Performance of Orthogonal CDMA Codes for Quasi-Synchronous Communication Systems *

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Abstract

In this paper, the performance of quasi-synchronous direct-sequence CDMA communication systems based on different sets of orthogonal codes is investigated. We search for sets of sequences that minimize the probability of bit detection error, given that there is imperfect synchronization among the signals. Orthogonal sequences based on the Sylvester-type Hadamard matrices (Walsh functions) are shown to provide a large improvement over the case where an orthogonal set is chosen at random. For these sequences we derive closed form expressions for the average bit error rate. Computer searches indicate that this set of codes has special properties with respect to minimizing the average cross-correlations between the different signals for small errors in chip synchronization. It appears that these codes are optimal. We also discuss a multi-carrier signalling scheme designed to help synchronize the CDMA signals at the chip level.

1 Introduction

Direct sequence spread spectrum transmission has received a large degree of consideration for applications in mobile and personal communications networks as a result of its potential to provide higher spectral efficiencies in comparison to conventional modulation techniques [1, 3, 4]. The potential increases in capacity over conventional modulation schemes are a result of the cellular structure of these systems and not a result of the efficiency of direct-sequence code-division multiple-access (DS CDMA) as a multiplexing scheme. In fact, in a one-cell system the efficiency of a direct-sequence CDMA scheme is considerably lower than that of a system utilizing TDMA or FDMA [2]. To improve on the capacity of a CDMA cellular network beyond the estimates in [1], more specialized signal processing techniques may be used. These techniques attempt to cancel the interference that arises as a result of non-zero cross-correlation for the different codes used [5]. Alternatively, we may attempt to synchronize the various transmissions at the chip level and to use orthogonal spreading codes, thus reducing the amount of interference. This approach is used in the forward channel for the CDMA proposal discussed in [6]. However, its implementation in the reverse channel is more difficult, especially in the case of small chip periods. In an indoor environment the delay spreads are not too large and, hence, for low data rates and moderate processing gains, it may be feasible to maintain chip synchronization even in the reverse channel. This is the approach taken in the system proposed by Omura [4].

We consider a one-cell system with one receiver base station and several mobile transmitters. While synchronization and orthogonality of signals can be maintained in the forward link, only quasi-synchronous conditions exist in the reverse link. If mobiles remained stationary all reverse link signals could be made to arrive at the base station in synchronized fashion. If mobile speeds are insignificant in comparison to the bit time, quasi-synchronous conditions can exist.

In this paper we analyze the performance of a quasi-synchronous DS CDMA system and discuss a set of orthogonal codes that attempt to minimize the bit error rate. We also propose a new system architecture based on multi-carrier signalling to help achieve quasi-synchronous operation.

2 Non-orthogonal Codes

We consider a CDMA system with N_u mobile transmitters where the total received signal is

$$r(t) = \sum_{i=1}^{N_u} A_i d_i(t-\tau_i) p_i(t-\tau_i)\cos(w_c t + \theta_i) + n(t) \quad (1)$$

where A_i, $d_i(t-\tau_i)$, $p_i(t-\tau_i)$, and θ_i are the signal amplitude, time-delayed data signal, time-delayed code sequence, and phase offset of the i^{th} user, respectively, and $n(t)$ is a background thermal noise process with power spectral density $N_0/2$. The bit period is T_b, the chip period is T_c, and the processing gain is $N = T_b/T_c$. We assume that a rectangular chip pulse is used. A correlator receiver structure is used to receive the k^{th} signal. The output of the correlator receiver is

$$y = \pm\frac{A_k T_b}{2} + \psi + \eta \quad (2)$$

depending on the data bit. The first term in (2) is the signal component, ψ is the interference due to the other users and η is a Gaussian random variable due to the background noise. We assume that the relative delays τ_i are random variables with a uniform distribution on the interval $[-\alpha_{\max}T_c, \alpha_{\max}T_c]$. For the completely asynchronous case we model the code sequences as completely random

