Distributed Cross-Layer Optimization of Wireless Sensor Networks: A Game Theoretic Approach

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***Abstract*— This paper proposes a distributed optimization framework for wireless multihop sensor networks base on a game theoretic approach. We show that the cross-layer optimization problem can be decomposed into two subproblems corresponding to two separate layers (the physical and the application layers) of the overall system. By modelling each subproblem as a noncooperative game, we aim to solve the noncovex application-layer rate-allocation and physical-layer power-allocation subproblems in a distributed manner. Further, we prove the existence, uniqueness, and stability of the Nash equilibria for both games under certain sufficient conditions. Finally, we show that by using a set of dual variables as the market prices to coordinate the physical layer supply and the application layer demand, the overall optimization process strikes a right balance between the two layers in an overall cross-layer design.**

I. Introduction

Wireless sensor networks have a wide range of applications, such as military security, traffic control, and environmental monitoring. A sensor network consists of a large number of sensors deployed in a field. Each sensor makes a local observation of some underlying physical phenomenon, quantizes its observation, and transfers the data back to a central estimation office (i.e., CEO). Due to the limited transmission power, sensors that are far away from the CEO deliver their quantization data through a multihop network as shown in Fig. 1.

The goal of the sensor network design is to estimate the underlying physical phenomenon as accurately as possible under the network resource limitation. Thus, the sensor network problem can be formulated as a network optimization problem, in which the objective is to the minimize the overall distortion, i.e., the difference between the true underlying field and its estimation at the CEO. However, due to the partial observation at each sensor, the overall estimation error at CEO is a coupled and nonseparable function of all sensors' data rates. In addition, due to the shared nature of the wireless medium, geographically close transmissions often interfere with each other. Because of the interference, the traditional 'bit-pipe' assumption on the logical link capacity no longer holds.

We address the above issues in this paper by considering the fundamental performance limits of sensor networks. We adopt a separate source-channel coding model and use information theoretical concepts such as rate-distortion region and capacity region to gain insights into the fundamental tradeoffs in wireless sensor network design. In our previous paper [1], we showed that the overall network optimization problem may be decomposed in the dual domain into two disjoint subproblems: a power control subproblem at the physical layer and a source coding subproblem at the application layer. A set of dual variables can then be used to coordinate the interaction between the layers.

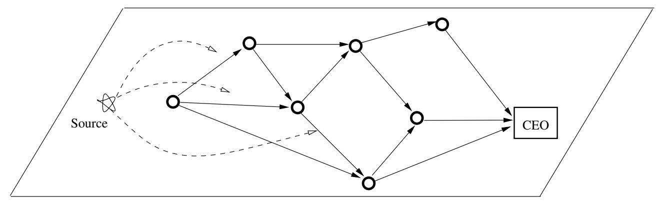

Fig. 1. Sensor Networks

This paper focuses on efficient and distributed solutions to each of these subproblems. At a first glance, neither subproblem appears to be easy to solve due to the inherent nonlinearity and nonconvexity of the problems. Further, even if an algorithm for finding the global solution is available, it may not be amendable to distributed implementation. As realistic sensor network deployment often encounters variations in both source statistics and physical layer channel characteristics, real-time and distributed algorithms are desired.

In this paper, we adopt a game-theoretic approach to solve each subproblem. Game theory has been widely applied to communications problems in the literature [2], [3], [4], [5]. However, existing formulations tend to focus on the physical layer exclusively. In this paper, we generalize the application of game theory to multiple layers in a cross-layer design of wireless sensor networks. Our main contributions are the following:

- We formulate a power control game at the physical layer and a source coding game at the application layer. Both games can be implemented in a distributed fashion.
- We prove sufficient conditions under which both games have unique and stable Nash equilibria.
- We generalize the pricing mechanism for the games by showing that:
  i) the interaction between two games can be coordinated by shadow prices (i.e., dual variables), where the law of demand and supply applies.
  ii) the social optimum of each subproblem can be achieved by proper design of tax/price in the games.

