Multi-Carrier DS-CDMA with Adaptive Sub-Carrier Hopping for Fading Channels

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Abstract— **A modified multi-carrier (MC) DS-CDMA system has been proposed for use over slow multi-path fading channels with frequency selectivity in the reverse link transmission of the cellular network. Instead of transmitting data sub-streams uniformly through sub-channels, data sub-streams hop over sub-channels with the hopping patterns adaptively adjusted to the channel fading characteristics. The problem of determining the optimal hopping pattern is formulated as a multi-objective optimization problem, for which an efficient algorithm is designed to solve the problem practically. Simulation results show that the performance in terms of the average system bit error probability (BER) is better than that of a single carrier Rake system, conventional MC CDMA systems applying moderate error protection and diversity systems with different combining techniques.**

I. Introduction

Various multi-carrier (MC) transmission schemes have been recently introduced into direct sequence code division multiple access (DS-CDMA) systems for benefits such as higher rate data transmission, bandwidth efficiency, frequency diversity, and interference reduction etc. [?; ?; ?; ?].

In one type of the conventional MC CDMA systems [?], the total bandwidth of W is divided into M equi-width sub-channels. The serial data stream of each user (mobile unit) is then converted into a corresponding number of parallel sub-streams with each being spread by the same sequence selected from a set of orthogonal sequences. The resultant sub-streams are then used to modulate M orthogonal carriers, and transmitted over different sub-channels. Compared with a single carrier DS-CDMA system, this system has the advantage in facilitating the synchronization process, and thus reducing the multiple access interference (MAI) in additive white Gaussian noise (AWGN) channels. However, when we are concerned with multi-path fading channels which commonly confronts broad band signal transmission from a mobile unit, the multi-carrier system is disadvantageous to a DS-CDMA system using a Rake receiver. This is because multi-paths are mostly not resolvable and thus can not be combined over sub-channels. Hence we have flat fading over each sub-channel, and these sub-channel fading parameters vary differently due to the frequency selectivity of the entire transmission channel. Data sub-streams transmitted over seriously deteriorated sub-channels may suffer a long burst of errors. While error control coding and interleaving techniques have been suggested to recover the performance loss [?], the complexity and time delay restrictions often make them either ineffective or very difficult to implement.

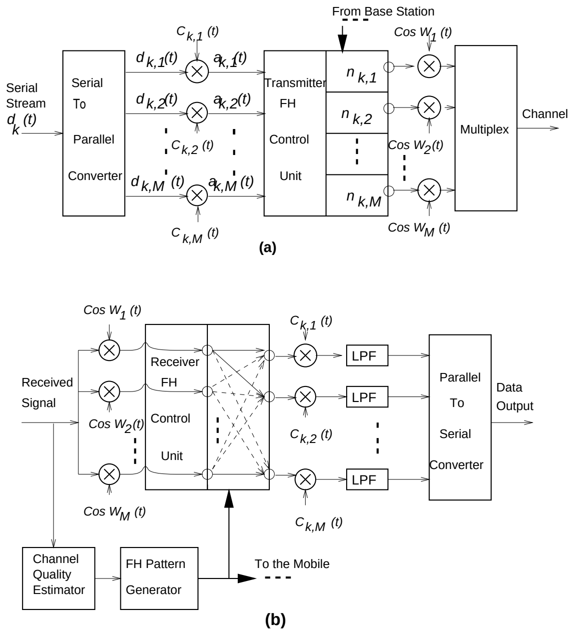

Fig. 1. A multi-carrier DS-CDMA system with adaptive frequency hopping for the k^{th} user: (a)Transmitter, (b) Receiver.

In this paper, we modify the conventional system in the following ways: We assign M (same as the number of carriers) pseudo-random noise (PN) sequences to each user so that every data sub-stream is uniquely identified by one spreading sequence; rather than sending one sub-stream over each sub-channel, sub-streams can hop to any of the sub-channels depending on the fading parameters. This modified system is especially suitable for operating in channels with very slow time variations as often encountered by slow moving vehicles or pedestrians. It also has the potential to accommodate users of different data rates.

