Multi-User Multi-Sensor Detectors for CDMA Networks

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Abstract— **We consider a Code Division Multiple Access (CDMA) cellular network and propose sub-optimal multi-user detectors which exploit a type of diversity inherent in the system and are based on the processing of signals received at multiple base stations. We investigate the performance of conventional matched-filter, linear, and decision-based detection schemes, using this diversity scheme, for both synchronous and asynchronous systems. The detection of user signals is based on tracking the signals at several base stations. The optimal detector would operate on all the signals received at all the base stations but would have a high degree of complexity. We discuss the case where the detection of a given signal is based on received signals at a small subset of base stations. We show that with judicious selection of this smaller set of base stations, good performance can be obtained without much increase in complexity.**

I. Introduction

In this paper we study the structures and performance of sub-optimal detectors for signals in CDMA networks where the signals are received at multiple base stations, or antennas. For the case of one antenna, many detectors have been proposed [1]-[5], where the signaling information from all the sources is used in estimating the data symbols. These detectors are referred to as multi-user detectors. Even though the complexity of these multi-user detectors is higher than that of the usual matched-filter detector (also referred to as the conventional detector), they may be cost-effective when all the users' data has to be estimated simultaneously (i.e. in multi-point to point channels) such as in the case of the reverse link of cellular or micro-cellular networks.

We suggest methods to improve the performance of such multi-user detectors (in terms of error probability) in a cellular network environment [6] [7] and in Personal Communication Services (PCS) systems [8]. In a cellular network an area, where the communication services are to be provided, is partitioned into cells. Each cell has a base station which is responsible for establishing and maintaining the link with all the users within that cell. Consider

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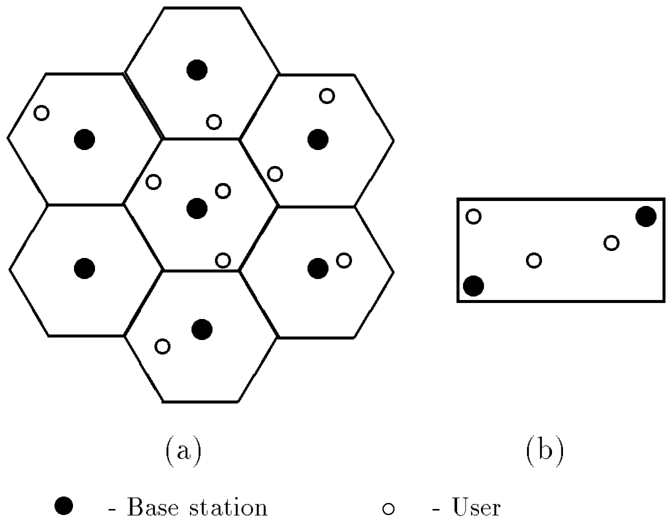

Fig. 1. Cellular Systems: (a) Outdoor (b) Indoor.

the scenario shown in Fig. 1. A user at the border of two cells is attempting to transmit data. Typically only one of the base stations accepts data from this user. However, there might be other users at close proximity to this base station that may cause significant interference (the near-far effect). This effect would be even more deleterious if the signal suffered from fading. The multi-user detectors mentioned above usually improve the performance in such settings. We examine some modified multi-user schemes in order to obtain additional improvement in performance. These modifications are based on exploiting the concept of *diversity.* Whenever a user is near the periphery of a cell, the user is roughly at the same distance with respect to at least one other base station. If all the base stations in the vicinity of one user keep track of this user, the signals received at *all* the base stations can be combined to obtain certain improvements in performance. The availability of one user's signal at many of the nearby stations constitutes *diversity.*

In section II, we present the system model and discuss how the improvements in performance can be achieved based on maximum likelihood detection theory. Based on the theory presented in section II, we suggest some detectors in section III. These diversity-based structures can be implemented with some increase in complexity, which can, however, be reduced by using a *partial tracking* strategy (section III.D). In section IV, we discuss the performance measures and give the expressions for error probabilities. We study the performance of these detectors by considering some numerical examples in section V. Section VI concludes the paper.

II. System Description and Detection Model

Suppose there are L base stations in the region of interest. The received signal, $r(t)$, at a given base station (say l^{th}) may be expressed as

$$r^{(l)}(t) = S^{(l)}(t;\mathbf{b}) + n^{(l)}(t), \qquad -\infty \le t \le \infty \quad (1)$$

with

$$S^{(l)}(t;\mathbf{b}) = \sum_{i=-M}^{M}\sum_{k=1}^{K} b_k(i)\sqrt{E_k^{(l)}(i)}s_k(t-iT-\tau_k^{(l)}),$$

where $n^{(l)}(t)$ is a white Gaussian noise process with two-sided power spectral density given by σ_l^2, K is the number of users, T is the symbol interval, $s_k(t)$, $E_k^{(l)}(i)$, and $b_k(i)$ are the modulating signal, symbol energy, and data symbol respectively associated with the k^{th} user during the i^{th} symbol interval, with $s_k(t)$ being zero for $t \notin [0,T]$ and having unit energy. $\tau_k^{(l)}$ is the delay (symbol epoch) associated with the k^{th} user (without loss of generality, we assume $0 \le \tau_k < T$), $NT = (2M+1)T$ is the observation interval. We let $\mathbf{b} = \left[\mathbf{b}^T(-M),\cdots,\mathbf{b}^T(0),\cdots\mathbf{b}^T(M)\right]^T$, where $\mathbf{b}(i) = [b_1(i),\cdots,b_K(i)]^T$ and $\mathbf{b}(-M-1) = \mathbf{b}(M+1) = 0$ where T denotes the matrix transpose. Even though the analysis is valid for any linear modulation scheme[1], for simplicity, we let the data symbols $b_k(i)$ assume values from the set $A = \{-1,+1\}$ (i.e., binary signaling). In certain cases (for example, in a CDMA system where the differences in symbol epochs between the users' signals are much smaller than one chip time), we can assume that all the signals arrive at the receiver simultaneously (i.e., the symbol epochs occur at the same time. In other words, $\tau_1^{(l)} = \cdots = \tau_k^{(l)} = \tau^{(l)}$). We refer to these systems as synchronous systems. If this condition is not satisfied, we refer to such systems as asynchronous systems.

A multi-user detector uses the above signal to estimate the data symbols. Usually, the data would be estimated at the base station, and then the estimates alone will be sent to the switch on a high speed and highly reliable channel (for example, a fiber link). However, when a user's signal suffers from severe fading, the decisions made even by a multi-user detector may not be reliable. These difficulties can be circumvented with an increase in infrastructure as well as in computational complexity.

In cellular systems, when a user is crossing over the boundary of a cell into another, in order to have a smooth transfer (hand-off) between two base stations, it is proposed [7] that *both* base stations track the user when it is near the periphery (on either side of cell). This type of tracking is called soft hand-off. Since the users near the periphery of the neighboring cells are likely to cause significant interference, if *both* the base stations track all the users having considerable signal strength, then these signals can be combined to obtain a reduction in the average symbol error rate. Thus, we are trying to use the diversity which already exists in many schemes. Hence better performance can be achieved by detecting the signals at the switch using this "diversity". The amount of data that is to be sent from the base station to the switch is increased, since for the detection of a given symbol, all the sufficient statistics (that are required for the detection) have to be sent to the switch. This increase can be justified if there is a significant improvement in the overall performance. This is the issue we address in this paper. Thus a base station receiving the signals, instead of detecting the data, will calculate the sufficient statistic and send them to the switch. The switch will combine these statistics and obtains an estimate of the data. The procedure is depicted in Fig. 2.

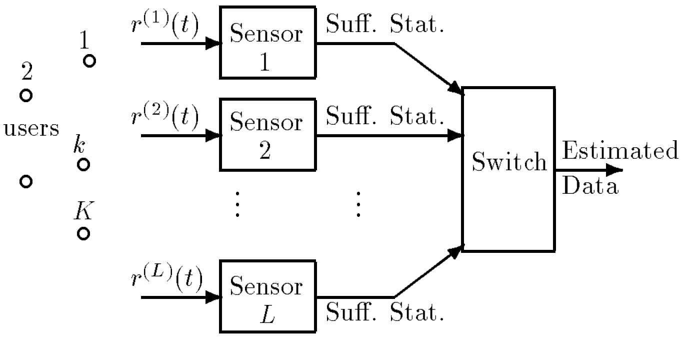

Fig. 2. A system using diversity; Suff. Stat. = Sufficient Statistics.