II. Multiple Games in Cross-Layer Optimization

A. Optimization Framework and Decomposition

In a multihop wireless sensor network, the design goal is to minimize the total distortion by jointly optimizing source coding and power allocation. Adopting the setup in [1], the joint optimization problem can be written as:

$$\begin{array}{ll} \text{minimize} & \alpha^T \mathbf{d} \\ \text{subject to} & \mathbf{s} \in \mathcal{R}(\mathbf{d}),\ \mathbf{c} \in \mathcal{C}(\mathbf{p}),\ A\mathbf{c} \geq \mathbf{s} \end{array} \tag{1}$$

where α is a vector representing the relative emphasis on different elements of the distortion vector $\mathbf{d}$; $\mathbf{s}$ is a set of source rates at each node; $\mathbf{c}$ is a set of link capacities; and $\mathbf{p}$ is the power consumption vector. $\mathcal{R}(\mathbf{d})$ is a fundamental concept in source coding, called rate-distortion region. The constraint $\mathbf{s} \in \mathcal{R}(\mathbf{d})$ models the inter-dependence of the distortion on the source rates. $\mathcal{C}(\mathbf{p})$ is a fundamental concept in channel coding, called capacity region. The constraint $\mathbf{c} \in \mathcal{C}(\mathbf{p})$ models the inter-dependence of the link capacity vector on the power consumption. The last inequality $A\mathbf{c} \geq \mathbf{s}$ reflects the fact that the source rate at each node must be less than the link capacity support. Here, A is an $N \times L$ node-incident matrix with N nodes and L links[1]. Using multi-commodity flow (routing) model [6], the matrix elements can be written as:

$$a_{nl} = \begin{cases} 1 & \text{if } n \text{ is the start node for link } l \\ -1 & \text{if } n \text{ is the end node for link } l \\ 0 & \text{else} \end{cases}$$

Applying dual decomposition technique [1], the joint optimization problem can be further decoupled into two distinct subproblems. A power control subproblem at the physical layer

$$\text{maximize } \left\{ \mu^T \mathbf{c} \;\middle|\; \mathbf{c} \in \mathcal{C}(\mathbf{p}) \right\} \tag{2}$$

and a source coding subproblem at the application layer:

$$\text{minimize } \left\{ \alpha^T \mathbf{d} + \lambda^T \mathbf{s} \;\middle|\; \mathbf{s} \in \mathcal{R}(\mathbf{d}) \right\} \tag{3}$$

where μ is related to the dual variable λ by the link price consistency equations : $\mu^T = \lambda^T A$. The Lagrange multipliers λ and μ have the interpretation of being the shadow prices coordinating the application layer *demand* and physical layer *supply*. Mathematically, the shadow price λ can be updated by subgradient method with stepsize ν_λ. This update reflects the law of demand and supply.

$$\lambda^{(k+1)} = \left[\lambda^{(k)} + \nu_\lambda^{(k)} (\mathbf{s} - A\mathbf{c}) \right]^+ \tag{4}$$

The main point of this paper is that both subproblems can be solved from a game theory perspective, where efficient solution and distributed implementation are possible, as is shown in the next subsections.

[1]For simplicity, we consider a network with only one CEO and set its corresponding node-link row as the last row in A. In the formulation of the optimization problem, this last row may be deleted without loss of generality because it is linearly dependent of previous rows.

B. Physical Layer: Power Control Game

The physical layer subproblem addresses the transmission interference among nearby sensors. Using an interference model, we explicitly write down the capacity region (more precisely the achievable region) constraints of (2) as follows:

$$\begin{array}{lll} \max\limits_{\mathbf{p}} & \sum\limits_l \mu_l c_l & \\ \text{s.t.} & c_l = \log(1 + \mathrm{SINR}_l) & \forall l \\ & \mathrm{SINR}_l = \dfrac{G_{ll} p_l}{\sum_{j \neq l} G_{lj} p_j + \sigma_l^2} & \forall l \\ & 0 \leq p_l \leq p_{l,\max} & \forall l \end{array} \tag{5}$$

where c_l is the capacity of link l; SINR_l is the signal to interference and noise ratio of link l; G_{ll} and σ_l^2 are the link gain and noise respectively, G_{lj} is the interference gain from link j to link l, and p_l is link l's power action that has a power constraint $p_{l,\max}$.