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and thus we may assume without loss of generality that $\alpha_{\max} = 1$. For the synchronous system $\alpha_{\max} = 0$, and for a quasi-synchronous system $0 < \alpha_{\max} < 1$. In (2) the variance of η is $N_0 T_b/4$, and the variance of ψ is

$$\sigma_\psi^2 = \overline{\psi^2} = \sum_{i=1, i\neq k}^{N_u} \frac{A_i^2}{8} \overline{R_{p_i p_k}^2(\tau_i)} T_b^2 \qquad (3)$$

where $\overline{R_{p_i p_k}^2(\tau_i)T_b^2} = \overline{\left(\int_0^{T_b} d_i(t-\tau_i)p_i(t-\tau_i)p_k(t)dt\right)^2}$. The overbar denotes expectation over the data and code symbols. We introduce the normalized delay $\alpha = \tau_i/T_c$. The variable α is assumed to be uniformly distributed on $[-\alpha_{\max}, \alpha_{\max}]$. If $p_i(t)$ and $p_k(t)$ are assumed to be random sequences then $\overline{R_{p_i p_k}^2(\tau_i)} = (1 - 2|\alpha| + 2\alpha^2)/N$. Since α is random, σ_ψ^2 is itself a random variable. For a moderate number of users we may use its expected value in the standard error formula [7]. Thus, the performance is determined by the variance of ψ,

$$\sigma_\psi^2 = \frac{NT_c^2}{8}(1 - \alpha_{\max} + \frac{2}{3}\alpha_{\max}^2) \sum_{i=1, i\neq k}^{N_u} A_i^2 \qquad (4)$$

The probability of bit error is a function of $\alpha_{\max}$ and is given by

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_k}{N_0 + \gamma \sum_{\substack{i=1 \\ i\neq k}}^{N_u} E_i}}\right) \qquad (5)$$

where

$$\gamma = \frac{1 - \alpha_{\max} + \frac{2}{3}\alpha_{\max}^2}{N} \qquad (6)$$

and $E_i = A_i^2 T_b/2$ is the bit energy of the i^{th} user.

3 Orthogonal Codes

The bit error probability given by (5) can be reduced if orthogonal spreading codes are used. Let us denote the code sequences for the signals $p_i(t)$ and $p_k(t)$ as $\{a_l\}$ and $\{b_l\}$, respectively. In this case $R_{p_i p_k}(\alpha T_c) = \frac{T_c}{T_b}\sum_{l=1}^N a_l((1-\alpha)b_l + \alpha b_{l+1})$ reduces to $\frac{T_c}{T_b}\sum_{l=1}^N \alpha a_l b_{l+1}$ since $\sum_{l=1}^N b_l a_l = 0$. Taking the expectation over the code and data symbols, we obtain

$$\overline{R_{p_i p_k}^2(\alpha T_c)} \approx \frac{T_c^2}{T_b^2}\sum_{l=1}^N \alpha^2 = \alpha^2/N. \qquad (7)$$

(7) is an approximation since it is obtained by assuming that cross-terms obtained after the squaring operation have an expected value of 0. In effect, we assume that we may model the elements of the code sequences as being random, even though the sequences themselves are orthogonal. The computer results of Section 5 indicate that (7) is a valid approximation.

Optimal conditions for a DS CDMA system based on orthogonal codes exist when perfect synchronization is maintained, that is, when $\alpha = 0$. In such a case $\sigma_\psi^2 = 0$ and the error probability is a minimum. When perfect synchronization exists between all transmitter clocks and the receiver clock, errors in bit detection arise solely from channel noise and there is no cross-interference created by simultaneous transmissions from different users.

As before, α is assumed to be uniformly distributed on $[-\alpha_{\max}, \alpha_{\max}]$ and the expected value for $\overline{R_{p_i p_k}^2(\alpha T_c)}$ is $\frac{\alpha_{\max}^2}{3N}$, thus the probability of bit error can be obtained by substituting

$$\gamma = \frac{\alpha_{\max}^2}{3N} \qquad (8)$$

into (5). The resulting probability of bit error is lower than the result for non-orthogonal codes given by (5) and (6) for any $0 \leq \alpha_{\max} < 1$.

4 Choice of Orthogonal Code Sequences

Given a specific set of orthogonal sequences, γ can be computed directly by averaging over all pairs of sequences. In this way we can compare the performance of one set of orthogonal sequences versus another set.

