correlation of the fading coefficients between transmit antennae p and q and receive antenna m is

$$[\mathbf{R}_T]_{p,q} = E[h_{mp}h_{mq}^*] \approx J_0\left(\Delta\frac{2\pi}{\lambda}d_T(p,q)\right), \tag{21}$$

where Δ is the angle spread, which is defined as the ratio of the radius of the scatterer ring around the receiver and the line-of-sight distance between the transmitter and the receiver, λ is the wavelength, $d_T(p,q)$ is the distance between the two transmit antennae, and $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. The correlation between two receive antennae l and m is

$$[\mathbf{R}_R]_{l,m} = E[h_{lp}h_{mp}^*] = J_0\left(\frac{2\pi}{\lambda}d_R(l,m)\right), \tag{22}$$

where $d_R(l,m)$ is the distance between the two receive antennae.

In practice, the angle spread Δ is usually small. As a result, from (21) and (22) we see that the receive correlation is usually small compared to transmit correlation. For instance, if the distance between two transmit antennae equals $\lambda/2$ and $\Delta = 0.1$, the correlation between these two transmit antennae is $J_0(0.1\pi) = 0.97$. But the correlation between two receive antennae with the same separation is just $J_0(\pi) = -0.30$.

In dealing with receive diversity, a correlation below 0.5 is considered negligible [16]. Therefore we can simplify our algorithm by ignoring the receive correlation. Under this approximation, the rows of $\mathbf{H}$ become independent and the channel correlation matrix can be written as $\mathbf{R} = \mathbf{I}_N \otimes \mathbf{R}_T$. In this case, (12) becomes

$$\begin{aligned} \mathbf{B}_{opt} &= \arg \max_{\substack{\mathbf{B}\succeq 0 \\ \mathrm{tr}(\mathbf{B})=\eta M}} \det[\mathbf{I}_N \otimes \mathbf{B} + \mathbf{I}_N \otimes \mathbf{\Lambda}_T^{-1}] \\ &= \arg \max_{\substack{\mathbf{B}\succeq 0 \\ \mathrm{tr}(\mathbf{B})=\eta M}} \det \left[\mathbf{B} + \mathbf{\Lambda}_T^{-1}\right]^N . \end{aligned} \tag{23}$$

Therefore, the solution is exactly the same as in (17), and generalized water-filling is avoided.

4.2 Switching Between Beamforming and STBC

The waterfilling scheme in Section 3.2 changes from beamforming to full diversity as a function of SNR. In the transition region the diversity order is determined by the number of the active eigen-channels, and the optimal power allocation is determined by waterfilling. This iterative process must be recalculated for each SNR. We can introduce a simplifying scheme to avoid waterfilling altogether by switching between beamforming ($\mathbf{W}$ is rank one) and O-STBC ($\mathbf{W} = \mathbf{I}$) at a pre-computed threshold SNR level. This threshold is found by equating the error probability performance with beamforming and O-STBC. In particular, for a MISO system we want to find the η that solves the equation

$$\det[\mathbf{Z}_{beam}\eta + \mathbf{R}_T^{-1}] = \det[\eta \mathbf{I}_M + \mathbf{R}_T^{-1}], \tag{24}$$

where $\mathbf{Z}_{beam}$ is the $\mathbf{Z}$ matrix for beamforming, i.e.

$$\mathbf{Z}_{beam} = \frac{1}{\eta}\mathbf{U}_T \mathrm{diag}[M\eta, 0, \ldots, 0]\mathbf{U}_T^\dagger = M\mathbf{u}_{t1}\mathbf{u}_{t1}^\dagger \tag{25}$$

where $\mathbf{u}_{t1}$ is the eigenvector corresponding to the largest eigenvalue of $\mathbf{U}_T$. With the solution of η, the SNR threshold can be set as

$$SNR_{th} = \frac{4\eta}{\mu_{min}}. \tag{26}$$

It is self-evident that the simplified strategy incurs a greater loss in performance relative to the full-complexity scheme when the transition region between beamforming and O-STBC grows. There are however cases when the transition region is so small that no difference in performance is discernible.

