Degrees of Freedom of MIMO Cellular Networks with Two Cells and Two Users Per Cell

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***Abstract*—This paper characterizes the spatially-normalized degrees of freedom of a 2-cell, 2-user/cell MIMO cellular networks with M antennas at each user and N antennas at each base-station. We show that the optimal DoF is a piecewise linear function, with either M or N being the bottleneck. Denoting the ratio M/N as γ, we show that the network has redundant dimensions in both M and N when $\gamma \in \{1/2, 1\}$ and that the network has no redundancy when $\gamma \in \{1/4, 2/3, 3/2\}$. We also show that not all proper systems are feasible and that the only set of feasible proper systems that lie on the proper-improper boundary are those with $\gamma \in \{1/4, 2/3, 3/2\}$. We make comparisons between the DoF achievable using strategies such as time sharing between users or cells and discuss their implications on user scheduling in such networks.**

I. Introduction

The notion of degrees of freedom (DoF) has emerged as a useful yet tractable metric for understanding the role of interference in wireless networks. For example, the celebrated result in [1] shows that the K-user MIMO interference channel with M antennas at each node is not interference limited and that $M/2$ DoF/user is achievable. This result has been extended to other MIMO networks, including the wireless X channel [2], [3], and the cellular networks [4], [5]. All the above works assume that all the nodes in the network have the same number of antennas. This significantly simplifies the analysis and enables the use of asymptotic alignment schemes.

Establishing the optimal DoF for networks with different numbers of antennas at different nodes has proven to be more challenging. In this realm, [6] establishes the DoF for the 3-user interference channel with M antennas at each of the transmitters and N antennas at each of the receivers. In [7], [8], the DoF of the K-user interference channel with M antennas at the transmitters and N antennas at the receivers is studied. A more detailed characterization of this same problem is provided in [9]. The achievable schemes used to establish these results include decomposing these MIMO networks to equivalent SISO networks [7] followed by the application of the schemes in [1], [10], or by careful design of linear transmit beamformers through techniques such as subspace alignment chains [6].

This paper goes beyond the interference channel and considers the DoF of a MIMO cellular network with multiple cells and more than one user per cell, where each user is equipped with M antennas and each base-station (BS) is equipped with N antennas. While several outer bounds [11]–[13] and achievable schemes [13]–[17] are known, an exact characterization of DoF for the case of arbitrary number of cells and arbitrary number of users per cell is expected to be quite challenging (as already evident from [6], [9]). It is in this context that we study the simplest such network having two cells and two users per cell, for which we are able to establish the optimal DoF/user in this paper. Similar in spirit to [6], we allow for spatial extensions of a given network and study the spatially-normalized DoF (sDoF). Spatial extensions are analogous to time/frequency extensions where spatial dimensions are added to the system through addition of antennas at the transmitters and receivers. Unlike time or frequency extensions where the resulting channels are block diagonal, spatial extensions assume generic channels with no additional structure—making them significantly easier to study without the peculiarities associated with additional structure.

The 2-cell, 2-user/cell network has been studied previously in [11], [13], [14]. In [11], [14] optimal interference alignment schemes are proposed for some specific M and N. In [13], a more general case of the network with different number of antennas at each node is studied and the DoF region is characterized in the uplink. In this work, we extend the work of [13] by presenting results for both uplink and downlink, but more importantly we focus attention to the case where all users (and respectively BSs) have the same number of antennas in order to provide analytic insights to the multicell network. Toward this end, we first establish an outer bound on the DoF/user for both the uplink and the downlink for any given M and N. This bound scales with the number of antennas and holds regardless of spatial normalization. We then show that this bound is tight for spatially-normalized DoF by establishing achievability of the bound through linear beamforming techniques. We then identify scenarios with redundant dimensions and comment on the feasibility of proper systems. Finally, we make comparisons to other scheduling choices such as time sharing between users or cells and make observations on when such strategies are optimal from a DoF standpoint.

A better understanding of the DoF/user achievable in MIMO cellular networks can have important consequences in user scheduling in cooperative cellular networks. In cooperative cellular networks where channel state information is shared across multiple base stations, the BSs can jointly beamform to partially or completely cancel inter-cell interference. The ability to completely cancel interference is particularly important at high SNRs. Since DoF is in essence the number of interference free directions available in a network, the achievable DoF informs us of the right number of users to simultaneously schedule and the number of data streams to deliver per user.