We seek an orthogonal set that minimizes the average value of $\overline{R_{p_i p_k}^2(\alpha T_c)} = \frac{T_c^2}{T_b^2}\overline{\left(\sum_{l=1}^N \alpha a_l b_{l+1}\right)^2}$. The problem is as follows. Define

$$\mu_{cor1} = \frac{1}{N(N-1)}\sum_{i=1}^N \sum_{\substack{k=1 \\ k\neq i}}^N \left(\sum_{j=1}^N v_{i,j} v_{k,j+1}\right)^2 \qquad (9)$$

where $v_{i,j+N} = v_{i,j}$ and define μ_{cor2} the same as μ_{cor1} except that now $v_{i,j+N} = -v_{i,j}$. We seek an $N \times N$ matrix V composed of elements $v_{i,j} \epsilon \{-1, 1\}$ such that

$$\mu_{cor} = \frac{1}{2}\mu_{cor1} + \frac{1}{2}\mu_{cor2} \qquad (10)$$

is minimized, subject to the constraint $\sum_{j=1}^N v_{i,j} v_{k,j} = 0$ for $1 \leq k \leq N$, $1 \leq i \leq N$, $i \neq k$.

The matrix rows are to be assigned as code sequences to a maximum of N users. We note that minimizing μ_{cor} corresponds to minimizing the average value of $\overline{R_{p_i p_k}^2(\alpha T_c)}$, where $p_i(t) = a(t)$ and $p_k(t) = b(t)$ are two row vectors of the matrix V. The 1/2 factors in (10) are weights and are chosen in accordance with the assumption that 1 and -1 data bits are equally likely.

To see the effect that $\mu_{cor}(N)$ has on the bit error rate directly, we write

$$\begin{aligned} \overline{R_{p_i p_k}^2(\alpha T_c)} &= \frac{T_c^2}{T_b^2}\alpha^2 \overline{\left(\sum_{l=1}^N a_l b_{l+1}\right)^2} \\ &= \frac{T_c^2}{T_b^2}\alpha^2 \mu_{cor}(N) \qquad (11) \end{aligned}$$

For this expression to be minimized $\mu_{cor}(N)$ should be a minimum, regardless of the distribution for α. If we take α to be uniformly distributed on $[-\alpha_{\max}, \alpha_{\max}]$ then using (3) we obtain

$$\sigma_\psi^2 = \frac{T_c^2 \alpha_{\max}^2 \mu_{cor}(N)}{24} \sum_{i=1, i\neq k}^{N_u} A_i^2 \qquad (12)$$

The average probability of bit error for a quasi-synchronous DS CDMA system with N_u simultaneous users is obtained by substituting

$$\gamma = \frac{\alpha_{\max}^2 \mu_{cor}(N)}{3N^2} \qquad (13)$$

into (5). Clearly, P_e is minimized if $\mu_{cor}(N)$ is minimized.

Since μ_{cor} is the same for nondistinct matrices, that is, matrices that can be obtained from one another by negating and interchanging rows, we are interested in investigating only distinct matrices when searching for an optimal matrix V that minimizes μ_{cor}, denoted V^*. We have no guarantee that a unique solution exists. If N is small, then it is a simple task to find all distinct $N \times N$ orthogonal matrices and compute μ_{cor} for each. For large N, however, we choose to search for a possibly sub-optimal V^* by generating a large number of orthogonal matrices randomly and keeping track of only those matrices that have a relatively low μ_{cor} value. We restrict our attention to the case where $N = 2^n$, where n is an integer.

For the case $N = 4$, there exist only two distinct orthogonal matrices, and $\mu^*_{cor} = 3.3333$, corresponding to the 4×4 Sylvester matrix. For the case $N = 8$ there are a total of 480 orthogonal distinct matrices, and $\mu^*_{cor} = 6$, which also corresponds to the 8×8 Sylvester-type Hadamard matrix. For higher values of N, orthogonal matrices were generated randomly. Orthogonal matrices were generated by interchanging an orthogonal matrix's columns and negating its columns randomly. Note that this method cannot generate all orthogonal distinct matrices for $N \geq 16$ in general, even if the process of permuting and negating columns is carried on indefinitely [8].