Further, in PCS systems, the cells would be very small (micro-cells) and numerous. Hence, the base stations should be inexpensive. In one of the schemes [8] that is being considered for such systems, the base station is just an antenna, where the signal from the user is received and being sent over to the switch (without any demodulation) on a fiber link. Since the demodulation is done at the switch, for detecting a given user's signal, it is again more advantageous to combine the signals received from all the base stations where this user's signal strength is significant.

Without loss of generality, we assume that there are K users and that all the users are tracked by the L base stations (henceforth, referred to as sensors) that are available in the vicinity. Throughout this discussion, we will be considering both asynchronous and synchronous systems. We further assume that the noise at each sensor is independent of the other sensors' noise.

1) *Asynchronous systems:* The expression for optimal estimate of $\mathbf{b}$ (in the maximum likelihood sense [9]) can be found easily and is given by

$$\hat{\mathbf{b}} = \arg\left\{\max_{\mathbf{b}}\left(\sum_{l=1}^{L}\gamma^{(l)}(\mathbf{b})\right)\right\} \quad (2)$$

[1]The above is a baseband model for any linear modulation and assumes perfect carrier frequency and phase synchronization.

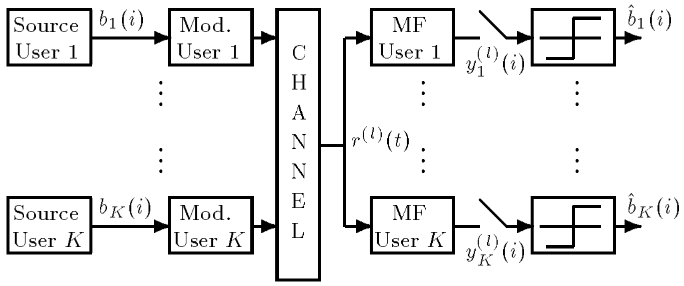

Fig. 3. A conventional detector; MF = Matched Filter, Mod. = Modulator.

where

$$\gamma^{(l)}(\mathbf{b}) = \frac{1}{\sigma_l^2}\left[2\int_{-\infty}^{\infty} r^{(l)}(t)\, S^{(l)}(t;\mathbf{b})\, dt - \int_{-\infty}^{\infty}\left(S^{(l)}(t;\mathbf{b})\right)^2 dt\right] \tag{3}$$

Note that $\gamma^{(l)}(\mathbf{b})$ is the required likelihood function for data detection at the l^{th} sensor when this sensor is considered alone for the purpose of detection [1]. Further, the likelihood function for this system is equivalent to the maximal ratio-combining of L received signals when there is only one user [10]. Since the required likelihood function is just a linear combination of $\gamma^{(l)}(\mathbf{b})$, one can invoke Verdú's [1] procedure (by using the Viterbi algorithm) to achieve optimal detection. However, since the complexity of this detector increases exponentially with the number of users, this method is impractical.

In [1], it is shown that the outputs of a bank of matched filters sampled at the symbol epochs - each filter matched to one of the users' signals - along with the symbol epochs and the users' signal energies, constitute the sufficient statistic [11]. Using a similar procedure, it can be again shown that the sampled outputs of the bank of matched filters and the symbol epochs, along with the users' signal energies do indeed provide a sufficient statistic, even when there is more than one sensor. Let $y_k^{(l)}(i)$ denote the output of the matched filter sampled at the symbol epochs (at $iT+\tau_k^{(l)}$) for the i^{th} symbol from the k^{th} user at the l^{th} sensor, i.e., (see Fig. 3)

$$y_k^{(l)}(i) = \int_{\tau_k^{(l)}+iT}^{\tau_k^{(l)}+iT+T} s_k(t-iT-\tau_k^{(l)})\, r^{(l)}(t)\; dt \qquad i=-M,\cdots,M; l=1,\cdots,L. \tag{4}$$

Let $\mathbf{y}^{(l)}(i) = [y_1^{(l)}(i),\cdots,y_K^{(l)}(i)]^T$, then

$$\mathbf{y}^{(l)}(i) = \mathbf{R}^{(l)}(-1)\mathbf{E}^{(l)}(i+1)\mathbf{b}(i+1) + \mathbf{R}^{(l)}(0)\mathbf{E}^{(l)}(i)\mathbf{b}(i) + \mathbf{R}^{(l)}(1)\mathbf{E}^{(l)}(i-1)\mathbf{b}(i-1) + \mathbf{n}^{(l)}(i) \tag{5}$$

with $\mathbf{n}^{(l)}(i) = [n_1^{(l)}(i),\cdots,n_K^{(l)}(i)]^T$, where $n_k^{(l)}(i)$ is the noise component of the k^{th} matched filter output, and with the assumption that $\mathbf{b}(-M-1)=\mathbf{b}(M+1)=0$. The $(k,j)^{\text{th}}$ element of the cross-correlation matrix of the signals at the l^{th} sensor, $\mathbf{R}^{(l)}(i)$, is given by

$$\begin{aligned} R_{kj}^{(l)}(i) &= \int_{-\infty}^{\infty} s_k(t-\tau_k^{(l)})\, s_j(t+iT-\tau_j^{(l)})\, dt \\ &= \begin{cases} 0 & \text{if } \;|\, i \,|>1 \\ R_{jk}^{(l)}(-i) & \text{if } \; i=0,\ \pm 1, \end{cases} \end{aligned} \tag{6}$$

and the energy matrix at the l^{th} sensor is given by

$$\mathbf{E}^{(l)}(i) = \operatorname{diag}\left(\left[\sqrt{E_1^{(l)}(i)},\sqrt{E_2^{(l)}(i)},\cdots,\sqrt{E_K^{(l)}(i)}\,\right]\right).$$

The cross-correlation matrix of $\mathbf{n}^{(l)}(i)$ is given by

$$\mathbb{E}\left[\mathbf{n}^{(l)}(i)\,\mathbf{n}^{(m)}(j)^T\right] = \sigma_l^2\,\mathbf{R}^{(l)}(i-j)\,\delta_{lm},$$

where $\mathbb{E}$ denotes the expected value and δ_{lm} is the Kronecker delta function. As was done in [2], for $l=1,\cdots,L$, we express (5) as

$$\mathbf{y}^{(l)} = \mathcal{R}^{(l)}\mathcal{E}^{(l)}\mathbf{b} + \mathbf{n}^{(l)}, \tag{7}$$

where

$$\begin{aligned} \mathbf{y}^{(l)} &= [\mathbf{y}^{(l)^T}(-M),\cdots,\mathbf{y}^{(l)^T}(M)]^T, \\ \mathbf{n}^{(l)} &= \left[\mathbf{n}^{(l)^T}(-M),\cdots,\mathbf{n}^{(l)^T}(M)\right]^T, \\ \mathcal{E}^{(l)} &= \operatorname{diag}\left(\left[\mathbf{E}^{(l)}(-M),\cdots,\mathbf{E}^{(l)}(M)\right]\right), \\ \mathbf{b} &= \left[\mathbf{b}^T(-M),\cdots,\mathbf{b}^T(M)\right]^T, \end{aligned}$$

and

$$\mathcal{R}^{(l)} = \begin{bmatrix} \mathbf{R}^{(l)}(0) & \mathbf{R}^{(l)}(-1) & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{R}^{(l)}(1) & \mathbf{R}^{(l)}(0) & \mathbf{R}^{(l)}(-1) & & \vdots \\ \mathbf{0} & \mathbf{R}^{(l)}(1) & \mathbf{R}^{(l)}(0) & \ddots & \mathbf{0} \\ \vdots & & \ddots & \ddots & \mathbf{R}^{(l)}(-1) \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{R}^{(l)}(1) & \mathbf{R}^{(l)}(0) \end{bmatrix}.$$

Equation (7) can be rewritten as

$$\mathbf{y} = \mathbf{H}_a\mathbf{b} + \mathbf{n}_a, \tag{8}$$

where $\mathbf{y} = [\mathbf{y}^{(1)^T},\cdots,\mathbf{y}^{(L)^T}]^T$, $\mathbf{n}_a = [\mathbf{n}^{(1)^T},\cdots,\mathbf{n}^{(L)^T}]^T$, and

$$\mathbf{H}_a = \begin{bmatrix} \mathcal{R}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathcal{R}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathcal{R}^{(L)} \end{bmatrix} \begin{bmatrix} \mathcal{E}^{(1)} \\ \mathcal{E}^{(2)} \\ \vdots \\ \mathcal{E}^{(L)} \end{bmatrix}, \tag{9}$$

with the cross-covariance matrix of $\mathbf{n}_a$ being given by

$$\mathbb{E}(\mathbf{n}_a{\mathbf{n}_a}^T) \;=\; \mathbf{Q}_a \;=\; \operatorname{diag}\left(\left[\sigma_1^2\mathcal{R}^{(1)},\cdots,\sigma_L^2\mathcal{R}^{(L)}\right]\right). \tag{10}$$

2) *Synchronous Systems:* For synchronous systems, since we assume that $s_k(t)$ is zero for $t\notin[0,T]$, we may drop the time dependency as we only need the information of symbols transmitted by the other users during that interval.