As an example of insight that can be gained from a DoF analysis, consider a 2-cell, 2-user/cell network with $(M, N) = (2, 3)$. This paper shows that by simultaneously scheduling both the users, we can achieve one interference-free data stream per user. This is a better strategy, from a DoF standpoint, than time sharing where only 1 user is scheduled per cell (with time sharing, only $3/4$ DoF/user are possible). It can also be shown that cooperation between the two users does not increase the DoF in this case. If however, the BSs had 4 antennas, i.e. $N = 4$, then time-sharing and simultaneous scheduling have the same number of DoF/user. Further, even in this case, cooperation among users does not alter the DoF/user. Interestingly, for this case, even time sharing between the two cells (thereby reducing the network to two non-interfering cells) is optimal from a DoF perspective.

II. System Model

We consider 2 interfering cells with 2 users in each cell. Each user is assumed to have M antennas and each BS is assumed to have N antennas. We denote the channel between the jth user in the ith cell and the kth BS as the $M \times N$ matrix $\mathbf{H}_{(ij,k)}$ and assume all channels to be generic. In the uplink, the jth user in the ith cell is assumed to transmit the $M \times 1$ signal vector $\mathbf{x}_{ij}(t)$. Thus, the received signal at the kth BS is given by

$$\mathbf{y}_k = \sum_{i,j \in \{1,2\}} \mathbf{H}_{(ij,k)} \mathbf{x}_{ij} + \mathbf{n}_k \tag{1}$$

where $\mathbf{n}_k$ is the $N \times 1$ vector representing circular symmetric additive white Gaussian noise $\sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$. The received signal is defined similarly for the downlink.

III. Main Results

We first restate the definition of spatially-normalized DoF as given in [6].

Definition 3.1 ([6]) *Denoting the DoF/user of a 2-cell, 2-user/cell network with M antennas at each user and N antennas at each BS as DoF(M, N), the spatially-normalized DoF/user is defined as*

$$\textit{sDoF}(M, N) = \max_{q \in \mathcal{Z}^+} \frac{\textit{DoF}(qM, qN)}{q}. \tag{2}$$

We define γ to be the ratio M/N. For any given pair (M, N), we define the function $D(M, N)$ as follows:

$$D(M, N) = \begin{cases} M & \gamma \leq 1/4 \\ N/4 & 1/4 < \gamma \leq 1/2 \\ M/2 & 1/2 < \gamma \leq 2/3 \\ N/3 & 2/3 < \gamma \leq 1 \\ M/3 & 1 < \gamma \leq 3/2 \\ N/2 & 3/2 < \gamma \end{cases} \tag{3}$$

We now state the following theorem characterizing an outer bound on the DoF/user of the network.

Theorem 3.1 *The DoF/user of a 2-cell, 2-user/cell MIMO cellular network with M antennas per user and N antennas per base-station is bounded above by*

$$\textit{DoF/user} \leq D(M, N). \tag{4}$$

Note that since this outer bound is linear in either M or N, this bound is invariant to spatial normalization and hence is also a bound on sDoF and not just DoF. This bound is derived from existing outer bounds for networks such as the 2-user MIMO interference channel [18] and the SISO X channel [2]. For the uplink, this bound can also be derived using results in [13]. Details of the proof are presented in Section V-A.

The next theorem characterizes the sDoF/user of the network under consideration.

Theorem 3.2 *The spatially-normalized DoF of a 2-cell, 2-user/cell MIMO cellular network with M antennas per user and N antennas per base-station is given by*

$$\textit{sDoF/user} = D(M, N). \tag{5}$$

This result states that when spatial-extensions are allowed, the outer bound presented in Theorem 3.1 is tight. Since Theorem 3.1 establishes the outer bound, we only need an achievable scheme to complete the proof of this theorem. The details of the proof along with the achievable scheme are presented in V-B.

Finally, the following theorem characterizes the achievable DoF/user when symbol extensions in space/time/frequency dimensions are not allowed.

Theorem 3.3 *For the 2-cell, 2-user/cell MIMO cellular network, with M antennas at each user and N antennas at each base-station, $\lfloor D(M, N) \rfloor$ DoF/user are achievable through linear interference alignment without the need for space/time/frequency extensions.*

The proof of this theorem is straightforward and is briefly outlined in Section V-C.