5 Computer Search

From (10), the expected value of μ_{cor} is

$$\overline{\mu_{cor}} = \frac{1}{2}(\overline{\mu_{cor1}} + \overline{\mu_{cor2}}) \approx N \tag{14}$$

using the same type of reasoning as in the derivation of (7). That is, if we randomly select $N \times N$ orthogonal matrices, and compute μ_{cor} for them, we expect that μ_{cor} will, on average, be approximately N. Substituting $\mu_{cor}(N) = N$ into (13) yields (8), as expected.

Orthogonal matrices were generated randomly on a computer and $\mu_{cor}(N)$ was calculated for each. There are 480 distinct 8×8 orthogonal matrices and, as mentioned before, the 8×8 Sylvester matrix produced the minimum μ_{cor}. Orthogonal matrices were generated at random for the cases $N = 16$, $N = 32$, $N = 64$, $N = 128$, and $N = 256$ and histograms for the N=16, 64, and 256 cases are shown in Figures 1-3. 40,000 16×16, 20,000 32×32, 2,000 64×64,

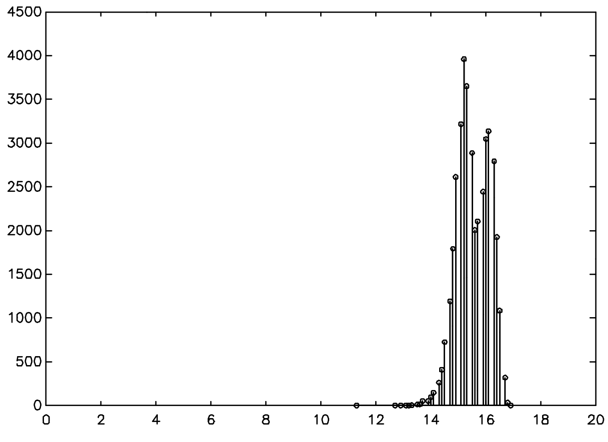

Figure 1: Number of occurrences of μ_{cor} versus μ_{cor} for the case N=16. 40,000 16×16 randomly generated orthogonal matrices were investigated.

1,000 128×128, and 500 256×256 randomly generated orthogonal matrices were investigated. In each of these cases

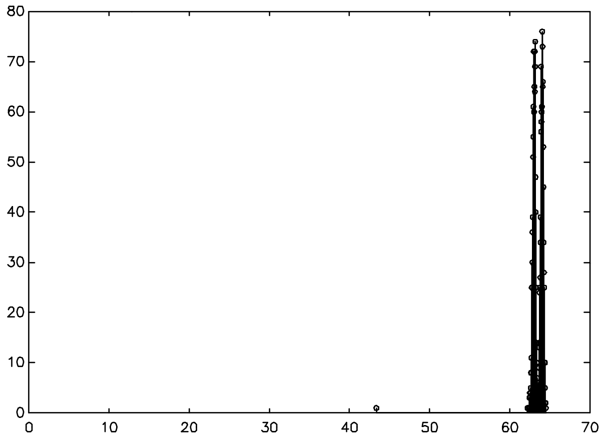

Figure 2: Number of occurrences of μ_{cor} versus μ_{cor} for the case N=64. 2,000 64×64 randomly generated orthogonal matrices were investigated.

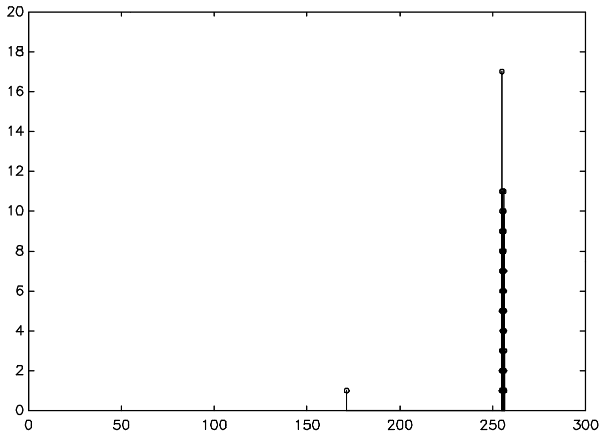

Figure 3: Number of occurrences of μ_{cor} versus μ_{cor} for the case N=256. 500 256×256 randomly generated orthogonal matrices were investigated.

the matrix that produced the smallest μ_{cor} value was the Sylvester matrix. Note from the figures the concentration of observations around $\mu_{cor} = N$. These histograms support roughly the statistical calculation of (14).