One example is when the correlation between antennae is low. In this case all the eigenvalues are close to 1, so the transition region is small. Another example is when the channel correlations are equal, in which case the eigenvalues of $\mathbf{R}_T$ take on only two values so that the transition region has zero width. To show this, consider

$$\mathbf{R}_T = \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \vdots & \vdots & \ddots & \vdots \\ \rho & \rho & \dots & 1 \end{bmatrix}.$$

This matrix has only two eigenvalues: $(1+\rho)$ and $(1-\rho)$ (repeated $(M-1)$ times). As a result, the waterfilling scheme has no transition region. In the low SNR region, only the eigen-channel corresponding to eigenvalue $(1+\rho)$ is used, so we have beamforming. All the other $M-1$ channels will come into use together when the SNR exceeds the threshold level, so the performance is quite close to STBC. Therefore, the switching scheme can achieve very good performance under this correlation model.

Although the switching scheme is designed for MISO systems to simplify the waterfilling process, it can be easily extended to MIMO systems by changing (24) into

$$\det[\eta\mathbf{I}_N \otimes \mathbf{Z}_{beam} + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}] = \det[\eta\mathbf{I}_N \otimes \mathbf{I}_M + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}]. \quad (27)$$

4.3 Equal Power Allocation (EPA) Scheme

The switching scheme cannot guarantee good performance for arbitrary channel correlation since it only provides a diversity order of 1 or M whereas the optimal scheme may require partial diversity order. As an alternative to the switching scheme, we propose the Equal Power Allocation (EPA) scheme. It automatically chooses the optimal diversity order, and assigns equal power to each active eigen-channel and so numerical waterfilling is avoided.

Similar to the switching scheme, the first step of EPA is to set SNR thresholds at the points where diversity order changes. These $M-1$ thresholds can be found by solving equations similar to (24). The ith threshold is obtained by solving

$$\det[\eta\mathbf{I}_N \otimes \mathbf{Z}_i + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}] = \det[\eta\mathbf{I}_N \otimes \mathbf{Z}_{i+1} + \mathbf{R}_R^{-1} \otimes \mathbf{R}_T^{-1}], \quad (28)$$

where $\mathbf{Z}_i$ denotes the $\mathbf{Z}$ matrix corresponding to equal power allocation over the i strongest eigen-channels, or

$$\mathbf{Z}_i = \frac{M\eta}{i}\mathbf{U}_T \begin{bmatrix} \mathbf{I}_i & \mathbf{0}_{i\times(M-i)} \\ \mathbf{0}_{(M-i)\times i} & \mathbf{0}_{(M-i)\times(M-i)} \end{bmatrix} \mathbf{U}_T^{\dagger}. \quad (29)$$

The SNR axis is then divided to M regions, each corresponding to a diversity order. The transmitter can check those thresholds to determine which region the true SNR belongs in. The corresponding diversity order for transmission is used. To reduce the complexity, instead of going through the waterfilling process to compute the power distribution, the transmitter now allocates

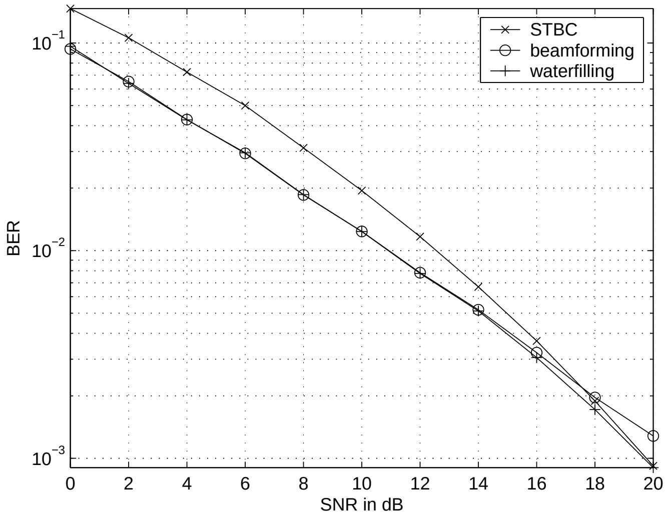

Figure 2: Waterfilling with $M = 2, N = 1$, and BPSK modulation.

power equally among all the active eigen-channels. We can expect this scheme to have better performance than the switching scheme in Section 4.2, but the complexity is also higher.