Fig. 1 captures the main results presented in the above theorems and plots sDoF/user normalized by N as a function of γ. Just as in the 3-user interference channel [6], there is an alternating behavior in the sDoF with either M or N being a bottleneck for a given γ. The figure also plots the sDoF/user for other strategies such as time sharing between users/cells, and when users are allowed to cooperate. Several interesting observations from Fig. 1 are discussed next.

IV. Key Observations

Redundant Dimensions: Analogous to the observation on redundant dimensions made in [6], we can see from Fig. 1 that

- For $M/N \in (1/4, 1/2) \cup (2/3, 1) \cup (3/2, \infty)$, the value of N is the bottleneck, while M has some redundant dimensions that can be sacrificed while preserving the sDoF.
- For $M/N \in (0, 1/4) \cup (1/2, 2/3) \cup (1, 3/2)$, the value of M is the bottleneck, while N has some redundant dimensions that can be sacrificed while preserving the sDoF.

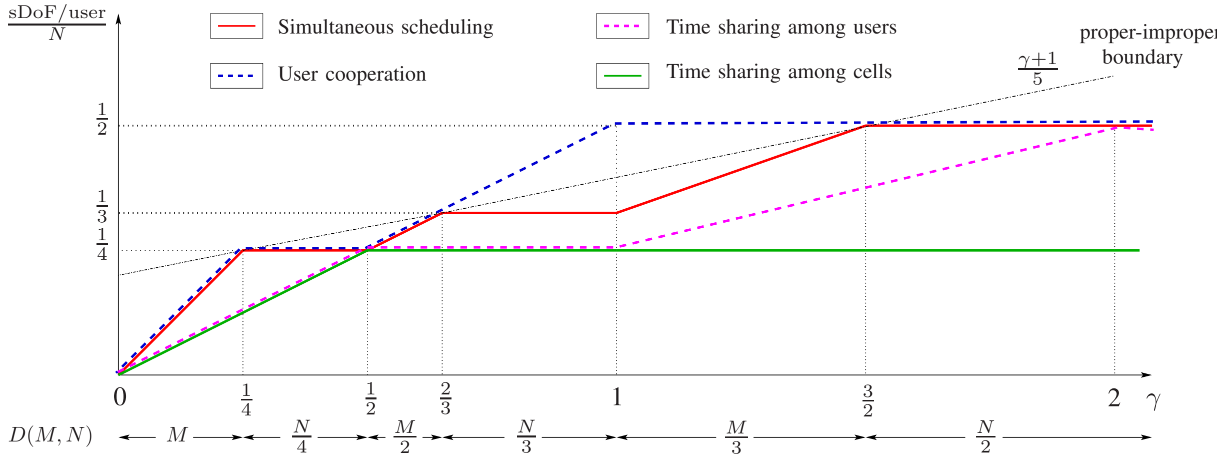

Fig. 1. Figure showing the sDoF/user of a 2-cell, 2-user/cell MIMO cellular network. Also plotted are the sDoF/user when user cooperation is allowed, and when time sharing among users or cells is used.

- For $M/N \in \{1/2, 1\}$, both M and N have redundant dimensions, and some dimensions from one of the two can be sacrificed without losing the sDoF.
- For $M/N \in \{1/4, 2/3, 3/2\}$, neither M nor N has any redundant dimensions, and decreasing either of them affects the sDoF.

For example, note that the networks with $(M, N) = (12, 12)$, $(M, N) = (8, 12)$ and $(M, N) = (12, 8)$ all have 4 DoF/user. This shows that there are 4 redundant antennas at the user as well as the BS for the network with $(M, N) = (12, 12)$. Note however that we cannot simultaneously decrease dimensions at both the user and the BS.