II. System Models

We consider the reverse link transmission of a CDMA system. An adaptive MC FH/DS system for the k^{th} user of a CDMA system is shown in Figure ??.

A. Transmitter Model

At the transmitter, the user's binary data stream $d_k(t)$ of rate R is converted into M interleaved sub-streams $D_k = (d_{k,1}(t), d_{k,2}(t), \cdots, d_{k,M}(t))$. The data sub-streams are transmitted over sub-channels with a reduced rate $\frac{R}{M}$, i.e., the data interval over sub-channels is $T = \frac{M}{R}$. The sub-streams are spread by random sequences $C_k = (c_{k,1}(t), c_{k,2}(t), \cdots$

, $c_{k,M}(t))$, respectively, resulting in another vector of direct sequence signals $A_k = (a_{k,1}(t), a_{k,2}(t), \cdots, a_{k,M}(t))$. Although for the purpose of reducing MAI, orthogonal spreading sequences are preferred, this is usually not achievable in practice. For a system of K users and M sub-channels, $N_s = K \times M$ spreading sequences are needed to uniquely identify each $d_{k,m} \in D_k$. Meanwhile the total number of orthogonal sequences available for the system is $N = \frac{W}{R}$, where N is the processing gain of the narrow-band DS signal. N_s is normally much larger than N in a practical system.

Assuming BPSK modulation, each $a_{k,i} \in A_k$ is to modulate one of the carrier frequencies in $\Omega = (\omega_1, \omega_2, \cdots, \omega_M)$. Carrier frequencies are equally separated by the chip rate $\frac{1}{T_c}$, and they are mutually orthogonal in the interval $[0, T]$. As it has been said, the k^{th} user's m^{th} sub-stream $d_{k,m}(t)$ is not necessarily transmitted over the m^{th} sub-channel. Letting $L_k = (l_{k,1}, l_{k,2}, \cdots, l_{k,M})$ denote the set of sub-channels which are actually occupied by D_k during one frequency hopping period denoted as T_{fh}, L_k indicates the mapping between D_k and Ω, i.e., $\omega_{l_{k,m}} \in \Omega$ is the carrier frequency modulated by $a_{k,m}(t)$. Note that since more than one of the user's sub-streams may hop to the same sub-channel, there could be equal elements in L_k. For the ease of operation, we impose that $\omega_{l_{k,1}} \leq \omega_{l_{k,2}}, \cdots, \leq \omega_{l_{k,M}}$, i.e., sub-streams enter the the sub-channels in the natural order. Mapping vector L_k is adaptively determined by the transmitter FH control unit (TFHCU) upon receiving instructions from the base station through a feed back control channel.

The transmitted signal from k^{th} user is written as

$$s_k(t) = \sum_{m=1}^{M} \sqrt{2P_{k,m}}\hat{a}_{k,m}(t) \cos \omega_{l_{k,m}} t, \qquad (1)$$

where $P_{k,m}$ is the power for transmitting $d_{k,m}(t)$, $p(t)$ is the chip pulse shape, $\hat{a}_{k,m}(t)$ is the data modulated PN pulse train. We assume every user has a total power of P_t, which is equally distributed among sub-streams, i.e., $P_{k,m} = \frac{P_t}{M}, \forall k \ \& \ m$. In the sequel $P_{k,m}$ will be simply written as P. To avoid the issue of inter-sub-channel interference which is not the focus here, $p(t)$ is chosen as frequency limited *sinc* chip pulse.

B. Channel Model

The broad band multi-path fading channel is considered as a dispersive Rayleigh fading channel with time variation and frequency selectivity. The k^{th} user's broad band channel is modeled as [?]:

$$h_k(t) = \sum_{l=0}^{L_p} \alpha_{k,l} \delta(t - \tau_{k,l}) \exp(j\phi_{k,l}), \qquad (2)$$

where L_p is the number of paths, $\alpha_{k,l}$, $\tau_{k,l}$ and $\phi_{k,l}$ are the path amplitude, time delay and signal phase in the l^{th} path, respectively. Fading amplitudes $\alpha_{k,l}$ are modeled as independent Rayleigh random variables (RVs) with unit second moment. $\phi_{k,l}$ and τ_k are independent RVs uniformly distributed in $[0, 2\pi)$ and $[0, T_d]$, respectively. The range of path delays T_d is defined as the channel delay spread. All the fading parameters are assumed constant over at least one hopping period.