Because of the interference structure, the power control subproblem (5) is a nonconvex optimization problem, which is inherently difficult to solve. In this paper, we explore ways of approximating the optimal solution using game theory. Inspired by the work of Saraydar, Mandayam, and Goodman [3], we introduce a tax mechanism into the game so that the players will have an incentive to intelligently avoid interference. More precisely, we formulate a *power control game (PCG)* at the physical layer, under which each link player maximizes its own payoff function

$$\begin{array}{ll} \max\limits_{p_l} & Q_l^{\mathrm{PHY}} = \mu_l \log\left(1 + \dfrac{G_{ll} p_l}{\sum_{j \neq l} G_{jl} p_j + \sigma_l^2}\right) - t_l p_l \\ \text{s.t.} & 0 \leq p_l \leq p_{l,\max} \quad \forall l \end{array} \tag{6}$$

where t_l is the tax rate for link l, and p_l is the action for link l. More power link l uses, more interference it will produce to others; therefore more tax (i.e., $t_l p_l$) it has to pay. One sensible choice of the tax rate is the following:

$$\begin{aligned} t_l &= \left| \frac{\partial \sum_{s \neq l} \mu_s c_s}{\partial p_l} \right| \\ &= \sum_{s \neq l} \frac{\mu_s G_{sl} G_{ss} p_s}{(G_{ss} p_s + \sum_{j \neq s} G_{sj} p_j + \sigma_s^2)(\sum_{j \neq s} G_{sj} p_j + \sigma_s^2)} \end{aligned} \tag{7}$$

where t_l is the rate at which other users' achievable data rates decrease with an additional amount of power.

In general, not every game has a Nash equilibrium, neither is the equilibrium necessarily stable. One of our contributions is the following sufficient condition, under which the game is ensured to converge to a unique and stable Nash equilibrium.

Definition 1: The strictly diagonal dominance (SDD) condition holds, if the channel gain G satisfies:

$$G_{ll} > \sum_{j : j \neq l} G_{lj}, \quad \forall l \tag{8}$$

Theorem 1: If channel gain satisfies the strictly diagonally dominant condition, given the tax rates, the power control game (6) always converges to a unique and stable Nash equilibrium.

Proof: We prove it in Appendix. ∎

Next, we propose the power control game algorithm.

Algorithm 1: **Power Control Game (PCG) Algorithm**

1) Initialize $\mathbf{p}^{(0)}$, $\mathbf{t}^{(0)}$. Set $k = 0$.
2) Set $\mathbf{p}^{(\tau_0)} = \mathbf{p}^{(k)}$. Set $i = 0$, iteratively update $\mathbf{p}^{(\tau_i)}$ as follows:

$$p_l^{(\tau_{i+1})} = \frac{\mu_l}{t_l^{(k)}} - \frac{1}{G_{ll}}\left(\sum_{j\neq l} G_{lj} p_j^{(\tau_i)} + \sigma_l^2\right)$$

project $p_l^{(\tau_{i+1})}$ into power constraint interval $[0, p_{l,\max}]$. Repeat until $\mathbf{p}^{(\tau_i)}$ converges. Set $\mathbf{p}^{(k+1)} = \mathbf{p}^{(\tau_i)}$.

3) Update tax rate t_l

$$\begin{aligned} t_l^{(k+1)} &= \sum_{f\neq l} G_{lf}\mathrm{bcm}_f \\ \mathrm{bcm}_f &= \mu_f \frac{\mathrm{SINR}_f^{(k+1)}}{G_{ff}p_f^{(k+1)}} \frac{\mathrm{SINR}_f^{(k+1)}}{1+\mathrm{SINR}_f^{(k+1)}} \end{aligned}$$

4) Return 2 until convergence.

