Multi-Carrier Orthogonal CDMA Signals for Quasi-Synchronous Communication Systems*

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Abstract

We propose a multi-carrier orthogonal CDMA signaling scheme for a multiple-access communication system, such as the reverse channel of a cellular network, as an alternative to the multi-user interference cancellation approach. The average variance of cross-correlations between sequences is used as a measure for sequence design. We search for sets of sequences that minimize the probability of symbol detection error, given that there is imperfect synchronization among the signals, that is, the signals are quasi-synchronous. Orthogonal sequences based on the Sylvester-type Hadamard matrices (Walsh functions) are shown to provide a significant improvement over the case where a Hadamard (orthogonal) matrix is chosen at random. Computer searches suggest that this set of codes is optimal with respect to the above measure. The issue of chip pulse shaping is investigated. Optimal pulses designed to minimize multiple-access interference in quasi-synchronous systems are obtained for various bandwidths and are shown to provide a large improvement over the raised cosine pulses. A multi-carrier signaling scheme is introduced in order to reduce chip level synchronization offsets between the users.

1 Introduction

Direct sequence spread spectrum transmission has received considerable attention for applications in mobile and personal communication networks as a result of its potential to provide higher spectral efficiencies in comparison to conventional modulation techniques [1, 2, 3]. The potential increase in capacity over conventional modulation schemes is a result of the cellular

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structure of these systems and not a result of the efficiency of direct-sequence code-division multiple-access (DS CDMA) as a multiplexing scheme. In fact, in a one-cell system the efficiency of direct-sequence CDMA is considerably lower than that of TDMA or FDMA [4]. To improve on the capacity of a CDMA cellular network beyond the estimates in [1], more specialized signal processing techniques may be used in order to reduce the interference from terminals in the same cell. These techniques attempt to cancel the interference that arises as a result of non-zero cross-correlation of the spreading codes [5]. Alternatively, we may attempt to synchronize the various transmissions at the chip level and to use orthogonal spreading codes, thus reducing the amount of multiple-access interference. This approach is used in the forward channel for the CDMA proposal discussed in [6]. However, its implementation in the reverse channel is more difficult, especially in the case of small chip periods. In an indoor environment the transmission delays are relatively small, hence, for low data rates and moderate processing gains, it may be feasible to maintain chip synchronization even in the reverse channel. This is the approach taken in the system proposed by Omura [3]. Assuming that there will be small errors in synchronization, orthogonal sequences with small cross-correlations for small synchronization offsets should be used. Some recent work that treats the problem of finding CDMA codes to reduce cross-interference is found in [7, 8, 9]. In [8] the 16×16 Sylvester matrix is found to minimize the mean square cross-correlation for one chip offsets. A band-limited quasi-synchronous CDMA system which uses raised cosine chip pulses is considered in [10].

We consider a one-cell system with one receiver base station and several mobile transmitters. While synchronization and orthogonality of signals can be maintained in the forward link, only quasi-synchronous operation can be maintained in the reverse link due to the differences in the transmitter clocks.

In this paper we analyze the performance of a quasi-synchronous DS CDMA system and discuss a set of orthogonal codes that attempt to minimize the probability of symbol error. Optimal chip pulses of various bandwidths that minimize multiple-access interference under quasi-synchronous operation are obtained. As a means to reduce the synchronization offsets we introduce a multi-carrier signaling scheme. With this scheme the chip period in each of the sub-carriers is increased thus reducing the synchronization offsets by a factor equal to the number of sub-carriers.

2 Single Carrier CDMA

We consider a CDMA system with N_u users. The received signal at the base station is

$$r(t) = \sum_{i=1}^{N_u} A_i s_i(t - \tau_i) \cos(\omega_0 t + \theta_i) + n(t) \tag{1}$$

where $s_i(t) = \sum_{k=-\infty}^{\infty} d^i_{\lfloor k/N \rfloor} \tilde{c}^i_k h(t - kT_c)$ is the spreading code signal modulated by the data sequence $d^i_l = \pm 1$, and $\tilde{c}^i_k$, A_i, θ_i, and τ_i are the spreading code sequence, amplitude, carrier phase, and delay of the i^{th} signal, respectively, and $n(t)$ is the background noise process which has power spectral density $N_0/2$. $\lfloor x \rfloor$ denotes the largest integer less than or equal to x, N is the processing gain, $h(t)$ is the chip pulse shape, T_c is the chip period, and $T_b = NT_c$ is the symbol period. To simplify notation we define a new data-modulated chipping sequence $c_k = d_{\lfloor k/N \rfloor}\tilde{c}_k$.

We assume a correlator receiver. The output of the correlator due to the k^{th} symbol is

$$y \approx \pm \frac{A_k E_s}{2} + \psi + \eta \tag{2}$$

where $E_s = \int_0^{T_b} s_k^2(t)dt$, ψ is the multiple-access interference, and η is a zero mean Gaussian random variable with variance $N_0 E_s/4$.

To determine the probability of symbol error we need the distribution of ψ which depends on the cross-correlation of the codes. Since we are assuming that the data symbols are equiprobable then by the central limit theorem ψ approaches a Gaussian random variable as the number of interferers contributing to ψ increases. We are interested in relatively large symbol error probabilities (e.g. 10^{-3}). In this case and for a moderate processing gain it has been shown that ψ can be approximated by a Gaussian distribution under quite general assumptions on the spreading codes (e.g. random codes, Gold codes). Under the Gaussian approximation the measure of interest is the variance of ψ, σ_ψ^2. We will use this measure to optimize the spreading code set. The variance of ψ is

$$\sigma_\psi^2 = \mathcal{E}\{\psi^2\} = \sum_{i=1, i\neq k}^{N_u} \frac{A_i^2}{8} \mathcal{E}\{R_{s_i s_k}^2(\tau_i)\} \tag{3}$$

where $\mathcal{E}\{\cdot\}$ denotes expectation over the data and code symbols and the relative delay τ_i, and $R_{s_i s_k}(\tau_i) = \int_0^{T_b} s_i(t-\tau_i)s_k(t)dt$.

The symbol error probability for the k^{th} user is given by (e.g. see [11] for similar result)

$$P_e = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_k}{N_0 + \gamma \sum_{\substack{i=1\\ i\neq k}}^{N_u} E_i}}\right) \tag{4}$$

where $E_i = A_i^2 \int_0^{T_b} s_i^2(t)dt/2 = A_i^2 E_s/2$ is the energy per symbol of the i^{th} user and

$$\gamma = \frac{\sum_{i=1,i\neq k}^{N_u} A_i^2 \mathcal{E}\{R_{s_i s_k}^2(\tau_i)\}}{2E_s \sum_{i=1,i\neq k}^{N_u} E_i} \tag{5}$$

is the normalized variance of the multiple-access interference.

In this paper we study the performance of a quasi-synchronous CDMA system where the reverse link transmissions from the different terminals arrive synchronized at the receiver base station to within a fraction of the chip period. We introduce the normalized offset $\Delta^i = \tau_i/T_c$ for the i^{th} user. We refer to this variable generically as Δ ($-1 < \Delta < 1$). For the different users we model Δ as a random variable with probability density function $f_\Delta(\cdot)$, which is nonzero in $[-\Delta_m, \Delta_m]$, $0 \leq \Delta_m < 1$.

For a synchronous system $\Delta_m = 0$ and, assuming orthogonal codes and an appropriate chip pulse, the bit error probability is given by (4) with $\gamma = 0$. For a quasi-synchronous system ($0 < \Delta_m < 1$) with rectangular chip pulse, random spreading codes, and Δ uniformly distributed on $[-\Delta_m, \Delta_m]$ we have $\gamma = (1 - \Delta_m + \frac{2}{3}\Delta_m^2)/N$; for $\Delta_m = 1$, $\gamma = \frac{2}{3N}$ and (4) becomes the well known result for asynchronous CDMA.

2.1 Orthogonal CDMA Codes

We assume that the spreading code period is equal to the symbol period and consider a rectangular chip pulse. The cross-correlation of the data-modulated code sequences for users i and k is

$$R_{s_i s_k}(\Delta T_c) \;=\; T_c \sum_{l=0}^{N-1} c_l^k((1-|\Delta|)c_l^i + |\Delta| c_{l\pm 1}^i). \tag{6}$$

Given a specific set of orthogonal sequences, we can assign each sequence as the spreading code of a user. The error probability for a given user will depend on the set of active users (up to a maximum equal to the number of orthogonal sequences), and on the chip offsets for the interfering users relative to the user of interest. In previous research the criterion for sequence set design has been based on the worst case cross-correlations (e.g. Gold codes). This criterion is applicable in the case of high SNR. The current trend, especially in mobile communications, is to use forward error correction codes and to design the system to operate at a relatively large symbol error rate (e.g. 10^{-3} and higher). In this case a criterion that is based on the mean square cross-correlation value is more appropriate.

Taking all users into account, we would like to use a set of sequences where the average of the squares of the cross-correlations over all pairs of sequences is minimized. If it turns out that the squared cross-correlations for the different users vary considerably with respect to the overall average then the performance of the different users can be made equal by having each mobile change its code cyclically over the whole set of codes such that no two mobiles use the same code simultaneously.

To minimize cross-interference, we seek an orthogonal set that minimizes $\mathcal{E}\{R^2_{s_i s_k}(\Delta T_c)\}$, where the expectation is taken over the random variable Δ and over the data-modulated spreading sequences from the orthogonal set (i.e. average over all pairs of codes in the set). We write

$$\mathcal{E}\{R^2_{s_i s_k}(\Delta T_c)\} = T_c^2 \mathcal{E}(\Delta^2)\mu_{cor}(N) \tag{7}$$

where

$$\mu_{cor}(N) = \mathcal{E}\left\{\left(\sum_{l=0}^{N-1} c_l^k c_{l+1}^i\right)^2\right\}. \tag{8}$$

To minimize (7) we need to minimize $\mu_{cor}(N)$. Given an $N \times N$ orthogonal matrix $C = [c_j^i]$ with $c_j^i \epsilon \{-1, 1\}$, (a Hadamard matrix) the rows are assigned as code sequences to a maximum of N users. The average of the squared cross-correlations of the sequences, under a one chip offset, is

$$\mu_{cor}(N) = \frac{1}{2}(\mu_{cor+} + \mu_{cor-}) \tag{9}$$

where

$$\mu_{cor+} = \frac{1}{N(N-1)} \sum_{k=1}^{N} \sum_{\substack{i=1 \\ i \neq k}}^{N} \left(\sum_{j=0}^{N-1} c_j^k c_{j+1}^i\right)^2 \tag{10}$$

and where $c_N^i = c_0^i$. We define μ_{cor-} the same as μ_{cor+} except that we set $c_N^i = -c_0^i$. Our problem is to find the Hadamard matrix C which minimizes (9).