Hence, the received signal, for the detection of the i^{th} symbol, can be expressed as

$$r^{(l)}(t) = \sum_{k=1}^{K} b_k(i)\sqrt{E_k^{(l)}(i)}s_k(t - iT - \tau^{(l)}) \ + \ n^{(l)}(t),$$
$$iT + \tau^{(l)} \leq t < (i+1)T + \tau^{(l)}, \tag{11}$$

where we set the symbol epochs $\tau_1^{(l)} = \cdots = \tau_k^{(l)} = \tau^{(l)}$. As with the asynchronous systems, the computational complexity of the optimal detector is exponential in number of users K. $\mathbf{y}^{(l)}$ (vector of the outputs of the bank of matched filters) is given by

$$\mathbf{y}^{(l)} = \mathbf{R}^{(l)}\mathbf{E}^{(l)}\mathbf{b} + \mathbf{n}^{(l)}, \tag{12}$$

where all the vectors are defined in analogous manner, $\mathbf{R}^{(l)} = \mathbf{R}^{(l)}(0)$, $\mathbf{E}^{(l)} = \mathbf{E}^{(l)}(0)$. Note that $\mathbf{R}^{(l)}$ will be same for $l = 1, \cdots, L$, as the system is synchronous, and the window of integration (in the matched filter) will be identical at all the sensors. The cross-correlation matrix of $\mathbf{n}^{(l)}$ is given by

$$\mathbb{E}\left(\mathbf{n}^{(l)}\mathbf{n}^{(m)^T}\right) = \sigma_l^2\mathbf{R}^{(l)}\delta_{lm}. \tag{13}$$

As before, (12) can be rewritten as

$$\mathbf{y} = \mathbf{H}_s\mathbf{b} + \mathbf{n}_s, \tag{14}$$

where $\mathbf{y} = [\mathbf{y}^{(1)^T}, \cdots, \mathbf{y}^{(L)^T}]^T$, $\mathbf{n}_s = [\mathbf{n}^{(1)^T}, \cdots, \mathbf{n}^{(L)^T}]^T$, and

$$\mathbf{H}_s = \begin{bmatrix} \mathbf{R}^{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^{(2)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}^{(L)} \end{bmatrix} \begin{bmatrix} \mathbf{E}^{(1)} \\ \mathbf{E}^{(2)} \\ \vdots \\ \mathbf{E}^{(L)} \end{bmatrix}. \tag{15}$$

The cross-covariance matrix of $\mathbf{n}_s$ is given by

$$\mathbb{E}\left(\mathbf{n}_s\mathbf{n}_s{}^T\right) \ = \ \mathbf{Q}_s \ = \ \mathbf{diag}\left(\left[\sigma_1^2\mathbf{R}^{(1)}, \cdots, \sigma_L^2\mathbf{R}^{(L)}\right]\right). \tag{16}$$

III. Sub-Optimal Detectors

A. Conventional Detectors

The most commonly used detector is the matched-filter receiver which we will refer to as the *conventional detector* (see Fig. 3). If there were no diversity, to estimate the i^{th} symbol of the k^{th} user, the decisions would be made based on the rule:

1) *Synchronous Systems:*

$$\hat{b}_k^c = \text{sgn}\left\{y_k^{(l)}\right\}, \tag{17}$$

2) *Asynchronous Systems:*

$$\hat{b}_k^c(i) = \text{sgn}\left\{y_k^{(l)}(i)\right\}, \tag{18}$$

where the superscript 'c' refers to the conventional detection technique. Note that (18) shows the time dependency explicitly, even though it is the same as (17).

When diversity is available, one would like to combine all the signals. There are two primary ways of achieving this: hard decoding and soft decoding. In this paper, we concentrate on soft decoding. In soft decoding, all the matched-filters' outputs are combined and then a decision is made. Further, we consider only the linear combinations of these outputs. Hence, the decisions would be based on the rule:

1) *Synchronous Systems:*

$$\hat{b}_k^c = \text{sgn}\left\{\sum_{l=1}^{L}\nu_k^{(l)}y_k^{(l)}\right\}, \tag{19}$$

2) *Asynchronous Systems:*

$$\hat{b}_k^c(i) = \text{sgn}\left\{\sum_{l=1}^{L}\nu_k^{(l)}(i)y_k^{(l)}(i)\right\}. \tag{20}$$

Here $\nu_k^{(l)}, k = 1, \cdots, K; l = 1 \cdots, L$ determine the combining rule. One would like to select $\nu_k^{(l)}$ such that the error probability is minimized. However, such a selection is not easy. One way to combine is by using the optimal combining rule as if there were only one user [10] [12]. Thus

$$\nu_k^{(l)}(i) = \frac{\sqrt{E_k^{(l)}(i)}}{\sigma_l^2}. \tag{21}$$

In this paper, for comparison purposes, we always consider the conventional detector with diversity which employs the combining rule in (21).

B. Linear Detector

Here, we determine linear detectors which are optimal in the sense of minimum mean square error (MMSE).

1) *Synchronous Systems:* We want to find a linear transformation $\mathbf{G}_s : \mathbb{R}^{LK} \to \mathbb{R}^K$, where $\mathbb{R}^K$ is the K-fold cross product of the field of real numbers $\mathbb{R}$, such that the estimate of the vector $\mathbf{b}$ is made through a "soft decision" on the vector $\mathbf{a} = \mathbf{G}_s\mathbf{y}$ and $\mathbb{E}\left((\mathbf{b} - \mathbf{a})^T(\mathbf{b} - \mathbf{a})\right)$ is minimized. In the absence of noise, this transformation is given by [13],

$$\mathbf{G}_s = \mathbf{H}_s{}^\dagger \tag{22}$$

where $\mathbf{H}_s$ is given by (15) and $\dagger$ denotes the generalized matrix inverse. Parenthetically, we note that one might want to take a Moore-Penrose inverse as it assures the uniqueness of the solution, even though other types of generalized inverses also satisfy the MMSE criterion [13] (this is needed only if the linear independence assumption [3] on the signals, in the absence of noise, is not valid - if it is valid, then all $\mathbf{R}^{(l)}$, $l = 1, 2, \cdots, L$ are identical and invertible and thus we do not need more than one sensor to detect the symbols). In the presence of noise, it can be easily shown that the transformation is given by

$$\mathbf{G}_s = \mathbf{H}_s^T\left(\mathbf{H}_s\mathbf{H}_s{}^T + \mathbf{Q}_s\right)^\dagger. \tag{23}$$

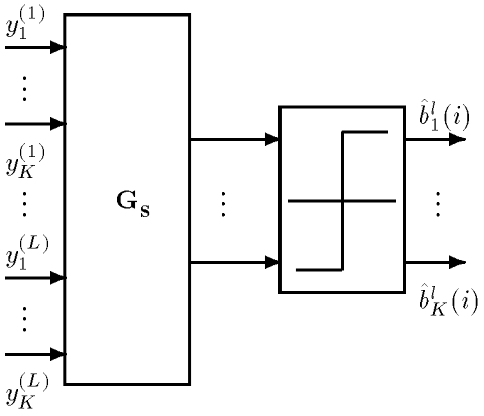

Fig. 4. Linear detector for synchronous systems: Equations (23) and (24).

Note that $\mathbf{Q}_s$, given in (16), would be positive definite if the signal set is linearly independent (not necessarily orthogonal) and the generalized inverse in the above equation becomes the regular matrix inverse. The symbols can then be estimated (since the alphabet is binary) as

$$\hat{\mathbf{b}}^l = \mathbf{sgn}\left\{\mathbf{G}_s\mathbf{y}\right\}. \tag{24}$$

The structure of the detector is shown in Fig. 4.