In our system, however, information is transmitted as narrow-band DS waveforms over sub-channels with bandwidth being a portion of the entire channel bandwidth. Defining W_s as the sub-channel bandwidth, i.e. $W_s = \frac{1}{T_c} = \frac{W}{M}$, a number of factors have to be considered in choosing W_s for fixed W. On one hand, for achieving quasi-synchronization so as to reduce MAI, a larger M is preferred so that T_d is a fraction of T_c. On the other hand, a larger M and thus a larger T means that, for a given coherence time, there are fewer data symbols transmitted during each hop, reducing the system efficiency. As a compromise between these two effects, we choose M such that $T_c \approx T_d$. In doing so, W_s is almost equal to the coherence bandwidth of the multi-path channel, and thus the correlation between the fading amplitudes of adjacent sub-channels should be less than 0.5.

We derive the sub-channel impulse response for the k^{th} user's m^{th} sub-channel (referred to as (k,m) sub-channel here) by applying (??) to M band-pass filters with each centered at one of the M carrier frequencies and having a bandwidth of W_s [?]. Let $H_{k,m}(j\omega)$ be the output of the k^{th} user's m^{th} band-pass filter, $H_{k,m}(j\omega) = |H_{k,m}(j\omega)| \exp(j\theta(\omega))$, $\omega \in [(m-1)W_s, mW_s]$. We write the sub-channel impulse response as

$$h_{k,m}(t) = \alpha_{k,m} \delta(t - \tau_k) \exp(j\phi_{k,m}), \qquad (3)$$

where $\alpha_{k,m}$ and $\phi_{k,m}$ are fading amplitude and signal phase for (k,m) sub-channel, and τ_k is the path delay. The parameters $\alpha_{k,m}$ and $\phi_{k,m}$ are obtained from (??) as follows:

$$\alpha_{k,m} = \mathcal{E}[|H_{k,m}(j\omega)|], \qquad (4)$$
$$\phi_{k,m} = \mathcal{E}[\theta(\omega)]. \qquad (5)$$

where $\mathcal{E}(\cdot)$ stands for the expectation over the sub-channel frequency band. The path delay τ_k is obtained as the average of multi-path delays, i.e., $\tau_k = \frac{1}{L_p} \sum_{l=0}^{l=L_p} \tau_{k,l}$.

C. Receiver Model

Conventional correlator detection is assumed at the receiver. The channel quality estimator (CQE) estimates the fading channel parameters periodically and passes the estimates to the FH pattern generator (FHPG). Based on the channel parameters, the FHPG generates a $K \times M$ matrix H with $n_{k,m} \in [0, M]$ denoting the number of sub-streams hopping to (k,m) sub-channel. The sum of each row of H is equal to M. The k^{th} row of H is sent to the TFHCU through a feed back control channel, based on which the carrier modulation is carried out in the transmitter. The same information is passed to the k^{th} user's receiver FH control unit (RFHCU) for completing the despreading process accordingly. Applying (??) to (??), the received signal is

$$r(t) = \sum_{k=1}^{K} \sum_{m=1}^{M} \sqrt{2P} \alpha_{l_{k,m}} \hat{a}_{k,m}(t - \tau_k) \cos(\omega_{l_{k,m}} t + \phi_{l_{k,m}}) + n(t), \qquad (6)$$

where $n(t)$ is the AWGN with two-sided power spectral density $N_0/2$.

The success of the proposed system depends heavily on the availability of a CQE providing accurate and timely estimation of the channel, a reliable feed back control channel and a FHPG promptly producing optimal frequency hopping patterns. We understand that the techniques for meeting the first two requirements have been intensively studied in general, and made available in a variety of practical systems. In the rest of this paper, we will focus on the algorithm design for FHPG and the system performance analysis.