For the variance of the multiple-access interference ψ to be minimized, (7) suggests that $\mu_{cor}(N)$ should be a minimum, regardless of the distribution for Δ. If we take Δ to be uniformly distributed on $[-\Delta_m, \Delta_m]$ the symbol error probability of the resulting quasi-synchronous CDMA system can then be obtained by substituting

$$\gamma = \frac{\Delta_m^2 \mu_{cor}(N)}{3N^2} \tag{11}$$

into (4), hence P_e is minimized when $\mu_{cor}(N)$ is minimized.

The bit error probability given by (4) is not the average bit error rate for all users. We interpret (4) as the bit error rate associated with a typical user subjected to an averaged cross-interference noise.

We wish to find Hadamard matrices C which minimize $\mu_{cor}(N)$. We restrict the search to the case $N = 2^n$. We note that interchanging and negating the rows of a Hadamard matrix produces an equivalent Hadamard matrix with the same $\mu_{cor}(N)$ value; non-equivalent matrices are referred to as distinct. If N is small, then it is a simple task to find all distinct $N \times N$ Hadamard matrices and to compute $\mu_{cor}(N)$ for each. However, for large N the set of distinct Hadamard matrices is unknown [12]. We have performed a search for Hadamard matrices with low values of $\mu_{cor}(N)$. In all cases of N considered, the best Hadamard matrices found were equivalent to the $N \times N$ Sylvester-type Hadamard matrix which is defined recursively as follows,

$$\mathbf{H}_N = \begin{bmatrix} \mathbf{H}_{N/2} & \mathbf{H}_{N/2} \\ \mathbf{H}_{N/2} & -\mathbf{H}_{N/2} \end{bmatrix} \tag{12}$$

where $\mathbf{H}_1 = 1$.

For the case $N = 4$, there exist two distinct Hadamard matrices, and the smallest correlation is $\mu^*_{cor} = 10/3$. For the case $N = 8$ an exhaustive search over all Hadamard matrices was performed and we obtained $\mu^*_{cor} = 6$ corresponding to the 8×8 Sylvester matrix. For higher values of N, Hadamard matrices were generated randomly by interchanging and negating the columns of a given Hadamard matrix. This process yields a large portion of the distinct matrices but not all of them in general [12, p. 48]. We generated 100,000 matrices for $N = 16$, 100,000 for $N = 32$, 10,000 for $N = 64$, 4,000 for $N = 128$, and 500 for $N = 256$. The histograms for the μ_{cor} values for two cases of N are shown in Figs. 1 – 2. In all cases the μ_{cor} values have a distribution concentrated around N. The spread of the distributions decreases, relative to N, as N increases except for an isolated point (corresponding to the Sylvester matrix) at a value of approximately $\frac{2}{3}N$. The exact value, derived in the Appendix, is as follows

$$\mu_{cor}(N) = \frac{2}{3}(N+1). \tag{13}$$

2.2 The Gaussian Approximation

Given that the spreading sequences are the rows of a Hadamard matrix, it is of interest to verify the Gaussian approximation in computing the probability of bit error (4) which was

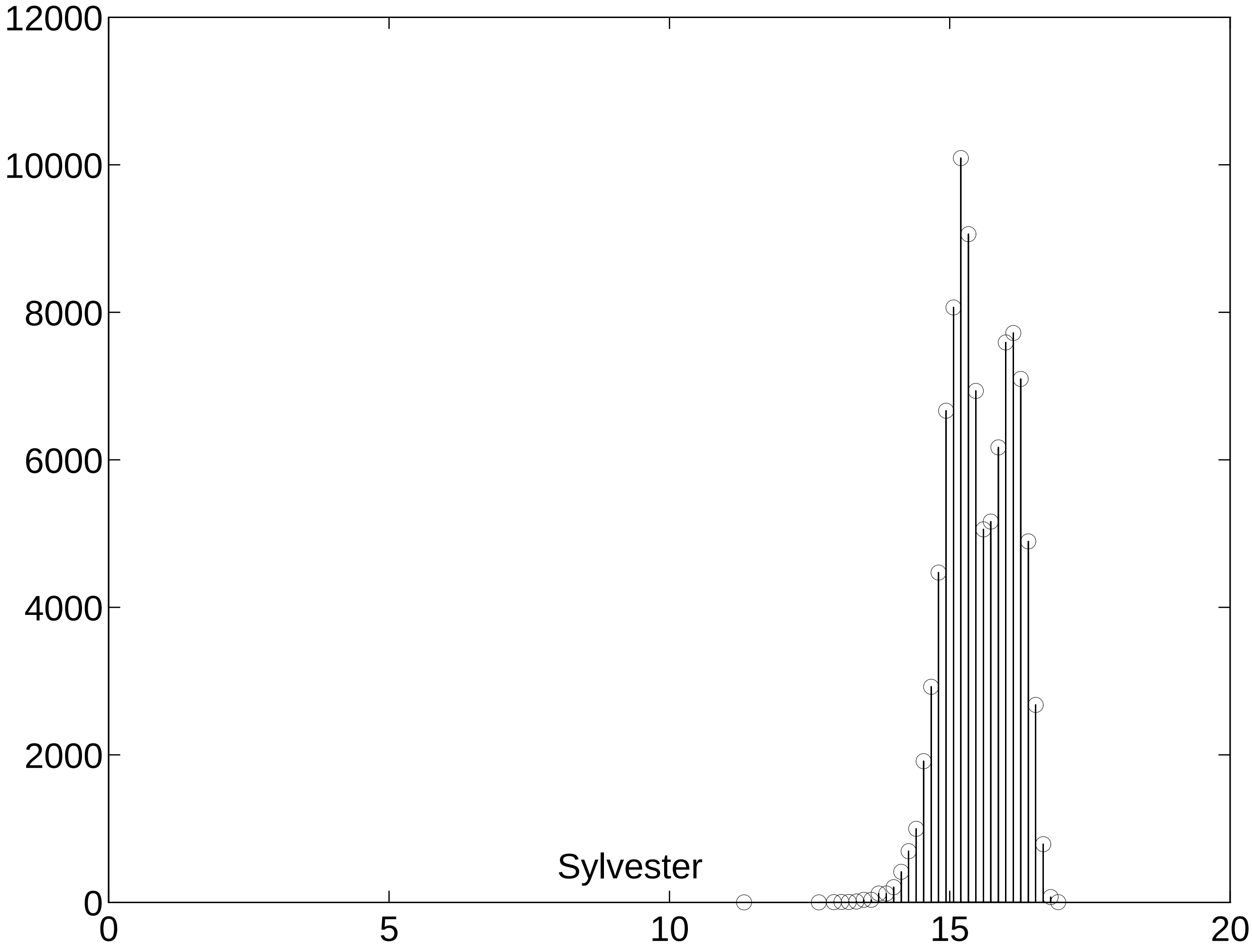

Figure 1: Number of occurrences of μ_{cor} versus μ_{cor} for the case $N = 16$. 100,000 randomly generated orthogonal matrices were investigated.

obtained under the assumption of random sequences. We have performed an exact computation for the case $N_u = N = 8$ assuming that Δ is uniformly distributed on $[-\Delta_m, \Delta_m]$, the signals are baseband signals, and that all users have the same bit energy $E_i = E_b$. It turns out that the probability of error is code dependent. There are four classes of spreading codes, with two codes per class, and each class has a different probability of error as shown in Fig. 3, for the case $\Delta_m = .5$. In Fig. 4b, we show the average probability of bit error over the four classes of codes. By using the Gaussian approximation for each class of codes we obtain error probability expressions given by (4) with four corresponding γ's; Fig. 4a shows the average of these. The curve obtained via the Gaussian approximation is pessimistic at an error probability of 10^{-3} by approximately $E_b/N_0 = 1.5$ dB. The approximation should be considerably better for larger values of N. We can modify the signaling scheme by having each user change the spreading code after each bit (cyclically through all codes) so that all users would have the performance shown in Fig. 4.

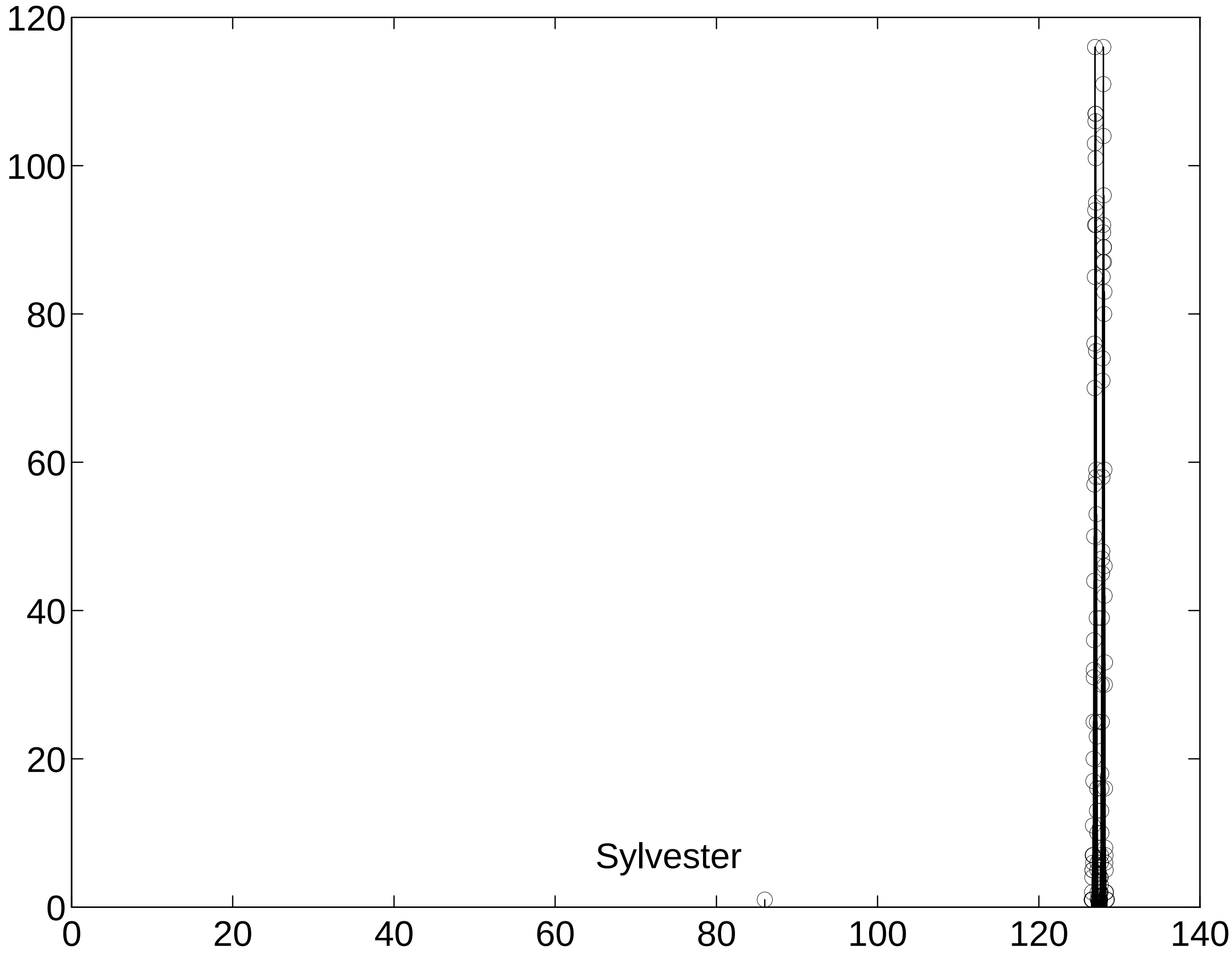

Figure 2: Number of occurrences of μ_{cor} versus μ_{cor} for the case $N = 128$. 4,000 randomly generated orthogonal matrices were investigated.