2) *Asynchronous Systems:* Here, the required linear transformation $\mathbf{G}_a : \mathbb{R}^{LNK} \rightarrow \mathbb{R}^{NK}$ is given by

$$\mathbf{G}_a = \mathbf{H}_a{}^T\left(\mathbf{H}_a\mathbf{H}_a{}^T + \mathbf{Q}_a\right)^{-1} \tag{25}$$

and the estimates are given by

$$\hat{\mathbf{b}}^l = \mathbf{sgn}\left\{\mathbf{G}_a\mathbf{y}\right\} \tag{26}$$

where $\mathbf{H}_a$ and $\mathbf{Q}_a$ are given by (9) and (10). The conditions of invertibility of the matrix in (25) are similar to those discused for (23) above. We note here that the linear equalizer and the weighted least squares detector proposed in [5] are basically modified versions of the above detector (25) when there is only one sensor.

C. Decision-Based Detector

The decision-based detector for one sensor (also referred to as multi-stage detector) has been described in [5] [4]. In these detectors, an initial set of estimates of the users' symbols are made, these estimates being used to estimate the interference. Then this "estimated" interference is removed and a new estimate is made. This procedure may be repeated until a more reliable estimate is obtained (based either on error probability or the convergence of the estimates [4]). Here, we generalize this scheme when L sensors are available. Suppose it is desired to estimate $b_k(i)$, and we assume that all the other data symbols estimates are available. Then, the optimal estimate (in the maximum-likelihood sense) of $b_k(i)$ can be obtained by (a simple modification to the single-user receiver with diversity [10]),

$$\hat{b}_k^d(i) = \operatorname{sgn}\left\{\sum_{l=1}^{L}\frac{\sqrt{E_k^{(l)}(i)}}{\sigma_l^2}z_k^{(l)}(i)\right\} \tag{27}$$

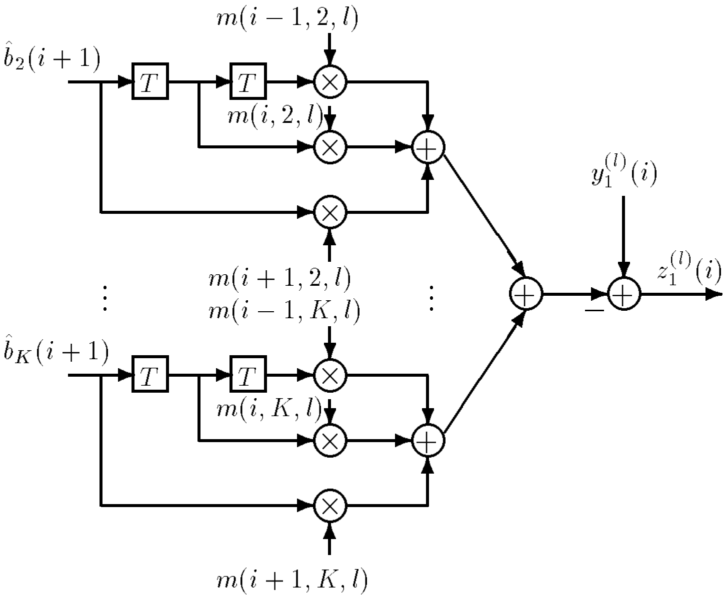

Fig. 5. Interference cancellation for the detection of $b_1(i)$ using a decision based detector: Equation (29); $m(i-j,k,l) = \sqrt{E_k^{(l)}(i-j)}R_{1k}^{(l)}(j)$.

where $z_k^{(l)}(i)$ is the result obtained by subtracting out the estimated interference from the matched-filter output. Hence, for

1) *Synchronous Systems:*

$$z_k^{(l)}(i) = y_k^{(l)}(i) - \sum_{\substack{n=1 \\ n \neq k}}^{K}\sqrt{E_n^{(l)}(i)}\hat{b}_n(i)R_{kn}^{(l)}(0), \tag{28}$$

while for

2) *Asynchronous Systems:*

$$z_k^{(l)}(i) = y_k^{(l)}(i)$$
$$-\sum_{j=-1}^{1}\sum_{\substack{n=1 \\ n \neq k}}^{K}\sqrt{E_n^{(l)}(i+j)}\hat{b}_n(i+j)R_{kn}^{(l)}(-j). \tag{29}$$

Thus the estimate of $b_k(i)$ depends on the estimates of other data symbols for that period as well as the preceding and the following periods (only for that particular period, if the system is synchronous). Fig. 5 depicts, for asynchronous systems and for the detection of $b_1(i)$, how the interference at l^{th} sensor is removed from $y_1^{(l)}(i)$, by using the initial estimates $\hat{\mathbf{b}}$ (computation of $z_k^{(l)}(i)$ as specified in (29)). Note that, in reality, only one delay element is required due to the time-boundedness assumption on $s_k(t)$. For users with $\tau_k^{(l)} > \tau_1^{(l)}$, the detection procedure uses $\hat{b}_k(i-1)$ instead of $\hat{b}_k(i+1)$. Fig. 6 shows how these (ideally) interference-free signals are combined to make the final estimates. Note that the above (optimal) combining rule is same as the maximal-ratio combining in a single-user case. Since the detection process is based on some initial values for the symbols, prior estimates should be

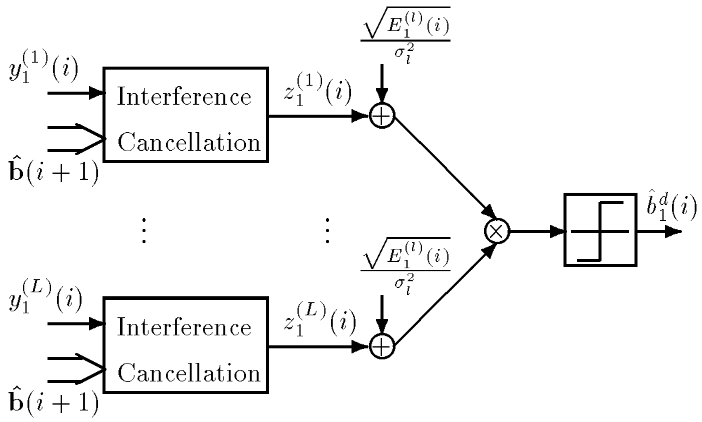

Fig. 6. Detection of $b_1(i)$ using a decision based detector: Equation (27).

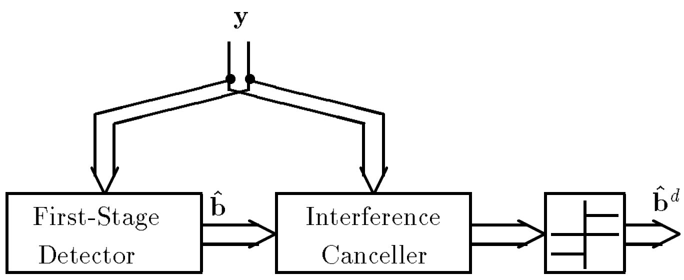

Fig. 7. A decision based detector.

made. Due to the dependence of the detector on the initial estimates, the performance is primarily governed by the accuracies of these initial estimates. If these estimates were perfect, we can obtain optimal performance. Since this is not possible, the performance would be slightly inferior to the optimal detector. The initial estimates can be obtained by using any of the above detectors. It should be noted that the overall performance is dependent on this first-stage detector. The structure of a decision-based detector is shown in Fig. 7. This detector can be thought of as a decision-feedback equalizer adapted to a system with diversity.

D. Partial Tracking of the Signals

In general, it is neither necessary, nor practical, to have *all* the users tracked by *all* the sensors. The energies of some of the users may be so low that neither these signals create much interference nor are they sufficiently strong for any meaningful detection. Hence, we can assume that the contribution of these signals to the detection process is negligible. This tracking can be handled very much in the same way as the hand-off. Every base station will decide to receive data or not from a given user based on the SNR (using some sort of threshold). If the SNR is higher than the threshold, the base station will receive data from the user even if there were some other base stations which are also receiving data from that receiver. If the SNR is less than the threshold it will not "track" this particular user's transmission. This would greatly reduce the complexity of the detector as the bank of matched-filters now will have a smaller number of matched filters as well as the amount of information that has to be sent to the switch would be reduced considerably.

For such a system, the detectors described in the previous sub-sections can be modified in a straight-forward manner. For the corresponding linear MMSE detector, the number of equations to be solved is reduced to a great extent.