The power update in step (2) is the best response of link player l given the tax rate and his assessment of others' action. Next, in step (3), the tax rates are updated according to (7). As the tax rates converge, the power control game Algorithm 1 converges to a unique and stable Nash equilibrium. Such power allocation equilibrium strikes a balance between maximizing rate and minimizing interference.

Furthermore, the PCG algorithm can be implemented in a distributed fashion. Specifically, inspired by the work of [7], we propose a two-phase message passing mechanism[2] in step (3). At the first phase, each link calculates its broadcast message (i.e., bcm_f) by local information (i.e., $\mu_f, \mathrm{SINR}_f, G_{ff}, p_f$); and broadcasts to the network. At the second phase, each link collects broadcast messages from others, and computes the tax rate t_l, where the interference term (i.e., G_{lf}) can be estimated, for example, by pilots.

For simplicity, we present the power control game for a scenario in which each link consists of a single channel. The same idea can be extended to the cases in which each link consists of multiple physical channels.

The proposed algorithm is similar to an algorithm proposed by Huang, Berry, and Honig in [5]. However, the authors of [5] focus on a utility $\log(\mathrm{SINR}(\mathbf{p}))$, while our analysis focus on $\log(1 + \mathrm{SINR}(\mathbf{p}))$, which is more realistic in low SINR scenarios. In addition, the proof of convergence is also different. The authors of [5] use the supermodular game theory, while we prove the convergence (Theorem 1) from the learning theory of games.

C. Application Layer: Source Coding Game

The source coding subproblem characterizes the interaction among sensor rates and estimation distortion. Consider an environment sensing application depicted in Fig. 2. The underlying physical phenomenon is denoted as θ, which is a vector of independent random variables. N sensors are deployed in

[2]The control overhead due to message passing should not be neglected. Such overhead may have impact on the scalability issue of sensor network. However, rigorous overhead analysis is out of the scope of this paper.

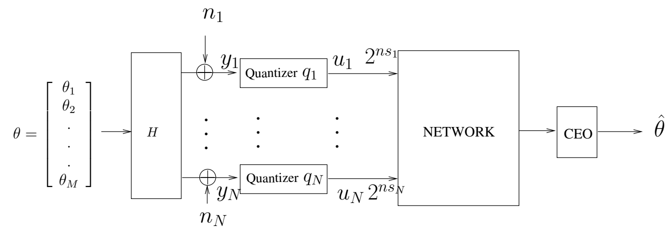

Fig. 2. Distributed Source Coding

the field, each making a local (and possibly partial) observation of θ, while being corrupted by independent observation noise n_i. The observation channel is characterized by a matrix H. At each sensor i, the noisy observation y_i is quantized into a codeword u_i. The quantized information from all sensors is transmitted back through the network to a remote central office (i.e., CEO) with source rates $(s_1, ...s_N)$. At the remote CEO, the decoder first jointly decodes the codewords $\mathbf{u}$, then estimates the source. The source estimation is denoted as $\hat{\theta}$. The performance criterion is to minimize the mean squared error, i.e., $D(\hat{\theta}, \theta) = ||\hat{\theta} - \theta||^2$.

Following the setup in [8], we tackle the source coding subproblem using rate-distortion theory. The rate-distortion region in (3) can be explicitly written as follows:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \alpha^T\mathbf{d} + \sum_{i=1}^N \lambda(i)s_i \qquad (9) \\ \text{s.t.} \quad & \alpha^T\mathbf{d} = \mathrm{tr}\left(R_\theta\right) - \mathrm{tr}\left(R_\theta H^T(HR_\theta H^T + R_w^{-1})^{-1}HR_\theta^T\right) \\ & s_i = \log\left(\frac{1+\sigma_{si}^2 w_i}{1-\sigma_{ni}^2 w_i}\right) \\ & 0 \leq w_i \leq \frac{1}{\sigma_{ni}^2}, \quad \sigma_{si}^2 = h_i^T R_\theta h_i \end{aligned}$$

In this paper, we assume equal weights on distortion elements (i.e., $\alpha = \mathbf{1}$), therefore, the first equality is the MMSE estimation distortion. Here, R_θ is the covariance matrix of underlying phenomenon; R_w is a diagonal matrix with the w_i as ith diagonal element. We further define a variable $\mathbf{w}$, which has the interpretation of *quantization effort*, i.e., the larger the $\mathbf{w}$, the smaller the distortion. The second equality characterizes the dependence of the source rate s_i on the quantization effort w_i. The larger the $\mathbf{w}$, the higher the source rates. Each sensor's observation noise has a variance σ_{ni}^2; h_i^T is the ith row of the channel observation matrix H.