Assuming large N, and substituting $\mu_{cor}(N) = 2N/3$ into (11), we obtain $\gamma = 2\Delta_m^2/(9N)$ for the $N \times N$ Sylvester matrix. For a typical non-Sylvester matrix $\mu_{cor} \approx N$ and $\gamma = \Delta_m^2/(3N)$; thus the multiple access interference is reduced by a factor of 3/2 in the case of a Sylvester matrix. Fig. 5 and Fig. 6 show plots of the bit error probability given by (4) (assuming $E_i = E_k = E_b$), for typical users (i.e. using the average squared cross-correlation values), for a) random sequences, b) typical non-Sylvester orthogonal sequences, and c) Sylvester sequences.

The improvement in performance of the Sylvester sequences over typical non-Sylvester sequences becomes more noticeable as Δ_m increases. The difference in the maximum number of simultaneous users for Sylvester and non-Sylvester Hadamard sequences, for a given probability of error, depends on the level of background noise N_0. Given a maximum operating probability of error, (4) may be solved for the maximum number of simultaneous users. Quasi-synchronous CDMA systems based on Sylvester sequences and non-Sylvester

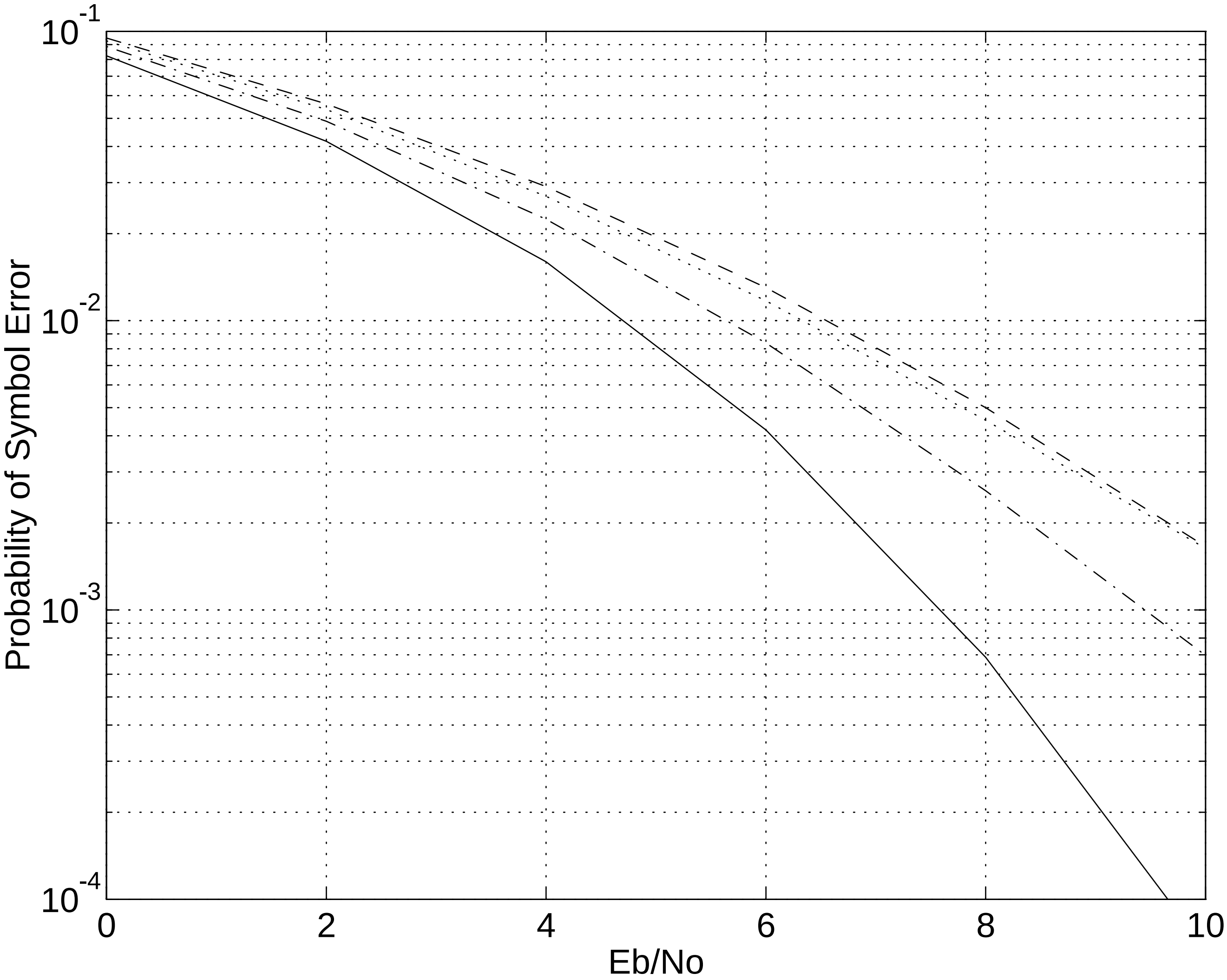

Figure 3: Exact probability of symbol error versus $\frac{E_b}{N_0}$ for a system using the 8×8 Sylvester matrix, $N_u = 8, \Delta_m = .5$.

Hadamard sequences may then be compared in terms of capacity, whereas Figs. 5 and 6 compare them in terms of SNR.

2.3 General Chip Pulse Shape

We now consider the case of a non-rectangular chip pulse shape. Let $g(t) = \int_{-\infty}^{\infty} h(u)h(u-t)du$. With perfect synchronization, orthogonality of signals can be maintained by using orthogonal spreading codes and a chip pulse which satisfies the Nyquist I criterion,

$$g(lT_c) = \begin{cases} E_h, & l = 0 \\ 0, & l = \pm 1, \pm 2, \pm 3 \ldots \end{cases} \tag{14}$$

Let us consider the case where the users are quasi-synchronous. The pulse that minimizes the cross-interference, will not necessarily satisfy (14). Consider a pair of baseband CDMA

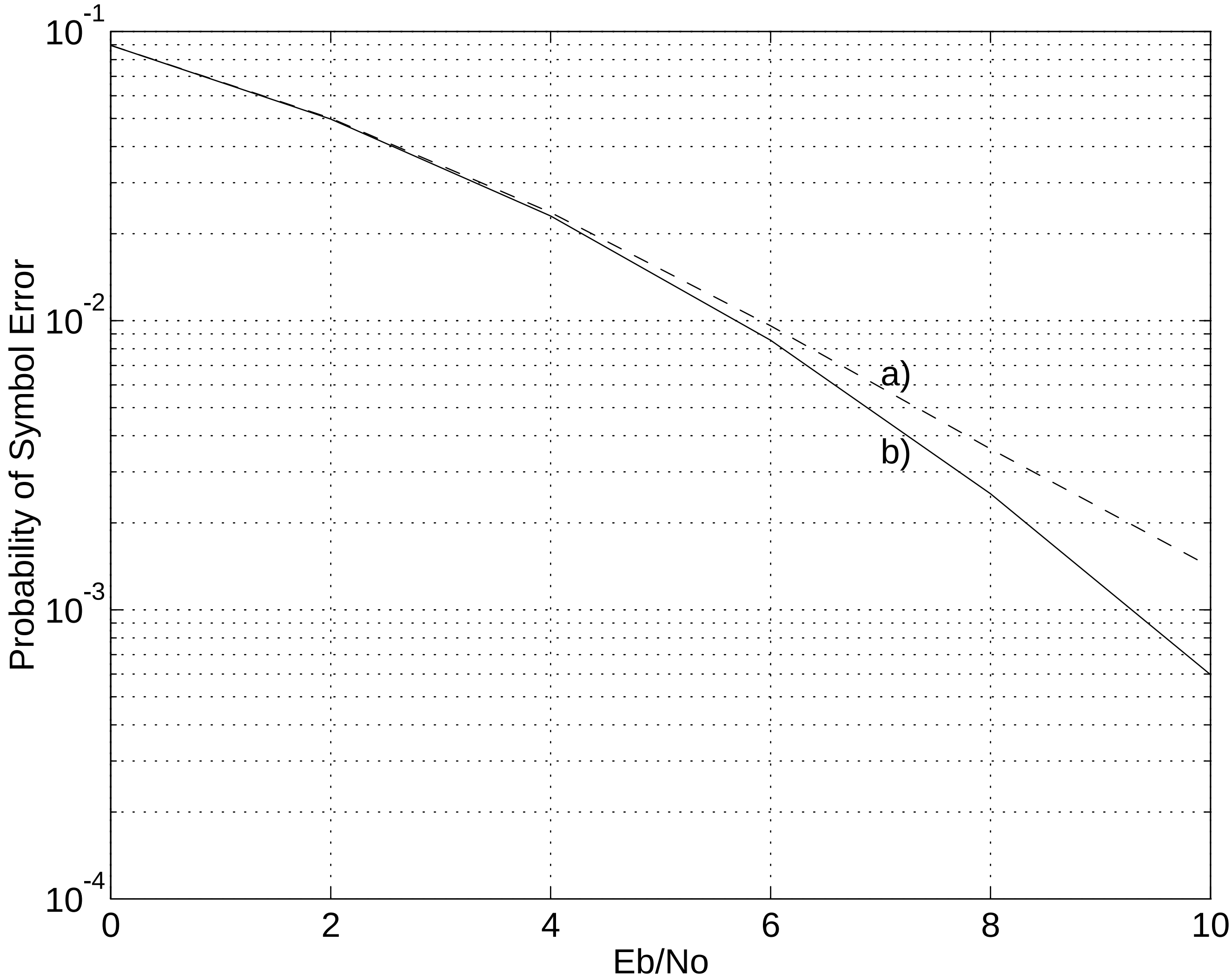

Figure 4: Probability of symbol error versus $\frac{E_b}{N_0}$, averaged over the users for a system using the 8×8 Sylvester matrix, $N_u = 8$, $\Delta_m = .5$: a) Gaussian approximation, b) exact computation.