III. Analysis and Algorithm Design

In the reverse link transmission, each user's signal goes through different propagation channels. These channels are statistically independent for two users separated geographically by at least half of the wave length which is normally the case. The idea behind our system is to take advantage of this diversity among users as well as the frequency selectivity in each channel. With coherent detection, the parameter which measures the channel quality is fading amplitude $\alpha_{k,m}$. Although it is seemingly desirable for one particular user to concentrate all its data sub-streams on the sub-channel which strong fading amplitude, it could be contributing large amount of interference and thus be harmful for other users who happen to be transmitting over the same sub-channel. The key issue is to find the best way for positioning user's sub-streams so that the lowest BER can be achieved in the system.

A. Analysis

We are concerned with very slow fading channels, i.e. $T_{fh} \gg T$. Without loss of generality, we detect the k^{th} user's m^{th} data sub-stream $d_{k,m}(t)$ transmitted over the $l_{k,m}^{\text{th}}$ sub-channel. Assuming bit '1' is transmitted, the low-pass filter (LPF) output at the sampling point is

$$y_{k,m}(T) = \frac{\gamma_{m,l_{m,k}}T}{2} + I_{m,k} + \eta_{m,k}, \tag{7}$$

where $\gamma_{k,m} = \alpha_{k,m}\sqrt{2P}$ (??), $I_{k,m}$ is the interference term, and $\eta_{k,m}$ is the noise term, which is a Gaussian RV with zero mean and variance $N_0T/4$.

It has been shown that for systems with large K and moderate to large N, Gaussian distribution with variable variance is a good approximation for modeling the MAI [?]. The average variance of $I_{k,m}$, then, is

$$\begin{aligned} V(I_{k,m}) \;&=\; \frac{NT_d^2}{8}\Big[\sum_{\substack{i=1\\ i\neq k}}^{K} n_{i,l_{k,m}}\gamma_{i,l_{k,m}}^2 \\ &\quad + (n_{k,l_{k,m}}-1)\gamma_{k,l_{k,m}}^2\Big]. \end{aligned} \tag{8}$$

The first term of (??) accounts for the interference from other users hopping to the same sub-channel, and the second term accounts for that from the user itself.

Conditioned on the channel parameters, the average BER for transmitting $d_{k,m}(t)$ during one hop period T_{fh} is the probability for $y_{k,m}(T)$ being greater than 0,

$$p_{k,m} = \frac{1}{2}\text{erfc}\sqrt{\lambda_{k,m}}, \tag{9}$$

and $\lambda_{k,m}$ is the signal-to-interference-plus-noise ratio (SINR)

$$\lambda_{k,m} = \frac{\upsilon_{k,l_{k,m}}}{N_0 + \frac{\bar{T}_d^2}{N}\big[\sum_{\substack{i=1\\ i\neq k}}^{K} n_{i,l_{k,m}}\upsilon_{i,l_{k,m}} + (n_{k,l_{k,m}}-1)\upsilon_{k,l_{k,m}}\big]} \tag{10}$$

where $\bar{T}_d = T_d/T_c$,the normalized delay spread, and $\upsilon_{k,m} = \frac{1}{2}\gamma_{k,m}^2 T$, the received energy per bit at (k,m) sub-channel. The mean value of $\upsilon_{k,m}$ is a constant E_b when open loop power control is applied. The probability $p_{k,m}$ is a function of both hopping pattern matrix H and the received bit energy matrix Υ. The average BER of the k^{th} user is $\bar{p}_k = \frac{1}{M}\sum_{m=1}^{M} p_{k,m}$ Letting $Pe = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K)$, we use the average BER among all users as the measure of our system performance. To decrease the system BER, minimizing each element of Pe should be equally important. This leaves us with a multi-objective optimization problem formulated as:

$$\begin{aligned} &\text{minimize}_{H\in\mathcal{I}^{\mathcal{K}\times\mathcal{M}}}\, Pe = (\bar{p}_1, \bar{p}_2, \cdots, \bar{p}_K), \quad \forall\Upsilon \\ &\text{subject to}: \textstyle\sum_{m=1}^{M} n_{k,m} = M, \quad \forall k \end{aligned}$$

where $\mathcal{I}^{\mathcal{K}\times\mathcal{M}}$ is the set of $K \times M$ matrices with integer entries.