Feasibility of Proper Systems: Designing transmit and receive beamformers for linear interference alignment is equivalent to solving a system of bilinear equations. A widely used necessary condition to check for the feasibility of linear interference alignment is to check if the total number of variables exceeds the total number of constraints in the system of equations. If a system has more number of variables than constraints then it is called a proper system, otherwise it is called an improper system [19]. For a fully connected wireless network with G cells, K users per cell, M antennas at each user, and N antennas at each BS, if d DoF are desired per user, the network is a proper system if it satisfies [17]

$$M + N \geq d(GK + 1). \tag{6}$$

Substituting $G = 2$ and $K = 2$, a 2-cell, 2-user/cell network is proper iff $\frac{M+N}{5} \geq d$. Equivalently, proper systems satisfy $\frac{1+\gamma}{5} \geq \frac{d}{N}$, and it is seen from Fig. 1 that not all proper systems are feasible. For example, consider designing an achievable scheme to deliver 6 DoF/user for the system with $(M, N) = (10, 20)$. Although such a system is proper, it is easy to see from Fig. 1 that only 5 DoF/user are feasible for such a system, so this is a proper system that happens to be infeasible. In fact, systems with $\gamma \in \{1/4, 2/3, 3/2\}$ are the only sets of systems that lie on the boundary between proper and improper systems and are feasible. Thus, just as in the case of the 3-user MIMO interference channel, most proper systems on the boundary are infeasible.

User Cooperation: If we allow users to cooperate, then the network reduces to a 2-user MIMO interference channel, for which the DoF has been characterized in [18]. For a 2-user interference channel with $2M$ antennas at the transmitter (and respectively receiver) and N antennas at the receiver (and respectively transmitter), the DoF is given by $\min(4M, 2N, \max(2M, N))$. Thus, the sDoF/user normalized by N is given by $\min(\gamma, 1/2, \max(\gamma/2, 1/4))$. As seen in Fig. 1, allowing users to cooperate does not increase the sDoF/user whenever $\gamma \leq 2/3$ or $\gamma \geq 3/2$.

Time Sharing Among Users: If we were to schedule only one user per cell per time slot, then the network reduces to a 2-user interference channel. The sDoF/user of such a network while accounting for time sharing is given as $\min(M/2, N/2, \max(M/4, N/4))$. As seen in Fig. 1, this is a sub-optimal strategy from a sDoF standpoint for almost all $\gamma \leq 2$ except when $\gamma = 1/2$. Note that at $\gamma = 1/2$, although the sDoF/user are the same for scheduling either one or two users, there are advantages for scheduling only one user as fewer cross-channels need to be estimated. Finally, as expected, when $\gamma \geq 2$, the number of BS antennas becomes the bottleneck and user side strategies such as simultaneous scheduling with and without user cooperation and time sharing among users all have the same sDoF/user.

Time Sharing Among Cells: Suppose we were to time share across the cells so that only one cell is active in any given time slot, then the network reduces to a MIMO multiple-access/broadcast channel, whose DoF are well known. It is seen that such a strategy is strictly sub-optimal for all γ, except again when $\gamma = 1/2$. When $\gamma = 1/2$, this strategy is as good as all preceding strategies while offering significant advantages such as not requiring the estimation of any cross-channels and not requiring any coordination between the BSs. With 2 antennas at the user side and 4 antennas at the BS being suggested in recent standards for future cellular networks, this insight is particularly useful when comparing strategies for mitigating interference from a dominant interferer.

V. PROOFS

A. Proof of Theorem 3.1

The required outer bound is derived from outer bounds established for simpler networks such as the 2-user MIMO interference channel [18] and the SISO cellular network [4], [5]. We divide the proof into 4 cases:

Case (a): $0 \leq \gamma \leq 2/3$: In the uplink, if we let the users in each cell cooperate, we obtain a 2-user MIMO interference channel with $2M$ antennas at the transmitter and N antennas at the receiver. In [18], the DoF of such a network is shown to be $\min(4M, 2N, \max(2M, N))$. Since user cooperation cannot reduce the DoF, this is also a bound on the total DoF of the 2-cell 2-user/cell network. The per user DoF bound can equivalently be written as

$$\text{DoF/user} \leq \begin{cases} M & \gamma \leq 1/4 \\ N/4 & 1/4 < \gamma \leq 1/2 \\ M/2 & 1/2 < \gamma \leq 2/3 \end{cases}. \quad (7)$$

By symmetry, the DoF bound on the 2-user interference channel remains unchanged if the transmitters and the receivers are switched, and hence the same bound also applies to the downlink.