5 Simulation Results

5.1 MISO Channels

This section examines the performance of the waterfilling scheme derived in Section 3.2. Figure 2 shows the performance of the proposed algorithm, O-STBC, and eigen-beamforming when there are two transmit and one receive antennae. The modulation scheme is BPSK and the vertical axis plots the bit error probability. SNR is defined as the ratio of the transmitted bit energy to power spectral density (i.e. E_b/N_0 at the transmitter). Figure 3 is for the case of four transmit antennae.

For the two simulation examples below, the transmit correlation matrices are chosen to be

$$\mathbf{R}_{T2} = \begin{bmatrix} 1 & 0.9755 \\ 0.9755 & 1 \end{bmatrix} \tag{30}$$

and

$$\mathbf{R}_{T4} = \begin{bmatrix} 1 & 0.9755 & 0.9037 & 0.79 \\ 0.9755 & 1 & 0.9755 & 0.9037 \\ 0.9037 & 0.9755 & 1 & 0.9755 \\ 0.79 & 0.9037 & 0.9755 & 1 \end{bmatrix} \tag{31}$$

respectively. They are obtained by using (21) from the extended "one ring" model. The distance between two adjacent antennae is $\lambda/2$, and the angle spread is $\Delta = 0.1$ radians.

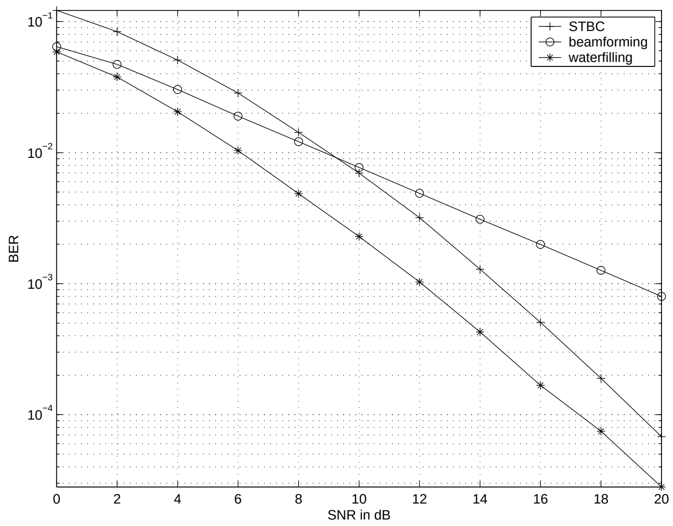

Figure 3: Waterfilling with $M = 4, N = 1$, BPSK modulation.

From the plots we can see that for very low SNR, the optimal transformation is equivalent to beamforming, as expected. For the other SNR regions, the performance of the optimal scheme is better than both beamforming and STBC. Furthermore, the optimal scheme approaches STBC as SNR increases, again as expected.

Figure 4 shows the performance of the optimal scheme with two transmitters when the channel correlation varies from 0 to 1. The SNR value is fixed at 5 dB. From this plot we can see that when the correlation coefficient is low ($\rho < 0.3$), the performance of the optimal scheme is a little better than STBC; while with high correlation ($\rho > 0.8$), the optimal scheme is the same as beamforming. In between, a relatively large performance improvement can be achieved by using the optimal scheme. This plot is remarkably similar to the corresponding plot in [17] which deals with a capacity analysis.

5.2 Numerical Solutions for MIMO Systems

As discussed in Section 3.1, the optimal transformation for MIMO system is found through a generalized waterfilling problem. No closed form solution has been found, but numerical methods, such as SQP can be used to solve (13) with a trace constraint. Here we use the MATLAB function `fmincon` to solve the problem.

Figures 5 and 6 show the performance curves obtained with the optimal transformation. In both cases the receive correlation is set to be

$$\mathbf{R}_{R2} = \begin{bmatrix} 1 & -0.3042 \\ -0.3042 & 1 \end{bmatrix}, \tag{32}$$

which is based on (22), and $\mathbf{R}_T$ is the same as in MISO cases. It is clear that the same conclusions