IV. Error Probabilities

In this section as well as the next, we will evaluate the performance of the detectors discussed in section III. The usual performance measures that are used in a CDMA network are the error probability and the signal-to-interference ratio. While the former measure directly determines the error rate, the latter one is an indirect measure which conveniently describes the system performance (mainly due to the fact that it is hard to compute the error probabilities in most cases). The other performance measures introduced in [1] are the asymptotic user efficiency as well as the near-far resistance. These parameters evaluate the performance of a detector in high signal-to-background noise environment. For the maximum-likelihood detector as well as for both the linear and decision-based detectors, these parameters have been evaluated in [1], [2], and [4], for the single-sensor case and are found to be optimum in the sense of minimizing the near-far effect. Since the single-sensor can be considered as a special case of the more generalized scheme of using diversity, the diversity based detectors using the concept of the multi-user detection are also optimum (in terms of the terminology used in [1], they have the same near-far resistance as that of the optimal maximum-likelihood receiver) in the sense of minimizing the near-far effect. Further, one would like to study how the detector performs when the signal-to-noise ratios are not too high. Hence, in this paper, we concentrate on the error probabilities. We briefly comment on the signal-to-interference ratio in section V.

Without loss of generality, unless specified, we assume that we are interested in the evaluating the error probability for user 1. As we assume that the symbols are equiprobable, to determine the average error probability, it is sufficient to compute the error probability when $b_1(i) = 1$. Define $\mathbf{j1}(n) = [j_1(n) \cdots j_{K-1}(n)]^T$, $\mathbf{j}(i,n) = [\delta_{ni},\ \mathbf{j1}(n)]^T$, where δ_{ni} is the Kronecker delta, and $j_k(n) \in A = \{-1, 1\}$. Further, let $\mathbf{C}^{(l)}(i,n) = \mathbf{E}^{(l)}(i)\mathbf{R}^{(l)}(i-n)\mathbf{E}^{(l)}(n)$.

A. Conventional Detectors

1) *Synchronous Systems:* If the interfering symbols were known, then $y_1^{(l)}(i)$ would be a Gaussian random variable with mean $\alpha_l/\sqrt{E_1^{(l)}(i)}$ and variance σ_l^2, where α_l is the element in the first row of the column matrix $\mathbf{C}^{(l)}(i,i)\mathbf{j}(i,i)$. Then, using the combining rule, given by (19) and (21), for

the detection of $b_1(i)$, it can be easily shown that the error probability is given by

$$P_{s,c}(\text{error}) = \mathbb{E}\left(\Pr(\hat{b}_1 = -1 \mid b_1 = 1)\right)$$

$$= \sum_{b_j \in A, j \neq 1} \Pr(\hat{b}_1 = -1 \mid b_1 = 1, b_2, \cdots, b_K) \Pr(b_2, \cdots, b_K)$$

$$= \sum_{\mathbf{j1}(i) \in A^{K-1}} \frac{1}{2^K} \operatorname{erfc}\left(\frac{\sum_{l=1}^{L} \frac{\alpha_l}{\sigma_l^2}}{\sqrt{2\sum_{l=1}^{L} \frac{E_1^{(l)}(i)}{\sigma_l^2}}}\right), \qquad (30)$$

where

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-x^2} dx,$$

and A^{K-1} is the $(K-1)$ fold cross product of $A = \{-1, 1\}$.

2) *Asynchronous Systems:* If the symbol epochs $\tau_k^{(l)}$, $l = 1, \cdots, L, k = 1, \cdots, K$ are known, then the average error probability for this case is similar to (30) with the summations being taken over $\mathbf{j1}(i-1)$, $\mathbf{j1}(i)$ and $\mathbf{j1}(i+1)$ and is given by

$$P_{a,c}(\text{error} \mid \tau_k^{(l)}, \forall k, l) =$$

$$\sum_{\substack{\mathbf{j_1}(i-1) \\ \in A^{K-1}}} \sum_{\substack{\mathbf{j_1}(i) \\ \in A^{K-1}}} \sum_{\substack{\mathbf{j_1}(i+1) \\ \in A^{K-1}}} \frac{1}{2^{3K-2}} \operatorname{erfc}\left(\frac{\sum_{l=1}^{L} \frac{\alpha_l}{\sigma_l^2}}{\sqrt{2\sum_{l=1}^{L} \frac{E_1^{(l)}(i)}{\sigma_l^2}}}\right) \qquad (31)$$

where α_l is the first element of the column matrix $\sum_{n=-1}^{1} \mathbf{C}^{(l)}(i, i-n)\mathbf{j}(i, i-n)$. Note that some of the terms in the numerator of (31) would be zero due to the time-boundedness of the signals (since some of the cross-correlations would be zero - for example, at least one of $R_{jk}^{(l)}(1)$ and $R_{jk}^{(l)}(-1)$ has to be zero). To determine the average error probability, one has to take the expected value (with respect to $\tau_k^{(l)}, \forall k, l$ or with respect to all the correlations) of the right hand side of (31). The direct evaluation of this expectation is not straight-forward. However, the average error probability can be evaluated by using the method of moments outlined in [14].

B. Linear Detector

For synchronous systems, the average error probability, for the detection of $b_k(i)$, for the linear detector (Fig. 4) is given by

$$P_{s,l}(\text{error}) = \frac{1}{2^K} \sum_{\mathbf{j1} \in A^{K-1}} \operatorname{erfc}\left(\sqrt{\frac{\eta}{2\sigma^2}}\right) \qquad (32)$$

with η being the k^{th} element of the column matrix $\mathbf{G}_s\mathbf{H}_s\mathbf{j}(i, i)$ and σ^2 being the (k, k) element of the matrix $\mathbf{G}_s\mathbf{Q}_s\mathbf{G}_s^T$, where $\mathbf{G}_s$, $\mathbf{H}_s$ and $\mathbf{Q}_s$ are given by equations (23), (15) and (16) respectively. The expression for asynchronous system is also similar with K being replaced by $K(2M+1)$ and averaging over all the possible users' symbols. Note that in this detector, for asynchronous systems, since all the symbol epochs are known, equation of the form (32) would give the exact error probability.

C. Decision-Based Detector

For the decision-based detector (Figs. 5 7), it is also possible to find the exact average error probabilities, if the detector used to make the initial decisions is simple enough (for example, the conventional detector or the linear detector). When the initial decisions are made by using the conventional detector (using the diversity), the average symbol error probability is given by

1) *Synchronous Systems:*

$$P_{s,d}(\text{error}) = \sum_{\mathbf{j1}(\mathbf{i}) \in A^{K-1}} \sum_{\hat{\mathbf{j1}}(\mathbf{i}) \in A^{K-1}} \frac{1}{2} \operatorname{erfc}\left(\frac{\sum_{l=1}^{L} \frac{\beta_l}{\sigma_l^2}}{\sqrt{2\sum_{l=1}^{L} \frac{E_1^{(l)}(i)}{\sigma_l^2}}}\right)$$

$$\cdot \operatorname{ERF}(K-1; \mathbf{j}(i,i); \hat{\mathbf{j}}(i); \mathbf{C_1}(i,i)) \qquad (33)$$

Here,

$$\operatorname{ERF}(K-1; \mathbf{j}; \hat{\mathbf{j}}; \mathbf{C_1}) = \frac{1}{(2\pi)^{\frac{K-1}{2}} \mid \mathbf{C_1} \mid^{\frac{1}{2}}}$$

$$\cdot \int_{c_1}^{d_1} \cdots \int_{c_{K-1}}^{d_{K-1}} \exp\left[-(\mathbf{z} - \bar{\mathbf{z}})^T \mathbf{C_1}^{-1} (\mathbf{z} - \bar{\mathbf{z}})\right] dz_1 \cdots dz_{K-1}, \qquad (34)$$

where $\mathbf{z} = [z_1, \cdots, z_{K-1}]^T$ is a Gaussian random vector with mean $\bar{\mathbf{z}}$ and covariance matrix $\mathbf{C_1}(i, i)$, $\hat{\mathbf{j}}(i) = [0, \hat{j}_1(i), \cdots, \hat{j}_{K-1}(i)]^T$ and let $\mathbf{C}(i, i) = \frac{1}{\sigma_l^2} \sum_{l=1}^{L} \mathbf{C}^{(l)}(i, i)$, then β_l is the element in the first row of the column matrix $\mathbf{C}^{(l)}(i, i)\left(\mathbf{j}(i, i) - \hat{\mathbf{j}}(i)\right)$. The k^{th} element of $\bar{\mathbf{z}} = [\bar{z}_1, \cdots, \bar{z}_{k-1}]^T$ is the sum of all the elements in the $(k+1)^{\text{th}}$ row of $\mathbf{C}(i, i)\mathbf{j}(i)$ and $\mathbf{C_1}(i, i)$ is the submatrix of the $(1, 1)$ element of $\mathbf{C}(i, i)$ and

$$c_k = \begin{cases} -\infty & \text{if } \hat{j}_k(i) = -1 \\ 0 & \text{if } \hat{j}_k(i) = 1 \end{cases},$$

$$d_k = \begin{cases} 0 & \text{if } \hat{j}_k(i) = -1 \\ \infty & \text{if } \hat{j}_k(i) = 1 \end{cases}.$$