Source coding subproblem aims to find an optimal balance between distortion and source rate. Here, we introduce a price mechanism into *source coding game (SCG)* such that it is easy for nodes to make a good tradeoff in a distributed manner. More precisely, we approximate the source rate as a linear function of quantization effort with a price indicator m_i, thus, each player maximizes its payoff:

$$\begin{aligned} \max_{w_i} \quad & Q_i^{APP} = \mathrm{tr}\left(R_\theta H^T(HR_\theta H^T + R_w^{-1})^{-1}HR_\theta^T\right) \\ & -\lambda_i m_i w_i \\ \text{s.t.} \quad & 0 \leq w_i \leq 1/\sigma_{ni}^2 \qquad (10) \end{aligned}$$

where m_i is the price indicator showing how expensive it is to quantize the source according to the following:

$$m_i = \frac{\partial s_i}{\partial w_i} = \frac{\sigma_{si}^2}{1+\sigma_{si}^2 w_i} + \frac{\sigma_{ni}^2}{1-\sigma_{ni}^2 w_i} \qquad (11)$$

Next, we present the source coding game algorithm.

Algorithm 2: **Source Coding Game (SCG) Algorithm**

1) Initialize
2) At round $(k+1)$, players sequentially update their best response. Repeat from $i=1$ to $i=N$.
   - 2.1 Each sensor i updates w_i as follows
   $$w_i^{(k)} = \text{argmax}_{w_i} Q_i^{\text{APP}}$$
   - 2.2 Update rate price m_i
   $$m_i^{(k+1)} = \frac{\sigma_{si}^2}{1+\sigma_{si}^2 w_i^{(k)}} + \frac{\sigma_{ni}^2}{1-\sigma_{ni}^2 w_i^{(k)}}$$
   - 2.3 Broadcast w_i
3) Repeat (2) until convergence

The update strategy in step (2) is the best response for each player i given its assessment of other players action. It is also possible to implement the algorithm in a distributed manner by a message passing mechanism [7]. This is done in step (2.3) of the SCG algorithm: each sensor broadcasts its quantization effort (w_i), therefore, the best response can be calculated locally in step (2.1).

We further proceed to examine the convergence and global optimality of the source coding game algorithm. According to the work of [9], the effect of quantizer q_i in Fig. 2 can be modelled as a Gaussian random variable with zero mean and variance σ_{qi}^2. This variance is related to the quantization effort and observation noise, i.e., $\sigma_{qi}^2 = 1/w_i - \sigma_{ni}^2$. We define the following condition.

Definition 2: The source coding optimality (SCO) condition holds, if any of the following is true:

- $$\sigma_{ni}^2 \geq \sigma_{si}^2 \qquad (12)$$
- $$\sigma_{ni}^2 < \sigma_{si}^2, \quad \text{and} \quad \sigma_{qi}^2 \in \left[0, \frac{\sigma_{si}^2 + \sigma_{ni}^2}{\sigma_{si}^2 - \sigma_{ni}^2} \sigma_{ni}^2\right] \qquad (13)$$

where $\sigma_{si}^2 = h_i^T R_\theta h_i$.

This is a reasonable set of conditions because of the following. The first condition characterizes a scenario in which the sensor noise variance is larger than the variance of the underlying physical phenomenon. The second condition characterizes a scenario in which the sensor noise variance is smaller and the quantization variance is smaller than sensor noise variance times a constant which is larger than 1. (Note that a high resolution quantizer has a small quantization variance.) In most practical quantizer design, the quantization resolution is almost always set to be below the sensor noise. Thus, the SCO condition almost always holds in a well designed sensor network.