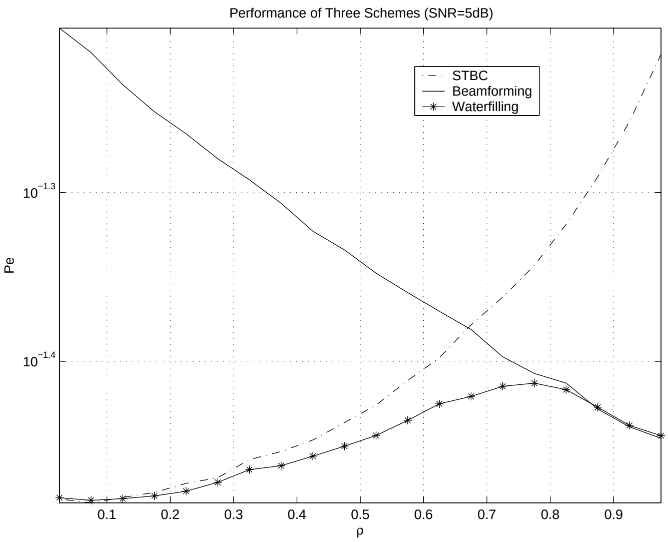

Figure 4: Bit error probability versus channel correlation ρ. $M = 2$, $N = 1$, SNR = 5 dB.

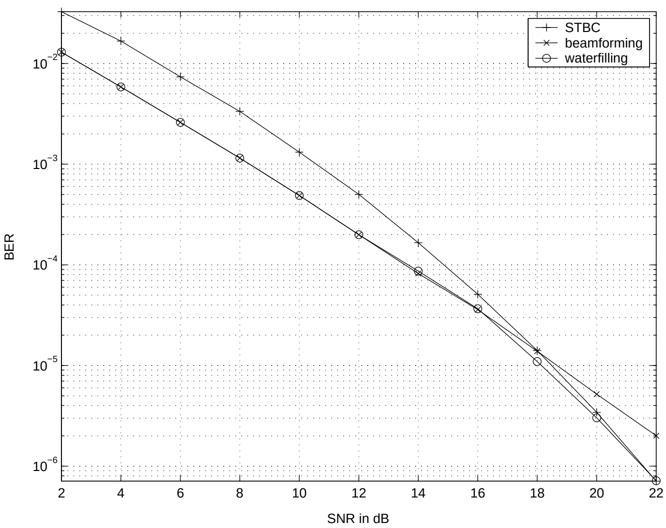

Figure 5: Optimal scheme for MIMO system. $M = 2, N = 2$.

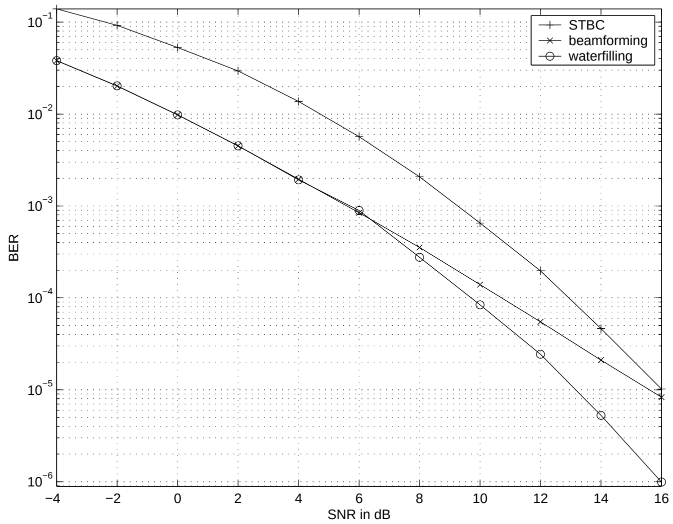

Figure 6: Optimal scheme for MIMO system. $M = 4, N = 2$.

about the optimality of waterfilling versus beamforming and O-STBC mentioned in the last section apply in this scenario as well.

5.3 Simplified Schemes

Figure 7 shows the performance when we ignore receiver correlation. A system with four transmit and two receive antennae is considered. The transmit correlation is given in Eqn. (31), and at the receiver side, the correlation between the two antennae is set to be a very high value of 0.7. From the figure we can find that there is nearly no performance loss when ignoring the receive correlation, even when the correlation is quite large.

Figure 8 shows the performance of the simplified switching scheme compared to the the waterfilling scheme for MISO systems with two or four transmit antennae. The transmit correlation uses the "all equal" model and the correlation is set as $\rho = 0.8$. For $M = 4$, the SNR threshold was found to be 4dB; for $M = 2$, it was 6.5dB. As analyzed in Section 4.2, the switching scheme achieves the same performance as waterfilling in the low SNR region; in high SNR region, it should come very close to waterfilling. A relatively larger loss occurs in the intermediate SNR region, in the vicinity of the threshold SNR. But considering the much simpler transmitter structure and low computation complexity, the switching scheme can be seen as a good alternative to the waterfilling scheme, if the SNR is known at the transmitter.