In general a multi-objective optimization problem can be solved with the Goal Attainment (GA) method [?]. A sophisticated optimization algorithm applying such a method (referred to as the GA algorithm) has been attempted in our simulation. Since the algorithm was designed for general applications, it disregards the specific properties of the problem and does not even converge under specific circumstances. For a system with reasonably large K and M, the computational time needed and the uncertainty of convergence makes the GA algorithm an impossible choice to work with FHPG in real time.

B. Algorithm Design

Within this part, an alternative approach is taken in solving the optimization problem. Instead of trying to achieve low BER directly, we aim at maximizing average SINR of the system. Although high SINR is equivalent to low BER, its expression provides us with more insight into system behaviour.

Equation (??) is rewritten as

$$\begin{aligned} \lambda_{k,m} \;&=\; \Big[\frac{N_0}{\upsilon_{k,l_{k,m}}} + \frac{\bar{T}_d^2}{N\upsilon_{k,l_{k,m}}}\Big(\sum_{\substack{i=1\\ i\neq k}}^{K} n_{i,l_{k,m}}\upsilon_{i,l_{k,m}} \\ &\quad + (n_{k,l_{k,m}}-1)\upsilon_{k,l_{k,m}}\Big)\Big]^{-1}, \\ &=\; [\text{snr}_{k,m}^{-1} + \text{sir}_{k,m}^{-1}]^{-1} \end{aligned} \tag{11}$$

where $\mathrm{snr}_{k,m}$ and $\mathrm{sir}_{k,m}$ are the signal-to-noise ratio (SNR) and signal-to-interference ratio (SIR). $\mathrm{snr}_{k,m}^{-1}$ is defined as the normalized (by received bit energy) noise term (NNT), and $\mathrm{sir}_{k,m}^{-1}$ is the normalized interference term (NIT). It is noticed that while a large $\upsilon_{k,m}$ reduces $\mathrm{snr}_{k,m}^{-1}$, it increases $\mathrm{sir}_{i,m}^{-1}, i \neq k$. Our task is to find a proper H that produces the smallest possible NNTs and NITs over the sub-channels in use.

A simple recursive algorithm is designed as follows:

step 0. Define a matrix S and initialize matrices H and S: $n_{i,j} = 0, \;\; s_{i,j} = N_0/\upsilon_{i,j}, \; i = 1, 2, \cdots, K, j = 1, \cdots, M$.

step 0.1 Define a vector $q = (1, 2, \cdots, K)$.

step 1. Find (k, m) sub-channel such that: $s_{k,m} = \max S_{min}$, where $S_{min} = \{s_{min(i)} | i \in q\}$, and $s_{min(i)} = \min(s_{i,1}, s_{i,2}, \cdots, s_{i,M})$.

step 2. Assign one sub-stream from k^{th} user to (k, m) sub-channel and update S: $n_{k,m} = n_{k,m}+1$, and $s_{i,m} = s_{i,m}+\delta_{i,m|k}$, $i = 1, \cdots, K$, where $\delta_{i,m|k}$ is the NIT contributed to sub-channel (i, m) by adding one sub-stream to sub-channel (k, m), i.e. $\delta_{i,m|k} = \frac{\upsilon_{k,m}\bar{T}_d^2}{\upsilon_{i,m}N}$.

step 3. Delete number k from vector q.

step 4. If $q \neq \emptyset$, go back to step 1.

step 5. Repeat step 0.1 to 4 M times.