Case (b): $2/3 \leq \gamma \leq 1$: To establish the bound for $2/3 \leq \gamma \leq 1$, we first append antennas to each user to make the total number of antennas at each user equal to N. We now use a result from [5] for the SISO 2-cell, 2-user/cell cellular network which states that the DoF/user of such a network is equal to 1/3. This result is derived from a previous result established for the SISO X channel in [2] and is also extended to the MIMO case with N antennas at each node. Thus, $N/3$ is an outer bound on the DoF/user achievable for the modified network and since appending antennas cannot decrease the DoF, this is also an outer bound for the original network. Using an analogous result in [4] for the downlink, the same bound also applies in the downlink.

Case (c): $1 \leq \gamma \leq 3/2$: Using an approach similar to Case (b), we now append antennas at the BSs so that each BS has a total of M antennas. Reusing the outer bounds presented in [4], [5], we see that $M/3$ is an outer bound on the DoF/user of the original network for both uplink and downlink.

Case (d): $\gamma \geq 3/2$: To obtain the outer bound when $\gamma \leq 3/2$, we let the two base stations cooperate to form a 4-user single-cell network with M antennas at each user and $2N$ antennas at the BS. Thus the DoF/user in this case can be bounded as $\frac{1}{4}\min(4M, 2N) = N/2$. □

This establishes the required outer bound on the DoF/user for the 2-cell, 2-user/cell MIMO cellular network. Note that since the bound scales linearly in the number of transmit/receive antennas, this is also a bound on the sDoF/user for this network.

B. Proof of Theorem 3.2

We prove the achievability of the outer bound by considering the uplink and by designing linear transmit beamformers for the users using finite spatial extensions so as to ensure interference at the BSs is contained within a certain number of dimensions. Since assuming channels to be generic assures that signal and interference spaces have a zero dimensional overlap almost surely, existence of receive beamformers that selectively null out the aligned interference is guaranteed. Further, by reciprocity of linear interference alignment, this proves achievability for uplink and downlink. We denote the jth transmit beamformer of the kth user in the ith cell as $\mathbf{v}_{ikj}$.

We divide the proof of achievability into six cases each corresponding to the six distinct piece-wise linear regions in Fig. 1.

Case i: $0 < \gamma \leq 1/4$: Each user here requires M DoF. It is easy to observe that since $N \geq 4M$, random transmit beamforming suffices and no interference alignment is necessary. The BSs have enough antennas to resolve signal from interference. Note that no spatial extensions are required here.

Case ii: $1/4 < \gamma \leq 1/2$: The goal here is to achieve $N/4$ DoF/user. If $N/4$ is not an integer, we consider a space-extension factor of 4, in which case we have $4M$ antennas at the users and $4N$ antennas at the transmitter. Since we need N DoF/user and the BSs now have $4N$ antennas, we once again see that random transmit beamforming suffices and no interference alignment is necessary.

Case iii: $1/2 < \gamma \leq 2/3$: Since each user requires $M/2$ DoF/user, we consider a space-extension factor of 2 so that there are $2M$ antennas at each user and $2N$ antennas at each BS. The two users in the second cell each have access to a $2M$ dimensional subspace at the first BS. These two subspaces overlap in $4M - 2N$ dimensions. Note that since $\gamma > 1/2$, $4M > 2N$, so such an overlap almost surely exists. The two users in Cell 2 pick $4M-2N$ linear transmit beamformers so as to span this space and align their interference. Specifically, the transmit beamformers $\mathbf{v}_{21j}$ and $\mathbf{v}_{22j}$ for $j = 1, \ldots, (4M-2N)$ are chosen such that

$$\mathbf{H}_{(21,1)}\mathbf{v}_{21j} = \mathbf{H}_{(22,1)}\mathbf{v}_{22j}. \quad (8)$$

Adopting the same strategy for Cell 1 users, we see that at both BSs interference occupies $4M - 2N$ dimensions while signal occupies $8M - 4N$ dimensions, with $8N - 12M$ unused dimensions. Note that since $\gamma \leq 2/3$, $8N - 12M \geq 0$. Letting each user pick $2N - 3M$ random beamformers, the remaining $8N - 12M$ dimensions are equally split among interference and signal at each of the BSs. We have thus designed M transmit beamformers for each user while ensuring that at each BS, interference occupies no more than $(4M-2N)+2(2N-3M) = 2N - 2M$ dimensions, resulting in $M/2$ sDoF/user.