Recall that an order $2m$ Sylvester matrix is computed recursively as

$$H_{2m} = \begin{bmatrix} H_m & H_m \\ H_m & -H_m \end{bmatrix} \tag{15}$$

where $H_1 = 1$. The following equation applies for $N \times N$ Sylvester matrices

$$\mu_{cor}(N) = \frac{2N^2 - 2}{3(N-1)} \tag{16}$$

and is derived in the appendix. From (16), as N increases, $\mu_{cor}(N)$ approaches $2N/3$.

If orthogonal sequences are chosen from the row vectors of an $N \times N$ Sylvester matrix, we have $\mu_{cor}(N) = 2N/3$ for large N, as is typically the case in practice, so that the average probability of bit error is obtained by substituting

$$\gamma = \frac{2\alpha_{\max}^2}{9N} \tag{17}$$

into (5). Comparing (17) to (8) we see the effect that the reduction of $\mu_{cor}(N)$ from N, the statistical average, to $2N/3$, for $N \times N$ Sylvester matrices, has on the performance of a DS CDMA system. Figures 4-5 are plots of the analytical expressions derived for P_e for three types of code sets, each code set consisting of $N = 128$ code sequences of length N chips. The number of simultaneous users is N_u=128. Each figure shows the performance assuming: a) non-orthogonal code sequences, b) the set of orthogonal code sequences is selected at random, c) the set of orthogonal sequences correspond to the row vectors of a Sylvester matrix. Case a) is given by (5) with γ given by (6), case b) has γ given by (8), and case c) has γ given by (17). In these figures, $E_i = E_k = E_b$, which may be possible if the CDMA system uses power control.

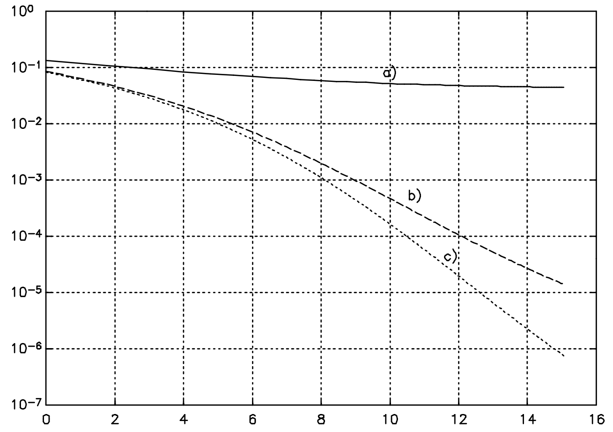

Figure 4: P_e versus E_b/N_o for case N_u=N=128, α_{max}=.5 a) Codes are non-orthogonal. b) Codes are orthogonal, random. c) Codes are orthogonal Sylvester sequences.

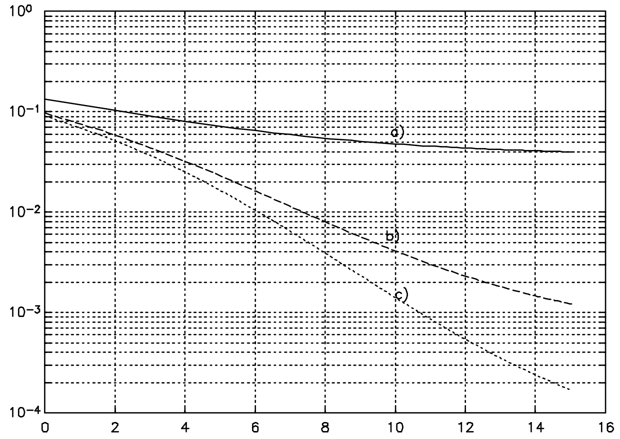

Figure 5: P_e versus E_b/N_o for case N_u=N=128, α_{max}=.75 a) Codes are non-orthogonal. b) Codes are orthogonal, random. c) Codes are orthogonal Sylvester sequences.