The design of this algorithm was motivated by the water-filling principle in information theory [?]. This principle states that given parallel channels with Gaussian noise, information should be first fed into channels with lower noise levels to achieve the maximum channel capacity. What prevents its direct application to our problem is the interaction between users through MAI. But the approximation of MAI with a Gaussian distribution has allowed us to treat it as part of the channel noise, and thus to apply the water-filling principle recursively. In what follows, we refer to the above algorithm as the Water-Filling (WF) algorithm.

It can be seen that although every user has an opportunity to access its "favourite" sub-channel during each iteration, priority is given to the user whose "favourite" is the worst among all for obvious reasons. The WF algorithm is simple to implement and guaranteed to converge after $K \times M$ iterations. As shown in the next section, the WF algorithm provides a performance nearly as optimal as the sophisticated optimization algorithm does, but with a much faster speed. One pitfall might lie in the case where one user's fading amplitudes are much larger than the average. While the more sophisticated algorithm may choose to discard some of those especially strong sub-channels from interference-consideration and still keep a lower than average BER for this user, the WF algorithm overlooks such a situation. However, since fading amplitudes are weakly correlated with equal average bit energy, such situation occurs rarely.

IV. Results and Discussion

The system performance is evaluated using Monte-Carlo simulation. The channel of interest is centered at 910 MHz with bandwidth 1.25 MHz, which is one CDMA channel in the IS-95 standard. We assume that the user data rate is R=8kbps equal to the output data rate of a full rate vocoder. M is set to 8 corresponding to a delay spread of about $T_d = 6\mu s$, which is common in urban areas [?]. PN sequences of length 156 is used. For a mobile with a velocity between 3km/h and 50km/h, the coherence time ranges from .02 sec to .4 sec, based on which T_{fh} is chosen. Since the data rate is 1kbps over each sub-channel, at least $24 \sim 400$ data symbols can be transmitted during each hop. The number of multiple paths is set to 3 in the simulation.

Figure ?? presents the BERs (averaged among all users) obtained with the Goal Attainment algorithm and the Water-Filling algorithm. It is seen that the performance obtained with the WF algorithm is slightly inferior. However, even with K assuming an unreasonably small number 5, the time taken to reach the solution by the GA algorithm is almost a thousand times longer than that of the WF algorithm. Hence the WF algorithm is applied hereafter to evaluate the system performance. K is set to 64 for the rest of the results.

The performance of the adaptive MC FH/DS system is compared with a single carrier DS-CDMA system applying a Rake receiver in Figure ??. The proposed system performs much better than the single carrier system. There are at least two reasons for explaining this. First, by achieving quasi-synchronization, our system effectively reduces the MAI which is a major performance constraint in an asynchronous single carrier CDMA system with considerable number of users. Second, instead of combining all paths as the Rake receiver, we tried to take full advantage of the strongest path.

In Figure ??, the system performance is compared with the conventional MC CDMA system. Even with the help of a $(32, 6)$ Reed Muller code, the conventional system's BER performance is still poorer. Though higher complexity is the price we pay for the improved performance, such a complexity should be quite affordable in a CDMA network where even more complicated power control algorithms have been proposed. In Figure ??, we also compare the performance of the proposed system with diversity systems which are frequently used in multi-path fading channels. Since data symbols are transmitted repeatedly with lower power, the diversity system's performance is also poorer. Besides the performance disadvantages, a coded or diversity system has the drawbacks of bandwidth expansion or data rate reduction.

V. Conclusions

A MC DS-CDMA system with adaptive frequency hopping is proposed in this paper. The problem of obtaining the optimal frequency hopping pattern was formulated as a multi-objective optimization problem. An efficient algorithm has been designed to make the system work practically. The system has shown significant performance improvement over some existing systems in terms of the average system BER, which can be directly translated into an increase in CDMA system capacity.