2) *Asynchronous Systems:* The expression for this case is also given by (33), with β_l being given by the first element of the column matrix $\sum_{n=-1}^{1} \mathbf{C}^{(l)}(i, i+n)[\mathbf{j}(i, i-n) - \hat{\mathbf{j}}(i-n)]$ with the summation taken over $\mathbf{j1}(i-1), \mathbf{j1}(i)$ and $\mathbf{j1}(i+1)$, as well as $\hat{\mathbf{j1}}(i-1), \hat{\mathbf{j1}}(i)$ and $\hat{\mathbf{j1}}(i+1)$, $\bar{\mathbf{z}}$ is a $3(K-1)$ vector with its k^{th} element being given by the sum of all the elements in the $(k+1)^{\text{th}}$ row of $\sum_{n=-1}^{1} \mathbf{C}(i, i+n)\mathbf{j}(i, i-n)$ and $\mathbf{C_1}(i, i)$ is the submatrix of the (1,1) element of $\sum_{n=-1}^{1} \mathbf{C}(i, i+n)$.

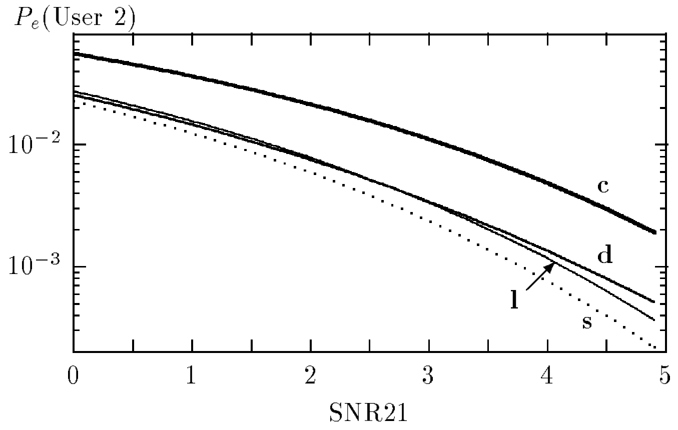

Fig. 8. Average error probability for three users as a function of SNR21 for SNR1 = 5 dB, and ρ = 0.3 (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

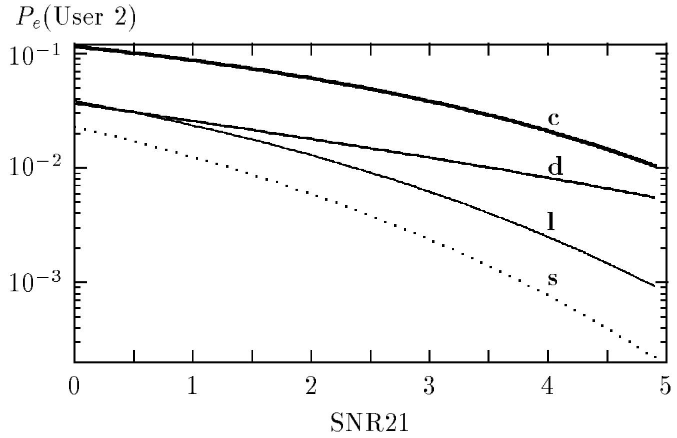

Fig. 9. Average error probability for three users as a function of SNR21 for SNR1 = 5 dB, and ρ = 0.5 (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

V. Numerical Results

To evaluate the system performance, we consider some examples. Consider a system comprising of three users and two sensors (K = 3, L = 2). We shall label the users 1, 2 and 3, while we label the sensors as 1 and 2. The first two users are tracked by sensor 1, while the last two are tracked by sensor 2 (i.e., all the users are not tracked by all the sensors). Let

$$\mathbf{R}^{(1)} = \mathbf{R}^{(2)} = \begin{bmatrix} 1 & \rho_1 & 0 \\ \rho_1 & 1 & \rho_2 \\ 0 & \rho_2 & 1 \end{bmatrix},$$

$$\mathbf{E}^{(1)} = \begin{bmatrix} E_1^{(1)} & 0 & 0 \\ 0 & E_2^{(1)} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{E}^{(2)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2^{(2)} & 0 \\ 0 & 0 & E_3^{(2)} \end{bmatrix},$$

and σ_1^2 and σ_2^2 be the two-sided power spectral density of the additive noise at the two sensors and we assume them to be equal. We are interested in the performance of the detector with respect to user 2. Let $E_2^{(1)}$ and $E_2^{(2)}$, the energies of the second user at the two sensors, be equal. Further, let us assume that $E_1^{(1)} = E_3^{(2)}$ and $\rho_1 = \rho_2 = \rho$. Define SNR_{ij} as the SNR of the i^{th} user at the j^{th} sensor; i.e. $\frac{E_i^{(j)}}{2\sigma_1^2}$, where $2\sigma_l^2$ is the two-sided noise power spectral density at the sensor. We consider the cases where SNR11 = SNR32 = SNR1 $\left(= \frac{E_1^{(1)}}{2\sigma_1^2} \right)$ is 5dB as well as 10 dB. Using the results of section IV, we plot in Figs. 8 - 13, the average error probabilities of the conventional detector, the linear detector, the decision based detector and a single-user detector (optimal detector used when there were no multi-user interference - see section III.A) as a function of the signal-to-noise ratio of the second user, SNR21 for various values of ρ, as well as a function of ρ for various values of SNR21. Even for higher values of ρ, the performance of linear detector and the decision based detector (Figs. 10 and 13) seem to be much better than the conventional detector. Their performance is actually very close to that of

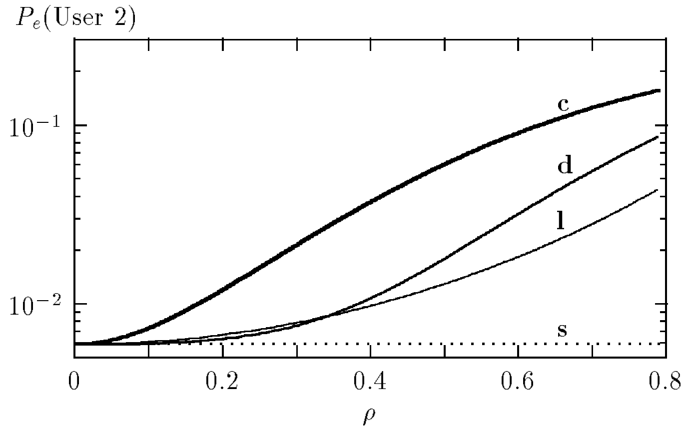

Fig. 10. Average error probability for three users as a function of ρ for SNR1 = 5 dB, and SNR21 = 2 dB (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

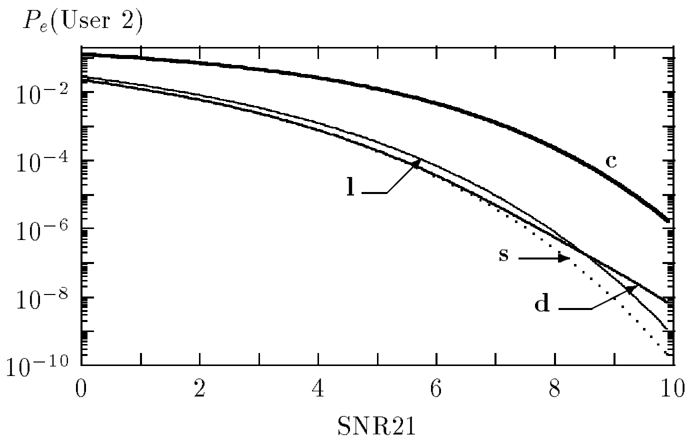

Fig. 11. Average error probability for three users as a function of SNR21 for SNR1 = 10 dB, and ρ = 0.3 (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