For large E_b/N_0, and $\alpha_{\max} \neq 0$, the N_0 term in (5) can be neglected and it is seen that the Sylvester sets offer better performance than the random orthogonal set by a factor of 10log3/2 = 1.76 dB. Equivalently, for a given P_e, usage of the Sylvester-type sets results in $1\frac{1}{2}$ times as many simultaneous users as the random orthogonal set would allow. For $\alpha_{\max} = 0$, which corresponds to the ideal situation where perfect synchronization between all transmitters and the receiver is maintained, all orthogonal code sequences offer exactly the same performance. For small $\alpha_{\max}$, a relatively large E_b/N_0 is needed before the differences in the bit error rates of synchronous CDMA systems with different orthogonal code sequences becomes noticeable. Note also the large gain in performance when orthogonal sequences are used rather than non-orthogonal sequences.

6 Multi-Carrier Modulation to Facilitate Synchronization Among the Different Signals

To reduce the error probability the mis-match in chip synchronization should be a small fraction of the chip period. For a given processing gain, an increase in the chip period results in an increase in the bit period, hence a decrease in the data rate. We can regain the original data rate by using a multi-carrier signalling scheme. The overall transmitted signal is comprised of several bit steams, each modulated by a separate carrier. Figure 6 illustrates the components of the overall power spectral density for a single user for the case of three subcarriers spaced at multiples of the chip rate.

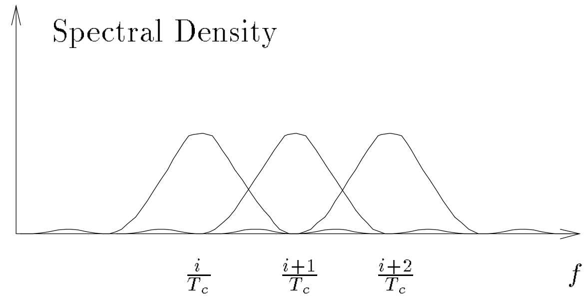

Figure 6: Spectral densities for three bit streams with different carriers.

In computing the probability of bit error for any given data stream we need to consider the interference from other data streams with the same carrier and interference from data streams using other carriers. For arbitrary carrier spacings the approach in [9] may be used to determine the variance of the noise. For carrier spacings equal to the chip rate the noise variance can be solved using a more direct approach which we use here. Assume that the chip pulse $h(t)$ is rectangular, so that it is 0 outside $[0, T_c]$. We assume that each bit stream corresponding to a single transmitter is multipied by the same spreading code - the different streams belonging to the same transmitter do not interfere with each other due to the orthogonality of the carriers. The variance of the cross-correlation between two DS CDMA signals with carriers spaced by $\Delta w = 2\pi i/T_c$ can be shown to be $\overline{\mu_{cor}(N)\left(\int_{-\infty}^{\infty} h(t+\tau-T_c)h(t)\sin(\Delta w+\theta)dt\right)^2}$ where the expectation is with respect to τ and θ. τ is modelled as uniform on $[-\alpha_{\max}T_c, \alpha_{\max}T_c]$ and θ is uniform on $[0, 2\pi]$. This cross-correlation expression reduces to $\mu_{cor}\frac{T_c^2}{(2\pi i)^2}(1 - \mathrm{sinc}(2\alpha_{\max}i))$, where $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$, compared to $\mu_{cor}T_c^2\alpha_{\max}^2/6$ which would have been the result if the two signals had the same carrier. Assuming M carriers, we can show that

$$\gamma = \frac{2\mu_{cor}(N)}{N^2}\left(\frac{\alpha_{\max}^2}{6} + \frac{1}{4\pi^2}\sum_{\substack{j=1\\ j\neq m}}^{M} f(j)\right) \qquad (18)$$

where $f(j) = (1 - \mathrm{sinc}(2\alpha_{\max}(m-j)))\,\frac{1}{(m-j)^2}$. The resulting probability of bit error obtained by substituting γ into (5) corresponds to the bit error rate of the k^{th} user's m^{th} bit stream.

7 Conclusions

The performance of quasi-synchronous direct-sequence code-division multiple-access communications systems was investigated. Analytical expressions for bit error rates of quasi-synchronous DS CDMA systems based on non-orthogonal and orthogonal code sets were derived and compared.