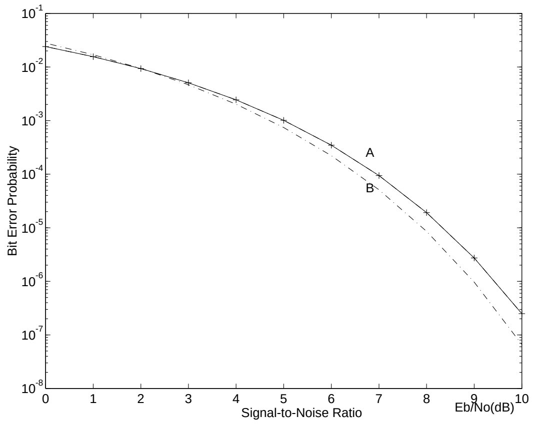

Fig. 2. Average system bit error probabilities obtained with the Goal Attainment algorithm and with the Water-Filling algorithm. $L_p = 3, T_d = 6\mu s$, $K = 5, M = 8$ and $N = 156$.

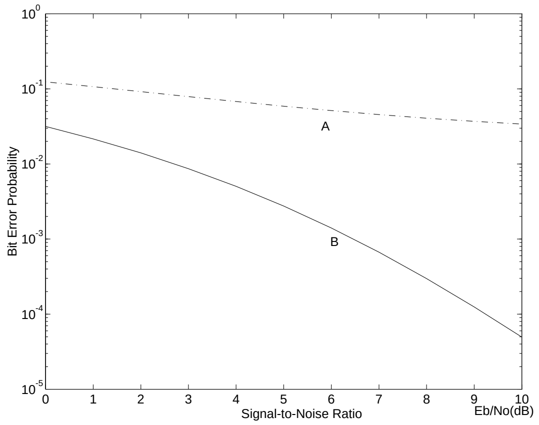

Fig. 3. Average system bit error probabilities obtained with the adaptive frequency hopping multi-carrier DS-CDMA system and a single carrier DS-CDMA system utilizing a Rake receiver. $L_p = 3, T_d = 6\mu s, K = 64, M = 8$ and $N = 156$.

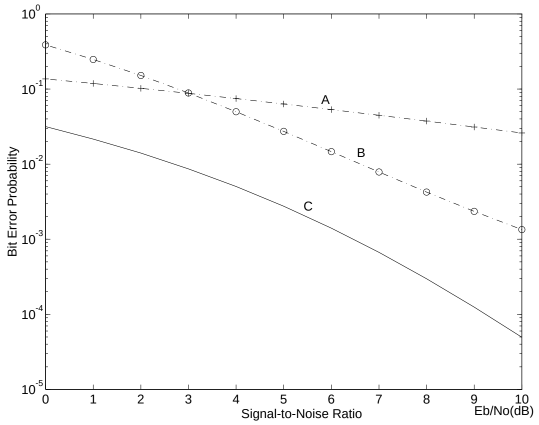

Fig. 4. Average system bit error probabilities obtained with the adaptive frequency hopping multi-carrier DS-CDMA system and uncoded/coded conventional multi-carrier CDMA system. The error control code used with conventional MC CDMA system is (32,6) Reed Muller code. $L_p = 3, T_d = 6\mu s, K = 64, M = 8$ and $N = 156$.

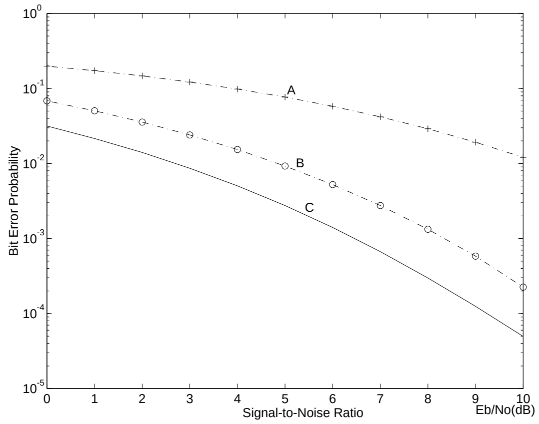

Fig. 5. Average system bit error probabilities obtained with the adaptive frequency hopping multi-carrier DS-CDMA system and diversity system using selection combining and maximal ratio combining techniques. $L_p = 3, T_d = 6\mu s, K = 64, M = 8$ and $N = 156$.