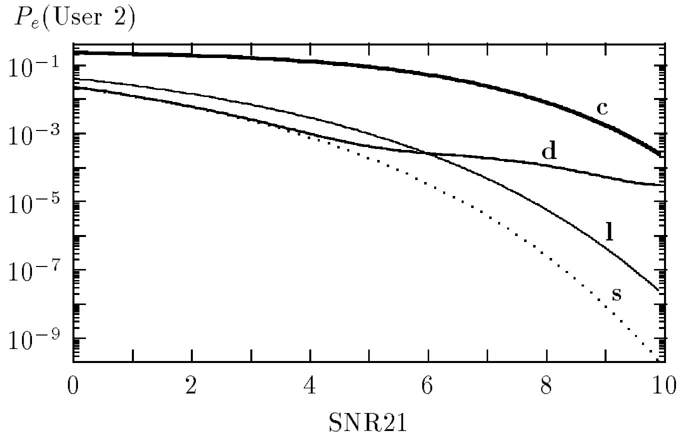

Fig. 12. Average error probability for three users as a function of SNR21 for SNR1 = 10 dB, and $\rho = 0.5$ (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

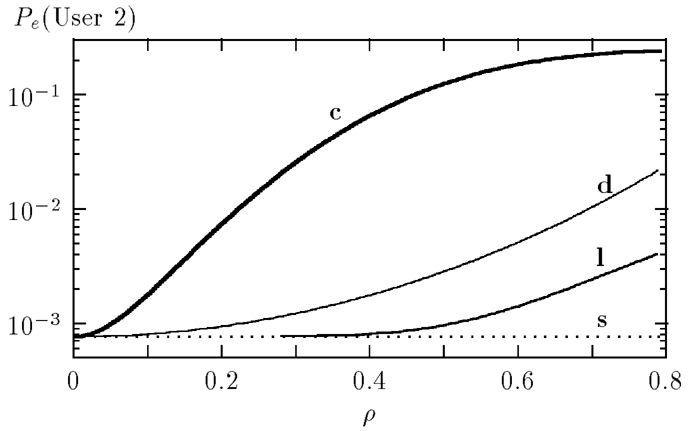

Fig. 13. Average error probability for three users as a function of ρ for SNR1 = 10 dB, and SNR21 = 4 dB (detectors: **c** - conventional, **d** - decision based, **s** - single user, **l** - linear).

the single-user detector for ρ's as high as 0.3 (Figs. 8 and 11) and gives a reasonable performance even for $\rho = 0.5$ (Figs. 9 and 12). Clearly, the mitigation of the near-far effect is demonstrated by these results. As expected [4], the decision based detector performs very well when SNR21 is either very low or very high compared to SNR1. However, when SNR21 is in the vicinity of SNR1 (Fig. 9), its performance is poor compared to that of the linear detector. This is due to the fact that as SNR21 is increased, it causes significant interference to the other users' signals and hence the initial estimates made by the conventional receiver are not as reliable as when SNR21 is very low or very high.

In the next example, we will illustrate the effect of diversity in a more complex system. Consider a synchronous system having six users with two sensors (combined) being able to track them. We use the sequences described in [15, Table 5.10] as the signature sequences. These sequences are not optimized in any sense and have (reasonably) high correlations. The correlation matrix (see (6)) is

$$\mathbf{R}^{(l)}(0) = \frac{1}{31}\begin{bmatrix} 31 & -9 & -9 & -9 & -9 & 11 \\ -9 & 31 & -9 & 11 & -9 & -9 \\ -9 & -9 & 31 & -9 & 11 & -9 \\ -9 & 11 & -9 & 31 & -9 & -9 \\ -9 & -9 & 11 & -9 & 31 & -9 \\ 11 & -9 & -9 & -9 & -9 & 31 \end{bmatrix}. \qquad (35)$$

In Table V, we consider the case when all the users have equal signal-to-noise ratios. In this table, we tabulate the error probabilities P_e for the conventional detector without and with diversity for various values of SNR. Also tabulated is the effective Signal-to-Interference ratio in dB that is required to obtain the same error probability if there were no interference, i.e., if the error probability for a given user using a given detector is P_e, then we define SIR as the effective signal-to-noise ratio of that user, given by the implicit equation,

$$P_e = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\mathrm{SIR}}\right). \qquad (36)$$

SIR is analogous to the signal-to-interference ratio that is commonly used in spread-spectrum systems [15]. Also tabulated is the average signal-to-interference ratio (ASIR), which we define, for a given user, as

$$\mathrm{ASIR} = \frac{\{\mathbb{E}[d(i)]\}^2}{\mathrm{var}[d(i)]}, \qquad (37)$$

where E denotes the expectation, var denotes the variance and $d(i)$ is the variable on which the decision is made for that user (applied to a threshold device). Note that the numerator would be same as the total signal energy as we have assumed equi-probable symbols. Hence, ASIR can be interpreted as the ratio of the signal energy of the user of interest to the sum of the variance of the additive white noise and the interference power from all the interfering users at the input of the threshold device. This would be a true measure of the interference, if the interference can be assumed to be a Gaussian variable [16]. However, in all the examples considered, this assumption cannot be invoked. Despite this fact, we find ASIR to be a convenient measure, since as can be seen from the table, ASIR is very close to SIR. In Table V, we tabulate the same for a linear detector.

Table V indicates[2] that, as can be expected, the performance of the conventional receiver using one sensor is very poor. Even when each of the users' signal-to-noise ratio is as high as 12 dB, the error probability is 0.08. The poor performance is also well illustrated by both SIR and ASIR which are in the vicinity of 0 dB. The performance does not really improve even when we use two sensors (the SIR's are still too close to 0 dB). When we use the linear detector with one sensor, the performance, again as can be expected, is improved to a great extent. However, still the SIR is about 2 dB less than SNR. When two sensors are used, the performance of a single-user detector is improved by 3 dB due to diversity. However, the SIR of the linear

[2] All the results in the tables were evaluated using the expressions in section IV.

TABLE I

Error Probabilities and Corresponding SIR's for Conventional Detector with 6 Users

SNR	**Conventional Without Diversity**			**Conventional With Diversity**		
	P_e	SIR	ASIR	P_e	SIR	ASIR
6dB	9.99×10^{-2}	-0.85	-0.71	8.84×10^{-2}	-0.40	-0.22
8dB	9.15×10^{-2}	-0.52	-0.35	8.30×10^{-2}	-0.18	-0.02
10dB	8.56×10^{-2}	-0.28	-0.11	7.74×10^{-2}	0.05	0.10
12dB	8.03×10^{-2}	-0.07	0.05	7.11×10^{-2}	0.32	0.19

TABLE II

Error Probabilities and Corresponding SIR's for Linear Detector with 6 Users

SNR	**Linear Without Diversity**			**Linear With Diversity**		
	P_e	SIR	ASIR	P_e	SIR	ASIR
6dB	1.28×10^{-2}	3.96	3.96	1.05×10^{-3}	6.75	6.75
8dB	2.90×10^{-3}	5.80	5.80	6.52×10^{-5}	8.65	8.65
10dB	3.08×10^{-4}	7.69	7.68	8.94×10^{-7}	10.58	10.57
12dB	9.86×10^{-6}	9.60	9.60	1.12×10^{-9}	12.53	12.52

detector is still approximately 2 dB less than that of the total (interference-free) signal-to-noise ratio (sum of SNR and a diversity gain of 3 dB).