When synchronous or quasi-synchronous conditions exist, orthogonal sets are preferred over non-orthogonal ones. When perfect synchronization is not present, different orthogonal code sequences result in varying degrees of cross-interference between the transmitted signals.

For a given orthogonal $N \times N$ matrix, there is associated with it a parameter, which we have denoted $\mu_{cor}(N)$. The larger the μ_{cor} value for a given orthogonal matrix, the worse the performance of a quasi-synchronous CDMA system based on the matrix. The problem investigated consisted of finding orthogonal matrices with minimum μ_{cor} values. It was shown analytically that $\mu_{cor}(N) = N$ for the average orthogonal matrix. Computer calculations supported this claim. For the Sylvester-type Hadamard matrices investigated, it was seen that $\mu_{cor}(N)$ tends to $2N/3$ as N gets large so that there is a large improvement factor in the performance of systems with spreading sequences based on these matrices over that of systems based on orthogonal sets chosen at random. For a large signal-to-noise ratio and quasi-synchronous conditions, the performance improvement is about 1.76 dB. Equivalently, for a given bit error rate, usage of the Sylvester sets results in $1\frac{1}{2}$ times as many simultaneous users as the average orthogonal set would allow.

A multi-carrier signalling scheme which is designed to help synchronize CDMA signals at the chip level was discussed. A quasi-synchronized CDMA system based on orthogonal codes is an alternative to an asynchronous scheme which attempts to use sophisticated interference cancellation techniques.

Appendix

In this appendix we derive (16).

Defining $X_{ij}^N = \sum_{k=1}^{N} v_{i,k}v_{j,k+1}$, where $v_{i,N+1} = v_{i,1}$, and $v_{i,k}$ is the $(i,k)^{\mathrm{th}}$ element of a Sylvester matrix, we use (9) to write

$$\begin{aligned} N(N-1)\mu_{cor1}(N) &= \textstyle\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}(X_{ij}^N)^2 \\ &= 8\textstyle\sum_{i=1}^{N/2}\sum_{\substack{j=1\\ j\neq i}}^{N/2}(X_{ij}^{N/2})^2 - 16\sum_{i=1}^{N/2}\sum_{\substack{j=1\\ j\neq i}}^{N/2}(X_{ij}^{N/2})v_{i,N/2} \\ &\quad +16\left(\tfrac{N}{2}\right)\left(\tfrac{N}{2}-1\right) \end{aligned} \qquad (A.1)$$

where (A.1) follows because X_{ij}^N equals $2X_{ij}^{N/2}$ if i and $j\epsilon\{1\ldots N/2\}$, 0 if $i\epsilon\{1\ldots N/2\}$ and $j\epsilon\{N/2+1\ldots N\}$, or $2X_{i-N/2,j-N/2}^{N/2} - 4v_{i-N/2,N/2}$ if i and $j\epsilon\{N/2+1\ldots N\}$. It can be shown that

$$\sum_{i=1}^{N/2}\sum_{\substack{j=1\\ j\neq i}}^{N/2}(X_{ij}^{N/2})v_{i,N/2} = 4\left(\frac{N}{4}\right)\left(\frac{N}{4}-1\right) \qquad (A.2)$$

where $v_{i+N/4,N/2} = -v_{i,N/4}$ was used. (A.1) reduces to

$$\mu_{cor1}(N) = \frac{8\left(\frac{N}{2}\right)\left(\frac{N}{2}-1\right)\mu_{cor1}\left(\frac{N}{2}\right) + 8N}{N(N-1)} \qquad (A.3)$$

This is a recursive formula for $\mu_{cor1}(N)$. We interpret (A.3) as a first order difference equation and solve it to obtain

$$\mu_{cor1}(N) = \frac{2N^2-8}{3(N-1)} \qquad (A.4)$$

for an $N \times N$ Sylvester matrix. The boundary condition $\mu_{cor1}(2) = 0$ applies to (A.3).

In a similar fashion, we can show that

$$\mu_{cor2}(N) = \frac{2N^2+4}{3(N-1)} \qquad (A.5)$$

(16) follows from (A.4), (A.5), and (10).

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