signals, $s_i(t+\tau) = \sum_{m=-\infty}^{\infty} c_m^i h(t+\tau-mT_c)$ and $s_k(t) = \sum_{n=-\infty}^{\infty} c_n^k h(t-nT_c)$, where $-T_c < \tau < T_c$. The design problem is as follows: for a given chip pulse bandwidth and p.d.f. for the synchronization offset, find the optimum set of orthogonal sequences and optimum chip pulse shape. The cost function to be minimized is the average of the squared cross-correlations for all pairs of signals. The cross-correlation between two signals is

$$\begin{aligned}
\psi &= \int_0^{T_b} s_i(t+\tau)s_k(t)dt \\
&\approx \sum_{n=0}^{N-1} \sum_{\substack{m=-\infty \\ m\neq n}}^{\infty} c_m^i c_n^k \int_{-\infty}^{\infty} h(t+\tau-mT_c)h(t-nT_c)dt \qquad (15) \\
&= \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \sum_{n=0}^{N-1} c_n^k c_{n+l}^i g(lT_c-\tau). \qquad (16)
\end{aligned}$$

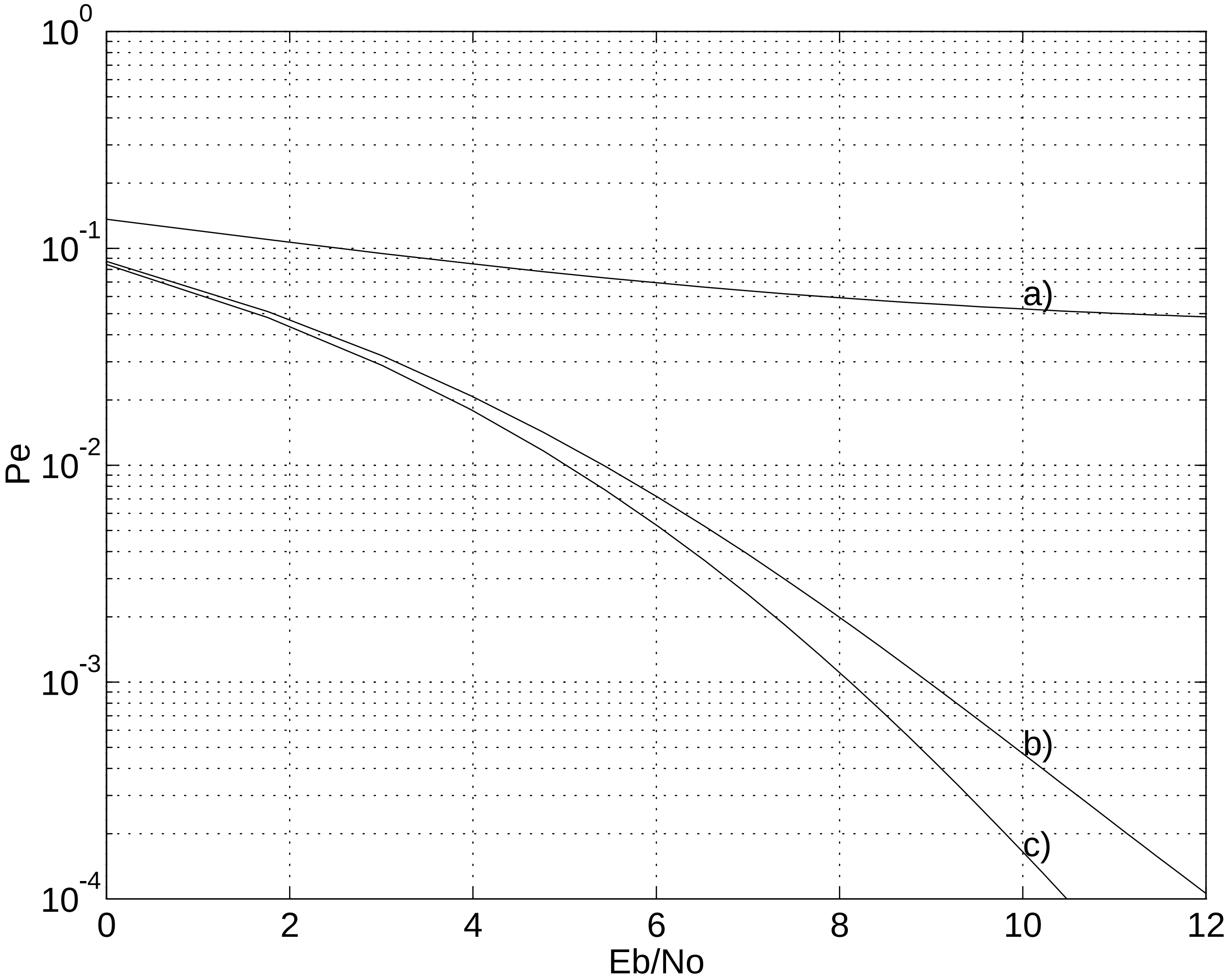

Figure 5: Error probability P_e versus $\frac{E_b}{N_0}$ for a typical user in the case of a) non-orthogonal sequences, b) non-Sylvester orthogonal sequences, and c) Sylvester sequences. $N_u = N = 128$, $\Delta_m = .5$.

where in the second step we have assumed that $h(t)$ contains most of its energy in the interval $t\epsilon[0, T_c]$, that N is large, and that the codes are orthogonal.

The variance of ψ may be computed as follows,

$$\sigma_\psi^2 = \mathcal{E}\{\psi^2\} \approx \sum_{\substack{l=-\infty \\ l\neq 0}}^{\infty} \mathcal{E}\left\{\left(\sum_{n=0}^{N-1} c_n^k c_{n+l}^i\right)^2\right\} \mathcal{E}\{g^2(lT_c - \tau)\} \tag{17}$$

where $c_{N+j}^i = \pm c_j^i$ depending on whether or not consecutive data symbols change sign.

Define

$$\mu_l = \frac{1}{2}(\mu_{l+} + \mu_{l-}) \tag{18}$$

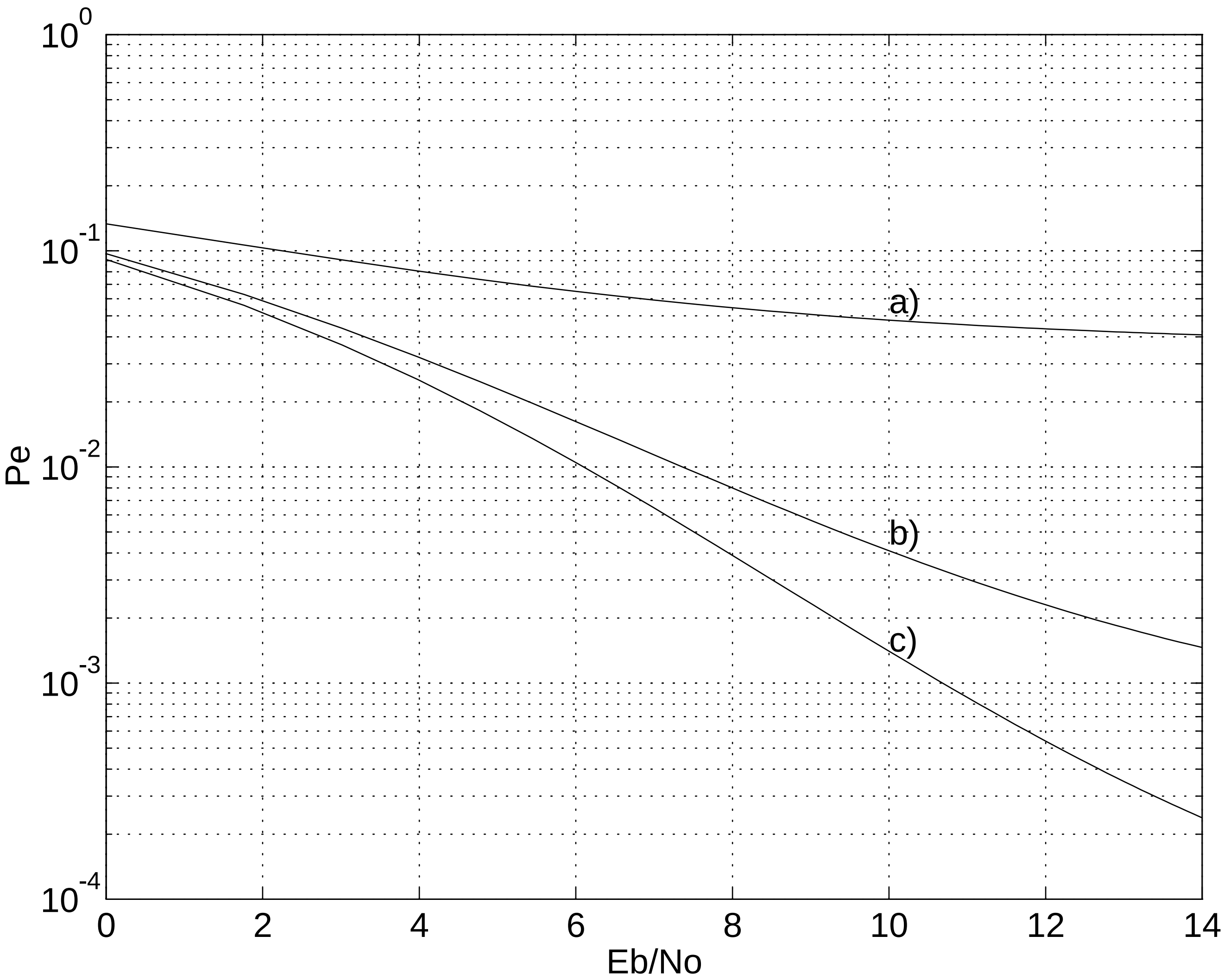

Figure 6: Error probability P_e versus $\frac{E_b}{N_0}$ for a typical user in the case of a) non-orthogonal sequences, b) non-Sylvester orthogonal sequences, and c) Sylvester sequences. $N_u = N = 128$, $\Delta_m = .75$.

where

$$\mu_{l+} = \mathcal{E}\left\{ \left(\sum_{n=0}^{N-1} c_n^k c_{n+l}^i \right)^2 \right\} = \frac{1}{N(N-1)} \sum_{k=1}^{N} \sum_{\substack{i=1 \\ i \neq k}}^{N} \left(\sum_{j=0}^{N-1} c_j^k c_{j+l}^i \right)^2 \tag{19}$$

and $c_{N+j}^i = c_j^i$, and μ_{l-} is defined similarly except that $c_{N+j}^i = -c_j^i$. Note that (18) with $l = 1$ is $\mu_{cor}(N)$ in (9). We note that $\mu_l {=} \mu_{-l}$ since the expectation is taken over all pairs of rows of an orthogonal matrix. Since $g(\cdot)$ and $f_\Delta(\cdot)$ are even functions, we rewrite σ_ψ^2 as

$$\sigma_\psi^2 = 2 \sum_{l=1}^{\infty} \mu_l \mathcal{E}\{g^2(lT_c - \tau)\}. \tag{20}$$

For a given bandwidth of $h(t)$ and a given processing gain $N = 2^n$, we wish to find the optimal set of orthogonal sequences and the optimal pulse shape such that (20) is minimized. Since the function $g^2(t)$ decays rapidly with increasing t we expect that the first few terms in the sum (20) dominate; hence we choose an orthogonal set of sequences with low μ_l for small l. For the Sylvester sequences $\mu_l = 2N/3$ for $l = 1, 2$, compared to $\mu_l = N$ for typical orthogonal sequences, hence we conclude that these are close to optimal and choose them. We have thus decoupled the problems for finding the optimum sequences and the optimum pulse shape.