Theorem 2: The source coding game Algorithm 2 converges to a unique and stable Nash equilibrium that is the global optimum for subproblem (9), provided that the SCO holds.

Proof: Due to the space limitation, we briefly outline the proof. According to the definition of rate price m_i, we claim that the Nash equilibrium of the source coding game (10) is precisely the local optimum of (9). It can be shown that under the SCO condition, the optimization problem of (9) is convex by checking that the Hessian is positive semi-definite. Therefore, (9) has a unique local optimum that is globally optimal. Hence, the Nash equilibrium is unique and globally optimal. ■

III. Primal-Dual Algorithm and Illustration

A. Distributed Primal-Dual Algorithm

In this section, we present a distributed primal-dual algorithm, which iteratively executes power control game and source coding game, and updates shadow prices.

Algorithm 3: Distributed Primal-Dual Algorithm

1) Initialize $\lambda^{(0)}$
2) At time (k), given the price $\lambda = \lambda^{(k)}$, set $\mu^T = \lambda^T A$
$$\begin{aligned} \text{PCG Algorithm 1} &\rightarrow \mathbf{c}^G, \mathbf{p}^G \\ \text{SCG Algorithm 2} &\rightarrow \mathbf{s}^G, \mathbf{d}^G \end{aligned}$$
3) In dual domain, update λ using the following rule:
$$\lambda^{(k+1)} = \left[\lambda^{(k)} + \nu_\lambda^{(k)} (\mathbf{s}^G - A\mathbf{c}^G)\right]^+ \qquad (14)$$
4) Return to step 2 until convergence.

The dual price update (14) reflects the law of demand and supply. For example, when the application layer demand $\mathbf{s}^G$ is greater than the physical layer supply $A\mathbf{c}^G$, the price will increase. Furthermore, the price update can be accomplished in a distributed way, because the λ_i update requires only the local source coding rate s_i and corresponding income and outcome link capacities. Therefore, combined with the distributed implementation of the games, the entire primal-dual algorithm can be implemented in a distributed manner.

B. Simulation Example

We simulate an example of a wireless sensor network in Fig. 3(a) to illustrate the main ideas. The underlying physical phenomenon to be observed is a two dimension Gaussian vector with an identity covariance matrix. For the sake of simplicity, we assume that only the nearest two nodes (e.g. sensor 1, 2) are active in sensing the field, while the rest nodes act as relays. The model for the physical layer is an interference channel, where each link consists of multiple sub-channels.

We use the proposed primal-dual algorithm to find an optimal solution for the joint source coding and power control problem in the multihop network. Both source coding game algorithm and power control game algorithm are implemented in a distributed fashion.

Fig. 3(b) shows the convergence process between the application layer source coding game and the physical layer power control game. At the beginning, the application layer demand of source rates is high, while the physical layer supply of link capacities is low. During the iterations, the shadow prices as shown in Fig. 3(c) coordinate both physical layer and application layer moving toward the market equilibrium. Under this market equilibrium, all link capacities exactly

