Optimizing Large-Scale MIMO Cellular Downlink: Multiplexing, Diversity, or Interference Nulling?

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***Abstract*—A base-station (BS) equipped with multiple antennas can use its spatial dimensions in three ways: (1) serve multiple users to achieve a multiplexing gain, (2) provide diversity to its users, and/or (3) null interference at a chosen subset of out-of-cell users. The main contribution of this paper is to answer the following question: what is the optimal balance between the three competing benefits of multiplexing, diversity and interference nulling? We answer this question in the context of the downlink of a cellular network in which each user chooses its best serving BS, and requests nearby interfering BSs for interference nulling. BSs are equipped with a large number of antennas, serve multiple single-antenna users using zero-forcing beamforming and equal power assignment, and null interference at a subset of out-of-cell users. The remaining spatial dimensions provide transmit diversity. We assume perfect channel state information at the BSs and users. Utilizing tools from stochastic geometry, we show that, surprisingly, to maximize the per-BS ergodic sum rate, at the optimal allocation of spatial resources, interference nulling does not bring tangible benefit. A close-to-optimal strategy is to use *none* of the spatial resources for interference nulling, while reserving 60% of spatial resources for achieving multiplexing and the rest for providing diversity.**

I. Introduction

The advent of the massive multiple-input multiple-output (MIMO) concept has brought myriad opportunities for optimizing transmissions in wireless cellular networks. Having a large number of base-station (BS) antennas at our disposal allows us to achieve multiple system objectives simultaneously: (1) serve multiple users in the same time-frequency slot to achieve a multiplexing gain, (2) provide diversity for the scheduled users, and (3) as often stated as a major advantage of massive MIMO systems, null interference at out-of-cell users. In fact, in the limit as the number of antennas goes to infinity, intercell interference can be completely eliminated, leaving pilot contamination as the only limiting factor [1].

This paper focuses on the design of large-scale MIMO (LS-MIMO) downlink systems [2], where each BS is equipped with a large but *finite* number of antennas. In this regime, it is crucial to understand the tradeoffs between multiplexing, diversity and interference nulling. This paper attempts to answer the following key question in such a design: between providing spatial multiplexing, diversity and mitigating intercell interference, which takes priority? Equivalently, what is the optimal tradeoff among the three?

Toward this end, we consider an LS-MIMO system where BSs are distributed according to an independent Poisson point process (PPP). Further, this paper advocates a user-centric clustering strategy. Each user receives the intended signal from the closest BS, while requesting interference nulling from a cluster of nearby BSs. We assume that perfect channel state information (CSI) is available without cost, at both the BSs and users. Each BS serves multiple single-antenna users simultaneously using zero-forcing (ZF) beamforming with equal power allocation across users. In addition, the remaining spatial dimensions at each BS can potentially be used to provide diversity for the scheduled users and/or to null interference at a subset of out-of-cell users.

This paper arrives at the following surprising conclusion: For a single-tier wireless cellular network, devoting spatial dimensions to interference nulling *does not provide tangible benefit*. In fact, it is fairly close to optimum, in terms of maximizing the per-BS ergodic sum rate, to use about 60% of the spatial dimensions for spatial multiplexing (i.e., with M antennas, serve $K \simeq 0.6M$ users), leaving the remaining dimensions for spatial diversity, while devoting *none* of the spatial dimensions for intercell interference nulling. This is despite the fact that (a) the BSs are densely deployed and the overall network is interference limited and (b) the analysis does not account for the cost of channel estimation. Thus, even with perfect CSI available for all direct and interference channels at all BSs without any cost, one may still wish to utilize spatial dimensions for providing spatial multiplexing and diversity for the intended users, rather than interference nulling for the neighboring users.

A. Related Work

The tradeoff between multiplexing and diversity for the single-user MIMO channel [3] and for the uplink single-cell multiuser system [4] are well known. However, the optimal allocation of spatial resources between multiplexing and diversity in a multicell setting has not been studied rigorously. The multicell setting is of particular interest because of the possibility of mitigating intercell interference through coordinating transmission strategies across multiple BSs, also known as interference coordination [5]. For example, the problems of joint design of beamformers across multiple BSs in order to minimize transmit power [6], [7] and to maximize SINR [8], [9] have been studied.