To solve for the optimum $g(t)$ we minimize (20) subject to a constraint on the bandwidth and the energy of the pulse $\int_{-\infty}^{\infty} h^2(t)dt = E_h = g(0)$. For small synchronization offsets we present an analytical solution. For larger offsets, we solve the problem using a numerical approach. Note that the optimum pulse will not necessarily be a Nyquist I pulse.

For small synchronization offsets we can use a two-term Taylor series expansion of $g(t)$ at lT_c to obtain $g(lT_c - \tau) \approx g(lT_c) - \tau g'(lT_c)$. Squaring, noting that the mean of τ is 0, computing the expectation over the random variable τ, and summing over l we express the cost function (20) as

$$\sigma_\psi^2 = 2\sum_{l=1}^{\infty} \mu_l (g^2(lT_c) + [g'(lT_c)]^2 \sigma_\tau^2) \tag{21}$$

where σ_τ^2 is the variance of τ. If we approximate the μ_l's by a constant then we obtain a cost function that is similar to a cost function obtained in [13] for an unrelated problem. From [13] the following pulse minimizes (21), if we assume that μ_l is a constant and that σ_τ^2 is small:

$$G(f) \approx \begin{cases} c, & |f| < \frac{1-\beta}{2T_c} \\ c(1 - |f|T_c), & \frac{1-\beta}{2T_c} \leq |f| \leq \frac{1+\beta}{2T_c} \\ 0, & |f| > \frac{1+\beta}{2T_c} \end{cases} \tag{22}$$

where c is a constant such that $\int_{-\infty}^{\infty} G(f)df = E_h$, and β is the fractional *excess bandwidth*.

Even if we make the approximation that μ_l is a constant, for larger synchronization offsets the above Taylor series method cannot be used. In this case since $g(t)$ is band-limited we expand it as a sum of weighted sinc$(\cdot)$ functions

$$g(t) = \sum_{k=0}^{L} x_k \phi_k(t) \tag{23}$$

where

$$\phi_k(t) = \text{sinc}\left(\frac{(1+\beta)t}{T_c} - k\right) + \text{sinc}\left(\frac{(1+\beta)t}{T_c} + k\right) \tag{24}$$

and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$. The bandwidth of $g(t)$ is $\frac{1+\beta}{2T_c}$. Substituting (23) into (20), and defining $f_{ijl} = \mathcal{E}\{\phi_i(\tau - lT_c)\phi_j(\tau - lT_c)\}$, we obtain the functional

$$F = \sigma_\psi^2 = 2\sum_{l=1}^{\infty} \mu_l \left(\sum_{i=0}^{L}\sum_{j=0}^{L} x_i x_j f_{ijl}\right) \tag{25}$$

to be minimized subject to the constraint $\sum_{k=0}^{L} x_k \phi_k(0) = E_h$.

The solution, based on Lagrange multipliers, reduces to the solution for the linear system

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{26}$$

where

$$\mathbf{A} = \begin{bmatrix} \sum_{l=1}^{\infty}\mu_l f_{00l} & \sum_{l=1}^{\infty}\mu_l f_{01l} & \sum_{l=1}^{\infty}\mu_l f_{02l} & \cdots & \sum_{l=1}^{\infty}\mu_l f_{0Ll} & \frac{1}{2} \\ \sum_{l=1}^{\infty}\mu_l f_{10l} & \sum_{l=1}^{\infty}\mu_l f_{11l} & \sum_{l=1}^{\infty}\mu_l f_{12l} & \cdots & \sum_{l=1}^{\infty}\mu_l f_{1Ll} & 0 \\ \sum_{l=1}^{\infty}\mu_l f_{20l} & \sum_{l=1}^{\infty}\mu_l f_{21l} & \sum_{l=1}^{\infty}\mu_l f_{22l} & \cdots & \sum_{l=1}^{\infty}\mu_l f_{2Ll} & 0 \\ \vdots & & & & & \\ \sum_{l=1}^{\infty}\mu_l f_{L0l} & \sum_{l=1}^{\infty}\mu_l f_{L1l} & \sum_{l=1}^{\infty}\mu_l f_{L2l} & \cdots & \sum_{l=1}^{\infty}\mu_l f_{LLl} & 0 \\ \frac{1}{2} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$

$$\mathbf{x}^{\mathrm{T}} = [x_0, x_1, x_2, x_3, \ldots, x_L, \lambda], \quad \mathbf{b}^{\mathrm{T}} = [0, 0, 0, 0, \ldots, 0, \frac{E_h}{4}],$$

and λ is a Lagrange multiplier.

2.4 Optimum Chip Pulse Shape

If the system attempts to maintain the signals synchronized we expect that the probability density function of Δ should have a maximum at zero. As an example, we solve the optimal chip pulse problem for the following density function.

$$f_\Delta(\Delta) = \begin{cases} \left(1 - \frac{|\Delta|}{\Delta_m}\right)\frac{1}{\Delta_m}, & |\Delta| \le \Delta_m \\ 0, & |\Delta| > \Delta_m \end{cases} \tag{27}$$

We discuss solutions to (26) for various values of Δ_m and fractional excess bandwidths β. In the frequency domain, the optimal pulse has the form

$$G(f) = \begin{cases} \frac{2T_c}{1+\beta}\sum_{k=0}^{L} x_k \cos(\frac{T_c 2\pi k f}{1+\beta}), & |f| \le \frac{1+\beta}{2T_c} \\ 0, & |f| > \frac{1+\beta}{2T_c} \end{cases} \tag{28}$$

Figs. 7 – 8 show $G(f)$ for two different bandwidths, and for each bandwidth, the three cases $\Delta_m = .7$, $.5$, and $.3$. We have used $L = 15$. Also, note that the results for $\Delta_m = .3$ and all the β values resemble the analytical solution given by (22). The values used for μ_l correspond to the limiting values (large N) for Sylvester sequences, and are as follows,

$$\begin{aligned} (\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6, \mu_7, \mu_8, \mu_8, \mu_9) &= N(\frac{2}{3}, \frac{2}{3}, \frac{5}{6}, \frac{2}{3}, \frac{7}{8}, \frac{5}{6}, \frac{7}{8}, \frac{2}{3}, \frac{7}{8}) \\ \mu_l &= N, \quad \text{for} \quad l \ge 10 \end{aligned} \tag{29}$$

The value for $l \ge 10$ is an approximation.

We now compare the performance of the optimal chip pulses versus the class of square root raised cosine pulses

$$G(f) = \begin{cases} E_h T_c, & |f| < \frac{1-\beta}{2T_c} \\ \frac{E_h T_c}{2}\left(1 + \cos\left[\frac{\pi T_c}{\beta}(|f| - \frac{1-\beta}{2T_c})\right]\right), & \frac{1-\beta}{2T_c} \le |f| \le \frac{1+\beta}{2T_c} \\ 0, & |f| > \frac{1+\beta}{2T_c}. \end{cases} \tag{30}$$

Table 1 lists the normalized costs $\sigma_\psi^2/(E_h^2 N)$ associated with the optimal pulses and the square root raised cosine pulses for six different bandwidths for the case $\Delta_m = .7$. We note