Case iv: $2/3 < \gamma \leq 1$: We need to achieve $N/3$ DoF/user. We consider a space-extension factor of 3, so that each user has $3M$ antennas and each BS has $3N$ antennas; and we need to design N transmit beamformers per user. The two users in the second cell each have access to a $3M$ dimensional subspace at the first BS. These two subspaces overlap in $6M - 3N$ dimensions. Since $\gamma > 2/3$, we note that $6M - 3N > N$, allowing us to pick a set of N transmit beamformers spanning this space such that interference is aligned at BS 1. Using the same strategy for users in Cell 1, we see that since interference spans only N dimensions, and since each BS has $3N$ antennas, we can separate the N dimensional signals from interference at both BSs. The transmit beamformers can be computed by solving the same set of equations as given in (8).

Case v: $1 < \gamma \leq 3/2$: In order to achieve $M/3$ DoF/user, we consider a space-extension factor of 3 and design M beamformers per user. Since we now have more transmit antennas than receive antennas, transmit zero-forcing becomes possible. Each user in Cell 2 picks $3M - 3N$ linearly independent transmit beamformers so as to zero-force BS 1, i.e., the beamformers are chosen from the null space of the channel $\mathbf{H}_{(2i,1)}$ and satisfy

$$\mathbf{H}_{(2i,1)}\mathbf{v}_{2ij} = 0 \; \forall \; i \in \{1,2\}, \; j \in \{1,2,\ldots(3M-3N)\}. \quad (9)$$

We let users in Cell 1 use the same strategy. Now, in order to achieve M DoF/user, we still need to design $3N - 2M$ transmit beamformers per user. So far, both BSs do not see any interference and have $6M - 6N$ dimensions occupied by the signals from their own users. The remaining $9N - 6M$ dimensions at each BS need to be split in a $2:1$ ratio among signal and interference to achieve M DoF/user. To meet this goal, we choose the remaining $3N - 2M$ transmit beamformers for users in Cell 2 such that the interference from these users aligns at BS 1. This is accomplished by solving for the transmit beamformers using (8) for users in Cell 2, and using a similar strategy for users in Cell 1, resulting in $(3M-3N)+(3N-2M) = M$ DoF/user over a space-extension factor of 3.

Case vi: $3/2 < \gamma$: Assuming a space-extension factor of 2, each user needs N transmit beamformers. The null space of the channel from a user in Cell 2 to BS 1 spans $2M - 2N$ dimensions and since $\gamma > 3/2$, $2M - 2N > N$. Choosing N transmit beamformers from such a null space and using the same strategy for users in Cell 1, we see that each BS sees no interference and hence is able to completely recover signals from both of its users. □

While we have considered spatial extensions in the above proof, the results are expected to hold with time/frequency extensions as well. However, for time/frequency extensions, subsequent to aligning interference, we need to additionally check that signal and interference are indeed separable. One way to do this is through a numerical test as outlined in [6]. Finally, we also observe that whenever $\gamma \notin \left(\frac{2}{3}, \frac{3}{2}\right)$, by allowing for an asymmetric distribution of the achievable DoF among the users and then averaging the DoF/user over multiple time slots, the optimal DoF/user can be achieved without requiring any channel extensions.

C. Proof of Theorem 3.3

Having established the achievability of Theorem 3.2, this result follows immediately using the same techniques. While achievability in cases (i), (ii) and (vi) is easy to see, we outline some differences in the remaining cases. In Case (iii), we pick $2M - N$ beamformers so as to align interference followed by $N-2M+\lfloor \frac{M}{2} \rfloor$ random beamformers. In Case (iv), without symbol extensions, the subspaces overlap in $2M - N$ dimensions. Noting that $(2M - N) > \lfloor \frac{N}{3} \rfloor$, we choose $\lfloor \frac{N}{3} \rfloor$ linearly independent beamformers within this space. In Case (v), we first pick $(M - N)$ zero-forcing beamformers followed by $N - M + \lfloor \frac{M}{3} \rfloor$ beamformers to align interference. □

VI. Conclusion

This paper studies the DoF of a 2-cell, 2-user/cell network with M antennas at each user and N antennas at each BS. We prove that linear beamforming and finite spatial extensions can achieve the optimal sDoF/user. We identify the scenarios with redundant dimensions and show that not all proper systems are feasible. Finally, we compare the achievable DoF using strategies such as time sharing between users/cells and comment on the consequences thereof.

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