The LS-MIMO system considered in this paper is the finite-dimensional version of non-cooperative massive MIMO systems, wherein each BS is equipped with an asymptotically large number of antennas and serves its scheduled users independently. In this asymptotic regime, the uncorrelated intercell interference completely vanishes; intercell coordination is therefore not required [1], [10]. However, with a finite number of antennas, it is not clear as to what extent this desirable feature of massive MIMO systems remains optimal. This paper deviates from the massive MIMO literature in that it explicitly accounts for interference cancellation using beamforming and aims to understand whether it is beneficial to use spatial dimensions for interference cancellation.

This paper considers a user-centric clustering strategy for interference nulling. The joint design of user-centric clusters and downlink beamformers for a fixed network topology has been studied in [11], [12]. To account for the randomness in BS and user locations, we use available tools from stochastic geometry [13]. Stochastic models for the analysis of user-centric clustering in wireless systems, where each BS serves a single user, have been introduced in [14], [15]. This paper carries out a stochastic analysis of an LS-MIMO system under user-centric clustering, where each BS serves multiple users, and aims at understanding the optimal allocation of spatial resources to multiplexing, diversity and interference nulling.

B. Summary of Contributions

The main contributions of this paper are as follows:

1) Stochastic analysis of LS-MIMO systems with user-centric clustering: We present a stochastic analysis of an LS-MIMO system under user-centric clustering. Using tools from stochastic geometry, we obtain a tractable expression for the per-BS ergodic sum rate as a function of average cluster size. This expression enables us to efficiently explore the performance of LS-MIMO systems with user-centric clustering under different system parameters.

2) Optimizing allocation of spatial resources for multiplexing, diversity and interference nulling: The analysis above allows us to answer our central question as to the optimal allocation of spatial dimensions. Our analysis shows that a close-to-optimal strategy is to use 60% of spatial dimensions for multiplexing, the remaining dimensions for providing diversity, while reserving *none* of the spatial dimensions for interference nulling.

II. System Model

We consider a time-division duplex (TDD) LS-MIMO system, where BSs are distributed according to a homogeneous PPP Φ with density λ over the entire $\mathbb{R}^2$ plane. Each BS is equipped with M antennas, and is constrained to use a maximum power of P_T. The single-antenna users are distributed according to an independent point process with density considerably larger than that of the BSs. Users form cells by associating with their closest BSs. Perfect CSI is assumed to be available at the BSs and the users.

The wireless fading channel is modeled as follows. Let $\mathbf{g}_{ij} = \sqrt{\beta_{ij}}\mathbf{h}_{ij} \in \mathcal{C}^{M\times 1}$ denote the channel between BS j and user i, where $\mathbf{h}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$ indicates a small-scale Rayleigh fading component, and $\beta_{ij} = \left(1 + \frac{r_{ij}}{d_o}\right)^{-\alpha}$ is the path-loss component; here, r_{ij} is the distance between user i and BS j, $\alpha > 2$ denotes the path-loss exponent, and d_o is the reference distance.

In a given time-frequency slot, BS b schedules a set of K users, denoted by $\mathcal{S}_b$, chosen from its associated users. The BS uses ZF beamforming with equal power allocated across the K users. Further, BS b uses its available antennas to cancel interference at a subset of out-of-cell users denoted by $\mathcal{O}_b$. The normalized ZF beam assigned to user i associated with BS b, while nulling the interference at the other $K - 1 + |\mathcal{O}_b|$ users, is given by

$$\mathbf{w}_{ib} = \frac{\left(\mathbf{I}_M - \mathbf{G}_{-ib}\mathbf{G}_{-ib}^{\dagger}\right)\hat{\mathbf{g}}_{ib}}{\left\|\left(\mathbf{I}_M - \mathbf{G}_{-ib}\mathbf{G}_{-ib}^{\dagger}\right)\hat{\mathbf{g}}_{ib}\right\|_2},\ \forall i \in \mathcal{S}_b$$

where $\mathbf{G}_{-ib} = \left[\hat{\mathbf{g}}_{1b}, \ldots, \hat{\mathbf{g}}_{(i-1)b}, \hat{\mathbf{g}}_{(i+1)b}, \ldots, \hat{\mathbf{g}}_{(K+|\mathcal{O}_b|)b}\right]$, $|\cdot|$ denotes the cardinality of a set, and $\hat{\mathbf{g}}_{jb} = \mathbf{g}_{jb}/\|\mathbf{g}_{jb}\|_2$.