Figure 9 shows the performance of the EPA scheme for a MISO system with 4 transmit antennae. The transmit correlation is again set as in (31). We can see that the switching scheme has a large performance loss in this unequal correlation case, while the EPA scheme performs very close to the optimal waterfilling scheme.

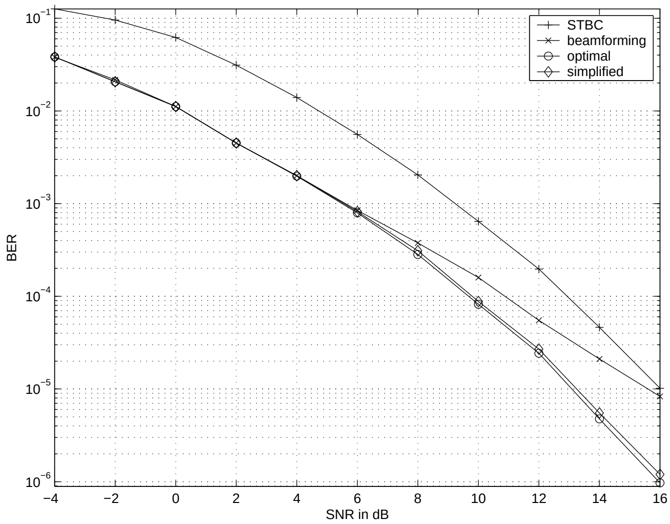

Figure 7: BEP curves when receive correlations are ignored. $M = 4, N = 2$.

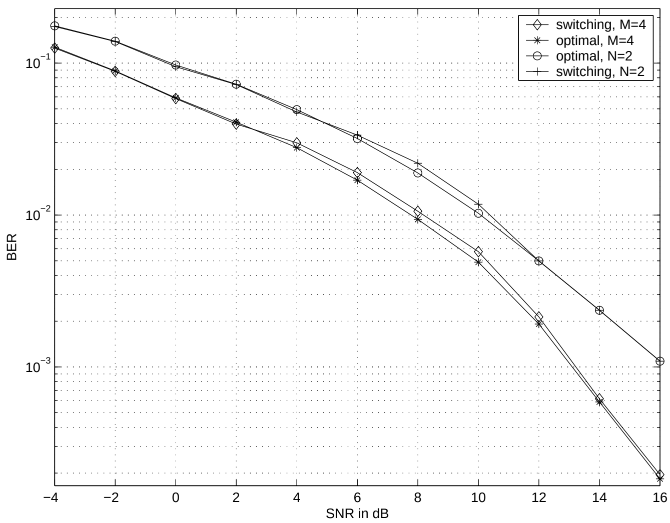

Figure 8: Switching Scheme vs. Waterfilling.

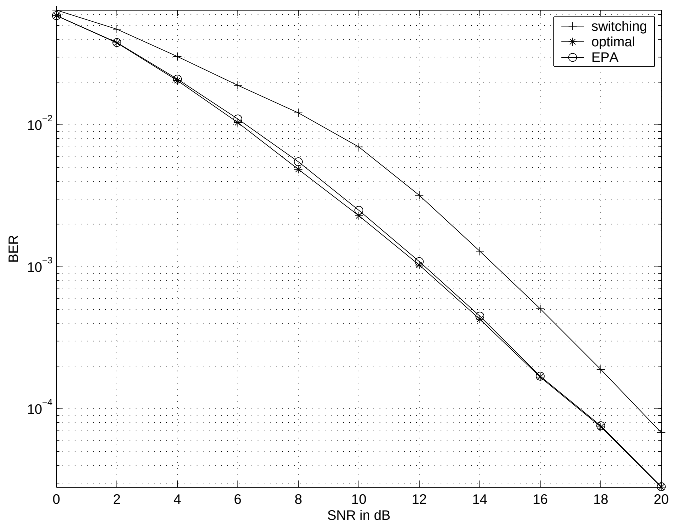

Figure 9: Performance of the EPA scheme, M=4, N=1.