In the final example, we will study the effect of partial tracking. Consider the case when there are 10 users and 2 sensors. Again, the signature sequence is of length 31. The first six sequences are the auto-optimal m-sequences given in [17, Fig. A1], while the other four have the least side-lobe energy phase with the best odd auto-correlation parameters from [17, Fig. B1]. With these sequences and for a synchronous system, the correlation matrix (i.e., $\mathbf{R}^{(l)}(0)$ of 6) is

$$\frac{1}{31}\begin{bmatrix} 31 & -1 & -1 & 7 & -1 & -1 & -1 & 7 & 3 & -1 \\ -1 & 31 & -9 & -9 & 7 & -1 & -1 & -1 & -1 & 3 \\ -1 & -9 & 31 & -1 & -1 & -5 & -1 & -1 & 7 & -1 \\ 7 & -9 & -1 & 31 & 7 & 7 & -9 & 7 & -1 & -1 \\ -1 & 7 & -1 & 7 & 31 & 7 & -9 & -1 & -1 & -1 \\ -1 & -1 & -5 & 7 & 7 & 31 & -1 & -9 & -9 & 7 \\ -1 & -1 & -1 & -9 & -9 & -1 & 31 & 7 & 3 & -1 \\ 7 & -1 & -1 & 7 & -1 & -9 & 7 & 31 & 7 & -5 \\ 3 & -1 & 7 & -1 & -1 & -9 & 3 & 7 & 31 & -1 \\ -1 & 3 & -1 & -1 & -1 & 7 & -1 & -5 & -1 & 31 \end{bmatrix}.$$

Even though the correlations are reasonably high, they are much smaller compared to the ones in the previous example. As before, we tabulate the SNRs of each user at each sensor and the corresponding error probabilities for both the conventional and linear detectors with and without diversity in Tables V and V. Note that for the case without diversity, the error probabilities tabulated for users 1-4 are based on the detection at sensor 1, while for users 7-10, they are based on detection at sensor 2. For users 5 and 6, we tabulate the error probabilities at both the sensors. Also, we tabulate both SIR and ASIR for each of these cases. From Table V, we can see the performance degradation due to the interference. For example, for user 2, even though the SNR is 8 dB, the effective SIR is only 2.79 dB. Further, a meaningful detection can be hardly performed for users 5 and 6. The situation is not much improved even when both the sensors are used.

The results tabulated in Table V are for the optimal linear detector (in the mean square sense), i.e., this detector requires information of all the users' whose energy is significant at both the sensors. As can be expected, better performance can be obtained by using the multi-user detector. However, due to the weakness of the original signals from users 5 and 6 at both the sensors, the performance is still not very satisfactory. By using both the sensors, we can again obtain a gain of about 3 dB.

Finally, we tabulate the results when a detector tracks only some of the users (whose energy is significant) in Table V. In this example, again, each sensor tracks 6 users (sensor 1 tracks users 1-6 and sensor 2 tracks users 5-10). When diversity is not used, this partial detector can be thought as the linear detector which would be used (as opposed to the one referred previously where all the users are tracked). This detector performs very well for some users, but is significantly inferior for the other users, even when they have significant energy. For example, for user 8, the performance is about 1 dB worse than that for the optimal linear detector. When diversity is used, the performance is inferior to that of the optimal linear detector (with diversity) by only about 0.1 dB.

VI. Conclusions

In this paper, we highlighted the diversity which can enhance performance and which will be available in most of the cellular networks and PCS systems. We further developed sub-optimum receivers for both synchronous and asynchronous networks using the concept of multi-user detection. These detectors combine the advantages of using both diversity as well as multi-user detection. This not only mitigates the near-far effect, but also promises to yield

TABLE III

Error Probabilities and Corresponding SIR's for Conventional Detector with 10 Users

User	SNR @1	SNR @2	Conventional Without Diversity			Conventional With Diversity		
			P_e	SIR	ASIR	P_e	SIR	ASIR
1	8 dB	-100 dB	1.79×10^{-3}	6.29	5.73	1.79×10^{-3}	6.29	5.73
2	8 dB	-100 dB	2.57×10^{-2}	2.79	2.59	2.57×10^{-2}	2.79	2.59
3	8 dB	-100 dB	5.25×10^{-3}	5.15	4.45	5.25×10^{-3}	5.15	4.45
4	8 dB	4 dB	3.31×10^{-2}	2.27	2.22	2.72×10^{-2}	2.67	2.59
5	5 dB	5 dB	7.73×10^{-2}	0.05	0.14	4.25×10^{-2}	1.71	1.71
			6.31×10^{-2}	0.68	0.72			
6	5 dB	5 dB	4.98×10^{-2}	1.32	1.33	3.89×10^{-2}	1.92	1.90
			1.19×10^{-1}	-1.59	-1.39			
7	4 dB	8 dB	1.41×10^{-2}	3.82	3.63	1.20×10^{-2}	4.06	3.65
8	-100 dB	8 dB	2.62×10^{-2}	2,74	2.67	2.62×10^{-2}	2,74	2.67
9	-100 dB	8 dB	7.55×10^{-3}	4.70	4.35	7.55×10^{-3}	4.70	4.35
10	-100 dB	8 dB	2.37×10^{-3}	6.01	5.72	$2,37 \times 10^{-3}$	6.01	5.72

TABLE IV

Error Probabilities and Corresponding SIR's for Linear Detector with 10 Users

User	SNR @1	SNR @2	Linear Without Diversity			Linear With Diversity		
			P_e	SIR	ASIR	P_e	SIR	ASIR
1	8 dB	-100 dB	2.89×10^{-4}	7.73	7.73	2.60×10^{-4}	7.80	7.80
2	8 dB	-100 dB	9.76×10^{-4}	6.81	6.81	6.72×10^{-4}	7.11	7.11
3	8 dB	-100 dB	4.30×10^{-4}	7.45	7.45	3.85×10^{-4}	7.53	7.53
4	8 dB	4 dB	1.24×10^{-3}	6.61	6.61	1.53×10^{-4}	8.14	8.14
5	5 dB	5 dB	1.23×10^{-2}	4.02	4.02	6.23×10^{-4}	7.17	7.17
			9.80×10^{-3}	4.35	4.35			
6	5 dB	5 dB	8.79×10^{-3}	4.50	4.50	6.89×10^{-4}	7.09	7.09
			1.60×10^{-2}	3.62	3.62			
7	4 dB	8 dB	8.13×10^{-4}	6.96	6.96	6.82×10^{-5}	8.62	8.62
8	-100 dB	8 dB	1.01×10^{-3}	6.78	6.78	5.68×10^{-4}	7.24	7.24
9	-100 dB	8 dB	3.83×10^{-4}	7.53	7.53	3.22×10^{-4}	7.66	7.66
10	-100 dB	8 dB	2.96×10^{-4}	7.71	7.71	2.60×10^{-4}	7.80	7.80

TABLE V

Error Probabilities and Corresponding SIR's for Partially Tracking Linear Detector with 10 Users

User	SNR @1	SNR @2	Partial Linear Without Diversity			Partial Linear With Diversity		
			P_e	SIR	ASIR	P_e	SIR	ASIR
1	8 dB	-100 dB	2.91×10^{-4}	7.73	7.73	2.86×10^{-4}	7.74	7.74
2	8 dB	-100 dB	1.07×10^{-3}	6.74	6.73	7.98×10^{-4}	6.98	6.98
3	8 dB	-100 dB	4.61×10^{-4}	7.40	7.40	3.91×10^{-4}	7.52	7.52
4	8 dB	4 dB	2.74×10^{-3}	5.87	5.68	1.28×10^{-3}	6.58	6.58
5	5 dB	5 dB	1.85×10^{-2}	3.38	3.38	7.24×10^{-4}	7.05	7.05
			1.04×10^{-2}	4.26	4.26			
6	5 dB	5 dB	9.15×10^{-3}	4.45	4.45	7.87×10^{-4}	6.99	6.99
			2.72×10^{-2}	2.67	2.62			
7	4 dB	8 dB	2.25×10^{-3}	6.06	5,77	8.06×10^{-4}	6.97	6.97
8	-100 dB	8 dB	2.99×10^{-3}	5.77	5.43	7.05×10^{-4}	7.08	7.08
9	-100 dB	8 dB	3.83×10^{-4}	7.53	7.53	3.23×10^{-4}	7.65	7.65
10	-100 dB	8 dB	3.08×10^{-4}	7.69	7.69	2.60×10^{-4}	7.80	7.80

increased network capacities. The complexity of these receivers is linear in both the number of users as well as the degree of diversity. Thus, by using the partial tracking strategy, it seems that the increase in complexity does not seem to be a big factor given the added advantage of diversity (whose degree will also not be very large, to begin with). Using the diversity, however, involves sending the sufficient statistics to the switch which leads to a higher traffic requirement on the base station-to-switch link. If the sufficient statistic is quantized and eight bits are used for each variable, then the traffic on the channel from the base station to the switch would be increased by a factor that is slightly greater than eight (due to overhead). Fortunately, most of these channels would be based on fiber links which can support high data rates for a low cost. Hence, the extra complexity as well as the high data rate between the sensors and the switch seems to be justified, as the multi-user, multi-sensor detectors promise to provide better network performance. The detectors discussed in this paper can be modified to obtain sub-optimal detectors for multi-path fading channels which yield good performance [11] [18].

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