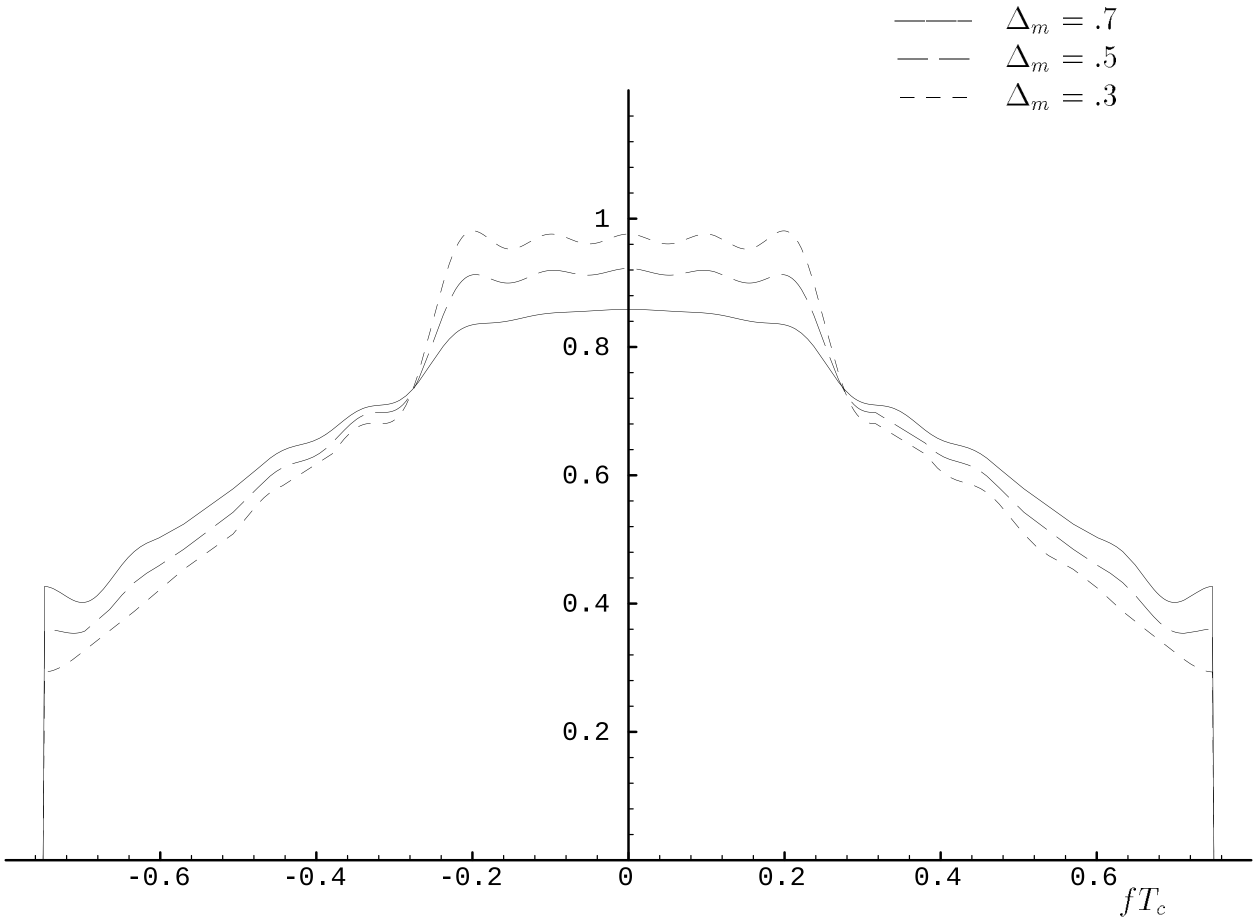

Figure 7: Optimum chip pulse, $\frac{G(f)}{E_h T_c}$, for $\beta = .5$.

that the improvement of the optimal pulse over the raised cosine pulse becomes significant as the bandwidth is increased.

We now consider the implications of Table 1 on the performance. Considering a CDMA system with N_u users, we can show that (5) reduces to

$$\gamma \;=\; \frac{F\sum_{\substack{i=1\\i\neq k}}^{N_u} A_i^2}{2E_s\sum_{\substack{i=1\\i\neq k}}^{N_u} E_i}. \tag{31}$$

Using the approximation $E_i = A_i^2\int_0^{T_b} s_i^2(t)dt/2 \approx A_i^2 N E_h/2$, that is, the bit energy is N times the chip energy, (31) reduces to,

$$\gamma \;=\; \frac{F}{N^2 E_h^2} \tag{32}$$

which is substituted into (4) to obtain the probability of error. In Fig. 9 we have plotted the probability of symbol error given by (4) for different pulse shapes and code sequences assuming Δ_m=.7 and β = .5. Three combinations are considered: a) randomly chosen

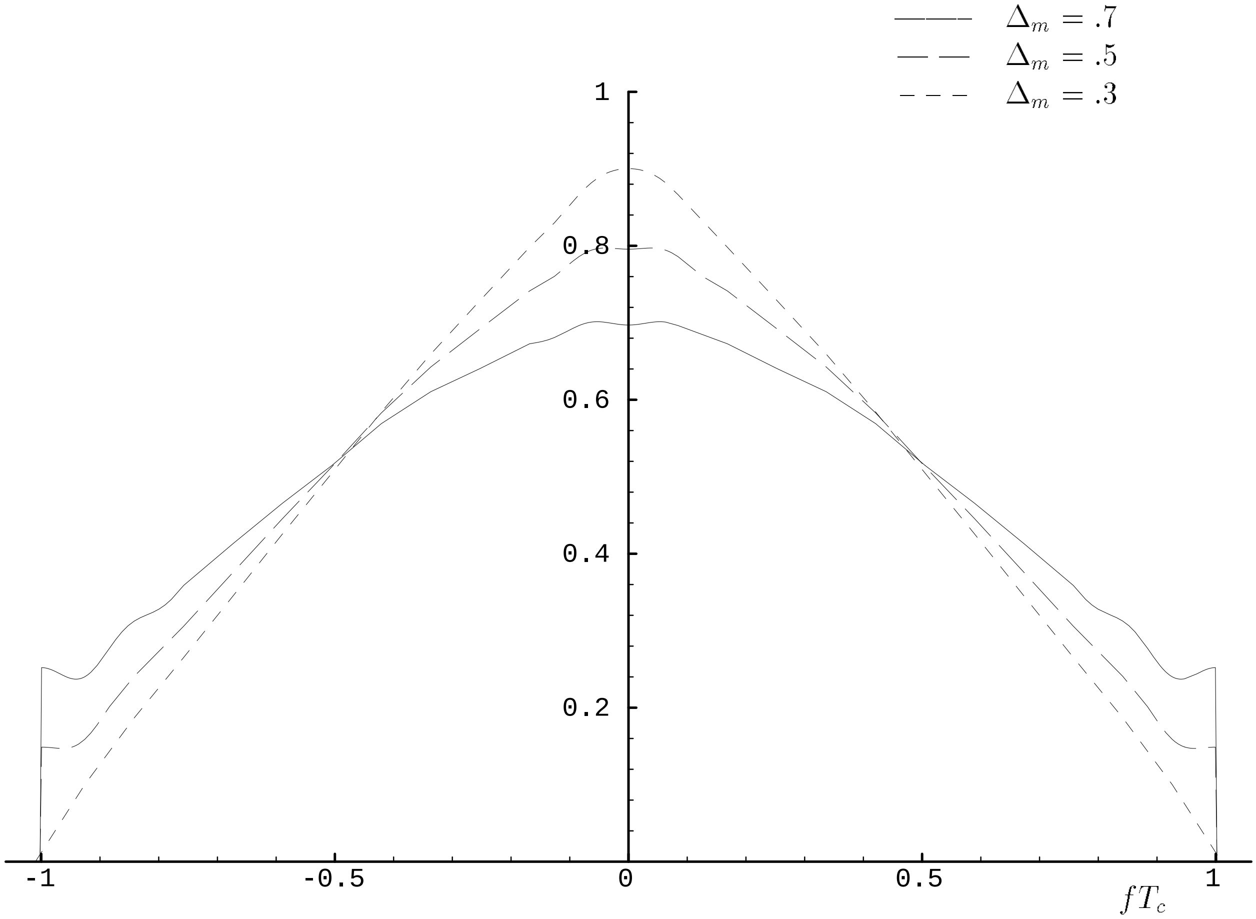

Figure 8: Optimum chip pulse, $\frac{G(f)}{E_h T_c}$, for $\beta = 1$.

orthogonal sequences and square root raised cosine chip pulses, b) Sylvester sequences and square root raised cosine pulses, and c) Sylvester sequences and optimal chip pulses. The b) and c) curves are computed using the two corresponding γ's computed from Table 1. The a) curve is computed assuming $\mu_l = N, l \geq 1$. At a probability of error of 10^{-3} there is a difference of approximately 3 dB between the curve corresponding to the Sylvester sequences and optimum chip pulse shape and that corresponding to a typical set of orthogonal sequences with square root raised cosine chip pulse shape.

3 Multi-Carrier CDMA

To reduce the probability of bit error the synchronization offsets should be a small fraction of the chip period. Increasing the chip period would make it easier to synchronize the signals. However, for a given processing gain, an increase in the chip period results in an increase in the symbol period, hence a decrease in the data rate. We can maintain the original data rate by using a multi-carrier signaling scheme. The overall transmitted signal is comprised of several different bit steams, each modulated by a separate sub-carrier.

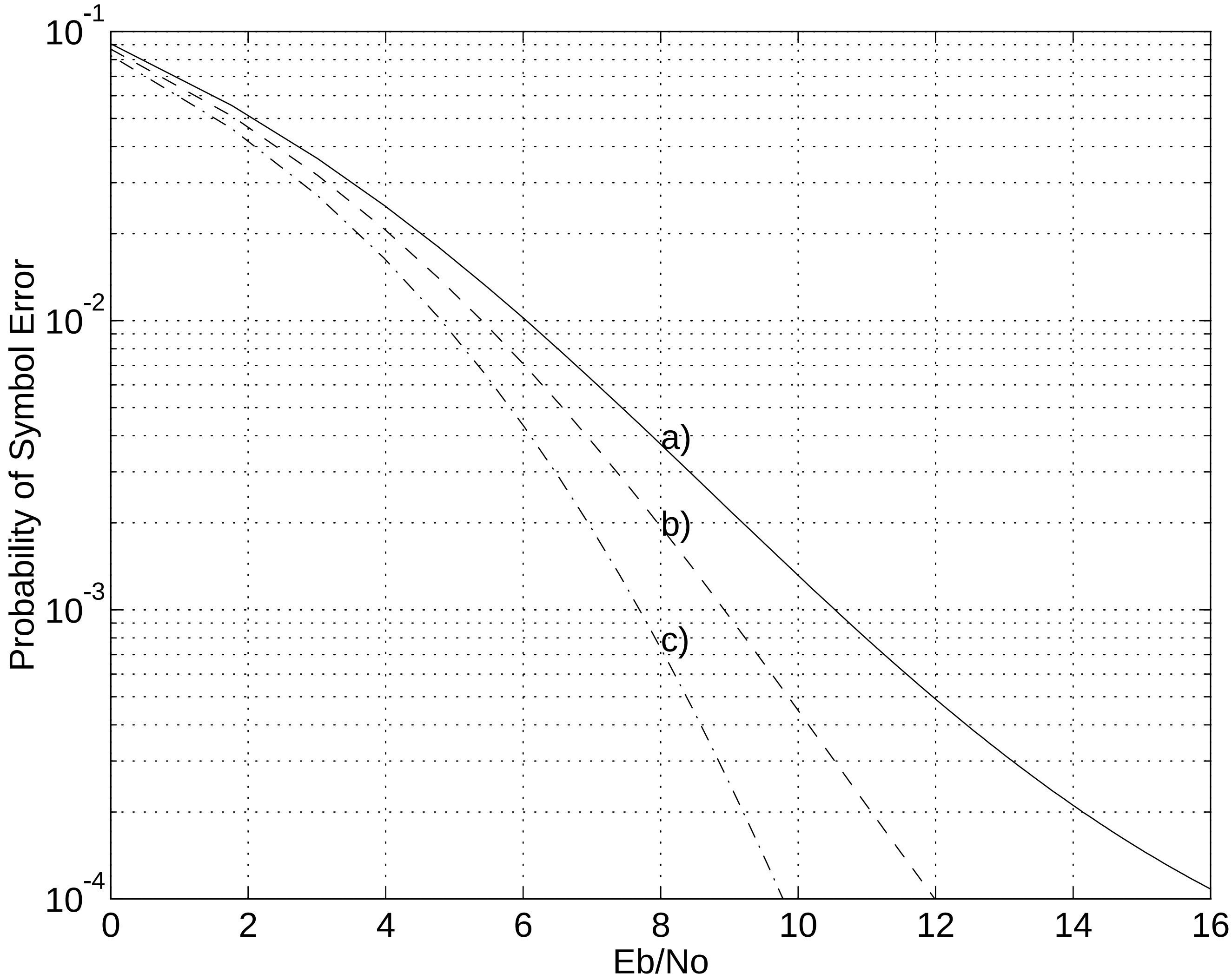

Figure 9: Probability of symbol error for different pulse shapes and orthogonal sequences; $\Delta_m = .7$, $\beta = .5$, and $N_u = N = 128$: a) raised cosine pulse and non-Sylvester Hadamard matrix, b) raised cosine pulse and Sylvester matrix, and c) optimum pulse and Sylvester matrix

Increasing the chip period by a factor of M, the number of sub-carriers, reduces the bandwidth of each of the sub-band signals by a factor of M relative to the bandwidth of the original spread spectrum signal so that the overall bandwidth is approximately the same as that of the single-carrier signal, assuming a spacing of $1/T_c$ for the carrier frequencies. Fig. 10 shows the block diagram of the transmitter for the k^{th} user of a 3-carrier DS CDMA system.