6 Conclusions

Orthogonal Space-Time Block Codes (OSTBC) are widely used in MIMO systems to achieve diversity gain, but the performance of the conventional OSTBC over correlated fading channels deteriorates rapidly with increasing channel correlation. With feedback of the channel correlation matrix, the transmitter can employ a linear transformation unit following the STBC encoder to improve performance. One such scheme chooses the transformation matrix which minimizes the maximum pairwise error probability.

Based on the performance criterion derived in previous work, we provide a waterfilling solution for the optimal transformation matrix for a MISO system. The same scheme is proven to be optimal for a receive-uncorrelated MIMO system. More generally, for arbitrary MIMO systems, we derive a "generalized waterfilling" solution which can be found using numerical algorithms such as Sequential Quadratic Programming.

Interestingly, the waterfilling scheme to minimize error probability is quite similar to capacity-achieving schemes. The best transmission strategy is allocating power over the eigen-channels of the transmit correlation matrices according to their eigenvalues. For both approaches, beamforming is shown to be optimal for low SNR or high correlation, while full diversity is best for high SNR and low correlation.

Based on the "one-ring" model, the correlations between receive antennae are much smaller than those between transmit antennae in the downlink of the cellular system. A simplified scheme for MIMO system is introduced by ignoring the receive correlation and using waterfilling scheme with the transmit correlation only. Finally, two schemes are introduced to reduce the complexity of implementing the optimal technique: The switching scheme uses STBC or beamforming directly based on the SNR level and channel correlation. It reduces the transmitter complexity dramatically. The EPA scheme uses the same diversity order as the optimal one, but all the active eigen-channels have the same power. We show that these schemes suffer from minimal performance loss in realistic scenarios.

References

[1] S. M. Alamouti, "A simple transmit diversity technique for wireless communication," *IEEE Journal on Select Areas in Communications*, vol. 16, pp. 1451–1458, October 1998.

[2] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block code from orthogonal designs," *IEEE Transaction on Information Theory*, vol. 44, pp. 744–765, July 1998.

[3] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744–765, March 1998.

[4] H. El Gamal, "On the robustness of space-time coding," *IEEE Transactions on Signal Processing*, vol. 50, pp. 2417–2428, October 2002.

[5] D. S. Shiu, G. J. Foschini, M. J. Gans, and J. M. Kahn, "Fading correlation and its effect on the capacity of multielement antenna systems," *IEEE Transaction on Communications*, vol. 48, pp. 502–513, March 2000.

[6] G. Jongren, M. Skoglund, and B. Ottersten, "Combining beamforming and orthoganal space-time block code," *IEEE Transactions on Information Theory*, vol. 48, pp. 611–627, March 2002.

[7] S. Jafar and A. Goldsmith, "On optimality of beamforming for multiple antenna systems with imperfect feedback," *IEEE Int. Symp. on Info. Theory*, p. 321, 2001.

[8] E. Visotsky and U. Madhow, "Space-time transmit precoding with imperfect feedback," *IEEE Transactions on Information Theory*, vol. 47, pp. 2632–2639, September 2001.

[9] J. H. Kotecha and A. M. Sayeed, "On the capacity of correlated MIMO channels with covariance feedback," *Proc. of IEEE International Symposium of Information Theory*, 2003.

[10] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block coding for wireless communications: Performance results," *IEEE Journal on Select Areas in Communications*, vol. 17, pp. 451–460, March 1999.

[11] R. A. Horn and C. R. Johnson, *Matrix Analysis.* Cambridge, 1985.

[12] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," *IEEE Journal on Selected Areas in Communications*, vol. 20, pp. 1211–1226, August 2002.

[13] A. Graham, *Kronecker Products and Matrix Calculus with Applications.* Ellis Horwood Ltd., 1981.

[14] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* John Wiley, 1991.

[15] P. E. Gill, W. Murray, and M. H. Wright, *Practical Optimization.* Academic Press, 1981.

[16] R. Janaswamy, *Radiowave Propagation and Samrt Antennas for Wireless Communications.* Kluwer Academic Publisher, 2001.

[17] S. H. Simon and A. L. Moustakas, "Optimizing MIMO antenna systems with channel covariance feedback," *IEEE Journal on Selected Areas in Communications*, vol. 21, pp. 406–417, April 2003.