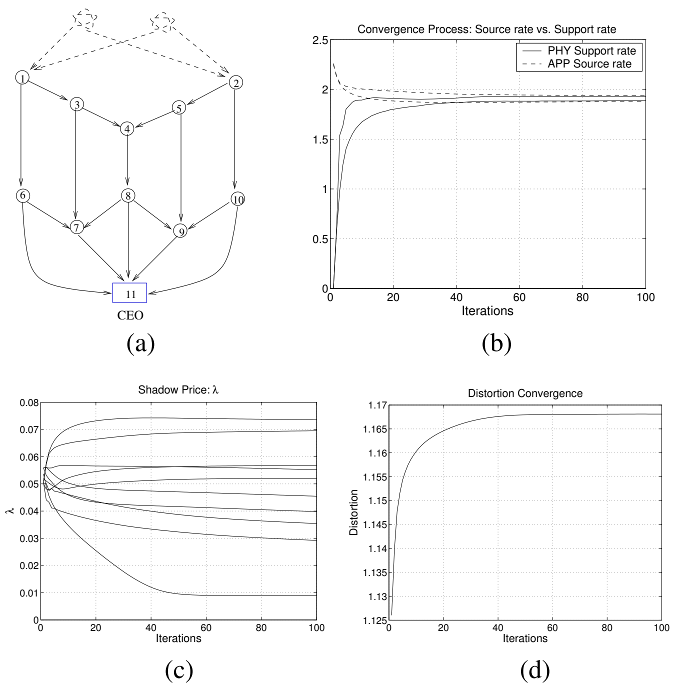

Fig. 3. (a) Sensor network topology; (b) Convergence process between source rates and capacity support, (c) Convergence process of dual variable λ, (d) Convergence process of network utility, i.e., estimation distortion.

support the source rates (i.e., $A\mathbf{c} = \mathbf{s}$) as shown in Fig. 3(b). Finally, Fig. 3(d) illustrates the graceful convergence of the overall distortion.

IV. Conclusions

In this paper, we tackle the general nonlinear and nonconvex optimization problem for wireless sensor networks from a game theoretic perspective. The incorporation of game theory in a cross-layer framework allows the overall network optimization problem to be solved approximately in a distributed manner.

Appendix: Proof of Theorem 1

We first prove the existence of Nash equilibrium (NE). The action profile set of player l is a nonempty compact convex set, i.e., $p_l \in [0, p_{l,\max}]$. Q^{PHY} is continuous in $\mathbf{p}$, and Q_l^{PHY} is strictly concave in p_l. According to Proposition 20.3 [10], the power control game (6) has at least one pure NE. The best response of link player l is:

$$\text{BR}_l(\mathbf{p}) = \frac{\mu_l}{t_l} - \frac{1}{G_{ll}}\left(\sum_{j \neq l} G_{lj} p_j + \sigma_l^2\right) \tag{15}$$

$$\sum_j \left|\frac{\partial \text{BR}_l(\mathbf{p})}{\partial p_j}\right| = \sum_{j:j\neq l} \left|\frac{\partial \text{BR}_l(\mathbf{p})}{\partial p_j}\right| = \sum_{j:j\neq l} \frac{G_{lj}}{G_{ll}} \tag{16}$$

According to the definition of SDD (8),

$$\sum_{j:j\neq l} \frac{G_{lj}}{G_{ll}} < 1 \;\Rightarrow\; \sum_j \left|\frac{\partial \text{BR}_l(\mathbf{p})}{\partial p_j}\right| < 1 \tag{17}$$

Therefore the best response is contractive. Due to Theorem 3.4 in [11], the game has a unique Nash equilibrium.

Definition 3: The dynamic stability (DS) matrix of the game is a square matrix, whose (l, j)th entry is defined as follows:

$$\text{DS}_{(l,j)} = \frac{\partial \text{BR}_l(\mathbf{p})}{\partial p_j}, \quad \forall\ l,\ j = 1, 2, ...L \tag{18}$$

According to Gersgorin theorem [12], all the eigenvalues of the dynamic stability (DS) matrix are located in the region

$$\bigcup_{l=1}^{L} \left\{ \left|z - \text{DS}_{(l,l)}\right| \leq \sum_{j:j\neq l} \left|\text{DS}_{(l,j)}\right| \right\}$$

Because the diagonal element of DS are all zeros, the region can be further simplified.

$$\bigcup_{l=1}^{L} \left\{ |z| \leq \sum_{j:j\neq l} \left|\text{DS}_{(l,j)}\right| = \sum_{j:j\neq l} \frac{G_{lj}}{G_{ll}} \right\} \subset \bigcup_{l=1}^{L} \left\{ |z| < 1 \right\}$$

Therefore, all the eigenvalues of DS fall into the unit circle. According to [13], a game is asymptotically stable, if the absolute value of eigenvalues of the dynamics stability matrix are all less than one. Hence, the power control game is asymptotically stable, and always converges to a unique, stable Nash equilibrium under SDD.

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