The received signal is

$$r(t) = \sum_{i=1}^{N_u} \sum_{j=1}^{M} A_i s_i^j(t - \tau_i) \cos(\omega_j t + \theta_i) \; + \; n(t) \tag{33}$$

where $s_i^j(t) = \sum_n c_n^{ij} h(t - nT_c)$, and M is the number of different sub-carriers. To compute the probability of bit error for a given data stream (sub-carrier) we need to consider the

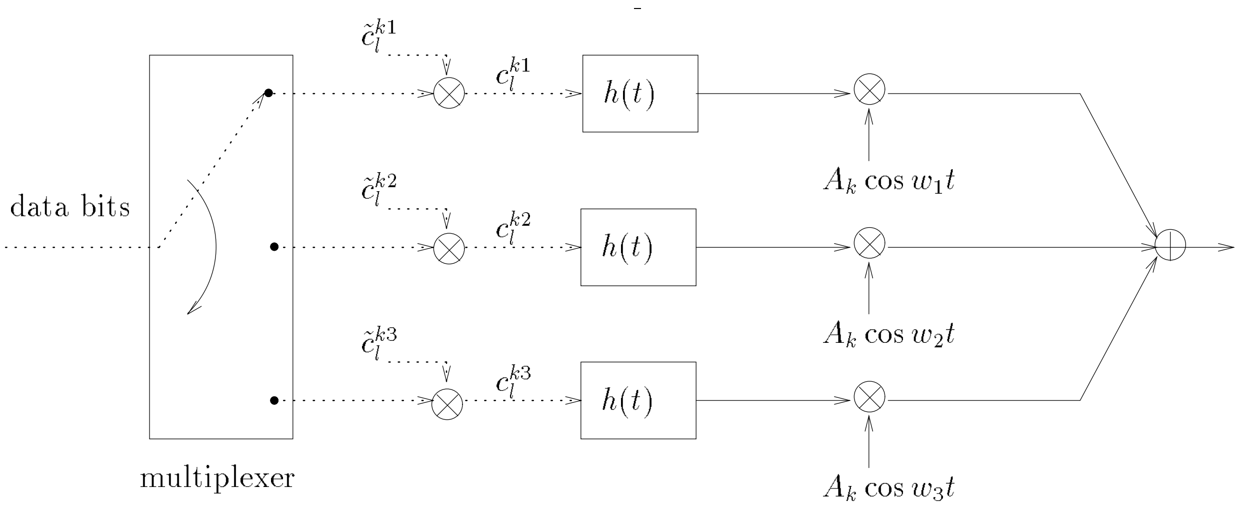

Figure 10: Block diagram of a 3-carrier DS CDMA transmitter.

interference from other data streams with the same carrier and interference from data streams using other carriers. For arbitrary carrier spacings the approach in [14] may be used to determine the variance of the multiple-access noise. For carrier spacings equal to the chip rate the noise variance can be solved using a more direct approach. If carriers are spaced at multiples of the chip rate, and if rectangular chip pulses are used, then all of the bit streams belonging to a single user do not interfere with one another due to the orthogonality (in time) of the carriers. For ease of implementation each bit stream corresponding to a single transmitter may be multiplied by the same spreading code. However in general the spreading codes for the sub-carriers may be chosen so as to minimize the peak amplitude of the signal.

3.1 Time-Limited Chip Pulses

First we consider the case of a rectangular chip pulse $h(t)$. We assume that the receiver demodulates the data in each stream independently. Assuming two users and reception of a particular bit stream, the variance of the out-of-band multiple-access interference at the integrator output can be shown to be $\frac{T_c^2}{(16\pi^2 i^2)}\mu_{cor}(N)[1 - \text{sinc}(2\Delta_m i)]$, where the frequency separation is $1/T_c$ Hz. This compares to the value of $T_c^2 \mu_{cor}(N)\Delta_m^2/24$ for case where the two signals have the same carrier. For this multi-carrier system we can show that the γ for the k^{th} user's m^{th} bit stream, corresponding to that in (4), is

$$\gamma = \frac{\mu_{cor}(N)}{N^2}\left(\frac{\Delta_m^2}{3} + \frac{1}{2\pi^2}\sum_{\substack{j=1 \\ j\neq m}}^{M} \nu_j\right) \tag{34}$$

where $\nu_j = (1 - \text{sinc}[2\Delta_m(m-j)])\,\frac{1}{(m-j)^2}$.

The question arises as to what value of M should be used. If chip pulses are not band-limited, we expect that as we increase the number of carriers, the in-band noise will decrease (since the chip period is increased by a factor of M) whereas the total interference from the other sub-bands will rise; hence there is a trade-off between interference from in-band signals and inter-sub-carrier (or out-of-band) interference. Also, the larger the M the greater the complexity of the system. To observe the effect of M on the interference, in Fig. 11 we have plotted the in-band interference, out-of-band interference, and the total interference for the case of a rectangular chip pulse, where the in-band and out-of-band interference are given by the first and second terms of (34) (after expanding), respectively. The curves have been suitably normalized, and plotted as a function of M where M is odd, and assuming $m = \lfloor \frac{M}{2} \rfloor + 1$, that is, the m^{th} band is the center band. To account for the increased chip period we have replaced Δ_m by $\frac{\Delta_m}{M}$ in (34) and have assumed $\Delta_m = 1$. As M increases the in-band interference decreases monotonically; on the other hand, the out-of-band interference increases to a maximum value and then decreases. The decrease in the out-of-band interference is a result of the fact that as the fractional synchronization offset decreases the sub-carrier signals become orthogonal, even though the number of interfering bands increases. In this particular example a good choice for M is approximately 5 since there is a negligible reduction in the total interference for larger M.

3.2 Band-Limited Chip Pulses

Let us now assume that the chip pulse is not rectangular and that it is band-limited. Consider a pair of passband CDMA signals, belonging to two different users, and corresponding to different sub-carrier frequencies, $s_i^j(t+\tau_i)\cos(w_j t + \theta_i)$ and $s_k^m(t+\tau_k)\cos(w_m t + \theta_k)$. We will generalize to the case of N_u users later. Assuming the base station correlator is receiving $s_k^m(t)$, the variance of the multiple-access interference at the integrator output is

$$\sigma_M^2 = \mathcal{E}\{\psi^2\} = \frac{1}{4} \sum_{\substack{l=-\infty \\ l \neq 0}}^{\infty} \mu_l \mathcal{E}\left\{ \left(\int_{-\infty}^{\infty} h(t+\tau-lT_c) h(t) \cos(w_s t + \theta) dt \right)^2 \right\}. \tag{35}$$

The expectation in (35) is with respect to θ, modeled as uniform on $[0, 2\pi]$, and τ.

Assuming that $h(t)$ has bandwidth equal to $\frac{1+\beta}{2T_c}$, $0 \leq \beta \leq 1$, we are interested only in the case where $w_s = \frac{2\pi}{T_c}$ since the cross-correlation is 0 for $w_s = \frac{2\pi k}{T_c}, k > 1$. Assuming $f_\Delta(\cdot)$ and $h(\cdot)$ are even, it can be shown that each term in the summation in (35) is even with respect to l. Using Parseval's relation, the above can be put into the following frequency domain form

$$\sigma_M^2 = \frac{1}{4} \sum_{l=1}^{\infty} \mu_l \int_{-\Delta_m}^{\Delta_m} P(\Delta, l) f_\Delta(\Delta) d\Delta \tag{36}$$

where

$$P(\Delta, l) = \left| \int_0^{\frac{1+\beta}{2T_c}} H(f) H(f - \frac{1}{T_c}) \exp(j2\pi(l-\Delta) f T_c) df \right|^2.$$

Generalizing to the case of N_u users, we assume the received signal is as in (33) and that the receiver is demodulating the data for the m^{th} stream of the k^{th} user, and that

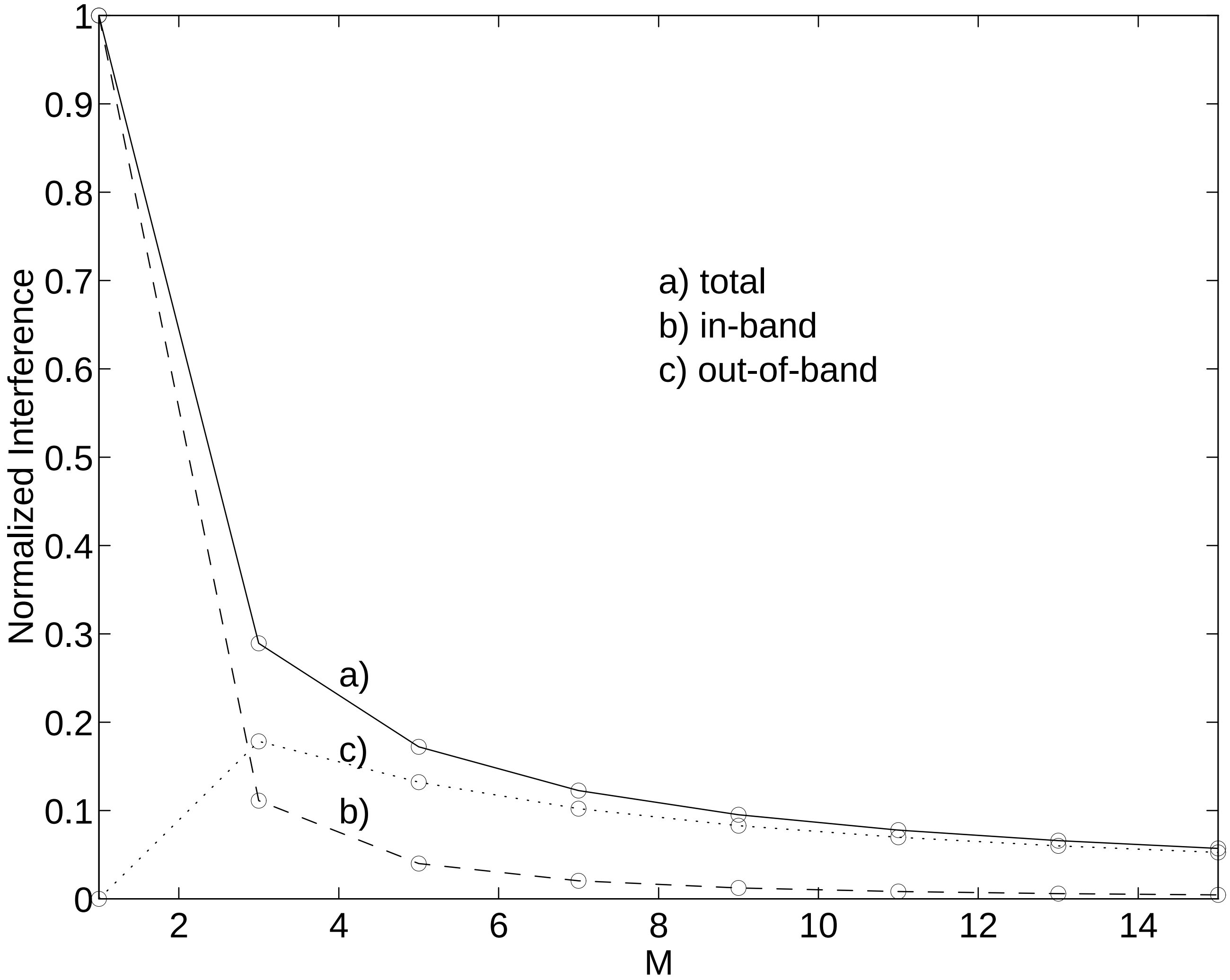

Figure 11: Normalized in-band interference, out-of-band interference, and total interference as a function of the number of sub-carriers M.

Fractional Excess bandwidth (β)	Normalized cost for sq. root R.C. pulse	Normalized cost for optimal pulse	Improvement factor
0	.141	.137	1.03
.25	.110	.072	1.5
.50	.082	.040	2.1
.67	.067	.027	2.5
.75	.061	.022	2.8
1	.045	.013	3.5

Table 1: Normalized cost $\frac{F}{E_h^2 N}$

$\theta_k = \tau_k = 0$. The integrator output is

$$y \approx \pm \frac{A_k E_s}{2} \; + \; \psi_{in} + \psi_{out} + \eta \tag{37}$$

where ψ_{in} is the in-band multiple-access interference and ψ_{out} is the out-of-band interference, due to the other $N_u - 1$ users. Assuming that the m^{th} band is an interior band, we can show that

$$\gamma \;=\; \gamma_1 + \frac{8\sigma_M^2 \sum_{\substack{i=1 \\ i\neq k}}^{N_u} A_i^2}{E_s \sum_{\substack{i=1 \\ i\neq k}}^{N_u} E_i} \;\approx\; \gamma_1 + \frac{16\sigma_M^2}{N^2 E_h^2} \tag{38}$$

where the rightmost expression in (38) was obtained assuming $E_s \approx N E_h$. γ_1 is the γ value for the single-carrier scheme (obtained from Table 1), and σ_M^2 is given by (36). Substituting (38) into (4) gives the bit error rate for the k^{th} user's m^{th} bit stream.

γ is comprised of two terms, the first resulting from the in-band interference and the second arising from the out-of-band interference. In the design of the optimum pulse shape we minimized the first term. In a multi-carrier system the criterion should be to minimize γ in (38). However, since the first term can be much larger than the second a criterion which minimizes the first should produce a pulse which is close to the optimal.

4 System Design Aspects

The multiple-access capability of a signaling scheme is an important system design issue. The allowable number of simultaneous users will depend on the background noise level N_0 which could be due to thermal noise or interference from other cells in the system, and on the required operating symbol error rate, P_e. For small N_0 and high P_e the system will support N users resulting in all codes being used. If it is not possible to support N users then users (spreading sequences) should be dropped until the multiple-access interference is reduced to an acceptable level. In doing so the correlation matrix for all codes should be examined so that sequences resulting in large cross-correlations are dropped first. As an example the Sylvester matrix has one row which is equal to a cyclic shift (by one) of another row. One of these should be the first to be dropped.

Even after dropping some codes the probability of error for the users using the remaining codes will differ. This difference may be removed by having the users interchange their codes after each symbol in a manner similar to frequency hopping (i.e. "code hopping"). The algorithms for the hopping should be designed so as to achieve a maximum degree of randomization. Such algorithms will not significantly increase the complexity of the transmission scheme. Even with randomization and relatively stable clocks there will be some dependence between successive symbol errors if the variable Δ holds relatively constant from symbol to symbol. This effect could be removed by randomizing Δ every few periods of the spreading code.

5 Conclusions

A multi-carrier orthogonal CDMA signaling scheme was introduced as a means to reduce interference in a multi-point to point CDMA network. Spreading codes and chip pulse shapes which minimize multiple-access interference in the case of quasi-synchronous operation were determined.

A sequence design criterion based on the mean square value of the sequence cross-correlations (denoted as $\mu_{cor}(N)$) was used. For a given $N \times N$ Hadamard matrix, there is associated with it a value of $\mu_{cor}(N)$. The larger the $\mu_{cor}(N)$ value, the worse the performance of a quasi-synchronous CDMA system based on the given matrix. We searched for orthogonal matrices with small $\mu_{cor}(N)$ values. Sylvester-type Hadamard matrices were shown to have a $\mu_{cor}(N)$ value of approximately $\frac{2}{3}N$ for large N. It appears that for moderate to large N most other Hadamard matrices have a $\mu_{cor}(N)$ value approximately equal to N.

Optimal chip pulses of various bandwidths that minimize multiple-access interference for quasi-synchronous systems were obtained. They were compared with the raised cosine pulses and were shown to have better performance at the higher excess bandwidths considered.

A multi-carrier signaling scheme, designed to reduce chip level synchronization offsets between the users, was introduced. This quasi-synchronous CDMA scheme, based on orthogonal spreading codes and multi-carrier modulation, is an alternative to an asynchronous scheme which utilizes sophisticated interference cancellation techniques.

Appendix

We derive (13). Defining $X_{ij}^N = \sum_{k=1}^{N} c_k^i c_{k+1}^j$, where c_k^i is the $(i,k)^{\text{th}}$ element of an $N \times N$ Sylvester matrix, and $c_{N+1}^i = c_1^i$, we use (10) to write

$$
\begin{aligned}
N(N-1)\mu_{cor+}(N) &= \sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}(X_{ij}^N)^2 \\
&= 8\sum_{i=1}^{\frac{N}{2}}\sum_{\substack{j=1\\ j\neq i}}^{\frac{N}{2}}(X_{ij}^{\frac{N}{2}})^2 - 16\sum_{i=1}^{\frac{N}{2}}\sum_{\substack{j=1\\ j\neq i}}^{\frac{N}{2}}(X_{ij}^{\frac{N}{2}})c_{\frac{N}{2}}^i + 16\left(\frac{N}{2}\right)\left(\frac{N}{2}-1\right) \qquad \text{(A.1)}
\end{aligned}
$$

where (A.1) follows because X_{ij}^{N} equals $2X_{ij}^{\frac{N}{2}}$ if i and $j \epsilon \{1 \dots \frac{N}{2}\}$, 0 if $i \epsilon \{1 \dots \frac{N}{2}\}$ and $j \epsilon \{\frac{N}{2}+1 \dots N\}$, or $2X_{i-\frac{N}{2},j-\frac{N}{2}}^{\frac{N}{2}} - 4c_{\frac{N}{2}}^{i-\frac{N}{2}}$ if i and $j \epsilon \{\frac{N}{2}+1 \dots N\}$. We also note that

$$\begin{aligned}
\sum_{i=1}^{\frac{N}{2}} \sum_{\substack{j=1 \\ j\neq i}}^{\frac{N}{2}} (X_{ij}^{\frac{N}{2}}) c_{\frac{N}{2}}^{i} &= \sum_{i=1}^{\frac{N}{4}} \sum_{\substack{j=1 \\ j\neq i}}^{\frac{N}{4}} (X_{ij}^{\frac{N}{2}}) c_{\frac{N}{2}}^{i} + \sum_{i=\frac{N}{4}+1}^{\frac{N}{2}} \sum_{\substack{j=\frac{N}{4}+1 \\ j\neq i}}^{\frac{N}{2}} (X_{ij}^{\frac{N}{2}}) c_{\frac{N}{2}}^{i} \\
&= 2\sum_{i=1}^{\frac{N}{4}} \sum_{\substack{j=1 \\ j\neq i}}^{\frac{N}{4}} (X_{ij}^{\frac{N}{4}}) c_{\frac{N}{2}}^{i} + \sum_{i=1}^{\frac{N}{4}} \sum_{\substack{j=1 \\ j\neq i}}^{\frac{N}{4}} (2X_{ij}^{\frac{N}{4}} - 4c_{\frac{N}{4}}^{i}) c_{\frac{N}{2}}^{i+\frac{N}{4}} \\
&= 4\left(\frac{N}{4}\right)\left(\frac{N}{4}-1\right) \qquad \text{(A.2)}
\end{aligned}$$

since $c_{\frac{N}{2}}^{i+\frac{N}{4}} = -c_{\frac{N}{4}}^{i} = -c_{\frac{N}{2}}^{i}$ for $i \epsilon \{1 \dots \frac{N}{4}\}$. (A.1) then reduces to

$$\mu_{cor+}(N) = \frac{8\left(\frac{N}{2}\right)\left(\frac{N}{2}-1\right)\mu_{cor+}\left(\frac{N}{2}\right) + 8N}{N(N-1)}. \qquad \text{(A.3)}$$

This is a recursive formula for $\mu_{cor+}(N)$ which may be interpreted as a first order difference equation and solved using the initial condition $\mu_{cor+}(2) = 0$ to obtain

$$\mu_{cor+}(N) = \frac{2N^2 - 8}{3(N-1)} \qquad \text{(A.4)}$$

In a similar fashion, we can show that

$$\mu_{cor-}(N) = \frac{2N^2 + 4}{3(N-1)}. \qquad \text{(A.5)}$$

Eq. (13) then follows from (A.4), (A.5), and (9).

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