A. User-Centric Clustering in LS-MIMO Systems

The choice of out-of-cell users for interference cancellation depends on the BS clustering strategy. This paper considers user-centric clustering, where each user identifies the interfering BSs with strong average channel magnitudes. These BSs form the cluster from which the user requests interference nulling. Ideally, interference from the BSs within the cluster would all be cancelled; only the interference from outside this cluster remains, and is treated as additional noise. However, in reality, because each BS has finite number of available spatial dimensions, it can only select a subset of out-of-cell users for interference nulling. In particular, each BS reserves sufficient antennas to serve its K scheduled users with diversity order ζ. Accounting for the orthogonal property of ZF beamforming, BS b must reserve $\zeta + K - 1$ dimensions for transmission to its own users, leaving a maximum of

$$O_{\max} = M - K - \zeta + 1$$

dimensions for interference nulling at out-of-cell users. If the number of users requesting interference nulling at BS b is larger than $O_{\max}$, then BS b selects the $O_{\max}$ users with the strongest channel magnitudes within the candidate set, i.e., some out-of-cell users' requests would have to be ignored. On the other hand, if the number of requesting users for interference cancellation from BS b is smaller than $O_{\max}$, then the extra antennas are used to provide users with diversity order larger than ζ.

In the system described above, let $\Phi_{S_i} \subset \Phi$ be the set of BSs located in the cluster of user i, and $\Phi_{I_i} = \Phi \setminus \Phi_{S_i}$ be the set of interfering BSs located outside the cluster. Further, let $\Phi_{S_i,\mathrm{Intf}} \subset \Phi_{S_i}$ indicate the BSs in Φ_{S_i} that do not have sufficient antennas to cancel interference at user i. The received signal at user i associated with BS b is the sum of the intended signal, residual intra-cluster interference, inter-cluster interference, and receiver noise; the received signal is given by

$$y_{ib} = \underbrace{\sqrt{\frac{P_T}{K}}\mathbf{g}_{ib}^{\mathsf{H}}\mathbf{w}_{ib}s_{ib}}_{\text{intended signal}} + \underbrace{\sum_{j\in\Phi_{S_i,\text{Intf}}}\sum_{k=1}^{K}\sqrt{\frac{P_T}{K}}\mathbf{g}_{ij}^{\mathsf{H}}\mathbf{w}_{kj}s_{kj}}_{\text{intra-cluster interference}}$$
$$+ \underbrace{\sum_{m\in\Phi_{I_i}}\sum_{u=1}^{K}\sqrt{\frac{P_T}{K}}\mathbf{g}_{im}^{\mathsf{H}}\mathbf{w}_{um}s_{um}}_{\text{inter-cluster interference}} + \underbrace{n_{ib}}_{\text{noise}}$$

where s_{ib} denotes the complex symbol intended for user i associated with BS b such that $\mathbb{E}\left[|s_{ib}|^2\right] = 1$, and n_{ib} denotes the additive white Gaussian noise with variance σ^2.

B. Analytical Model of User-Centric Clustering Strategy

This paper aims to provide a stochastic performance analysis of a user-centric LS-MIMO system. For tractability, the following simplified model is proposed for theoretical analysis:

$A1$: All BSs from which a user requests interference nulling are within a circle of radius R_c centered at the user.

$A2$: The number of BSs in the circular cluster is the same as the average number of BSs $\bar{B} = \lambda\pi R_c^2$.

$A3$: Each BS has enough antennas not only to serve its K scheduled users with a fixed diversity order of ζ, but also to grant all interference nulling requests. Together with $A1$, this means that each BS must cancel interference at all users located within distance R_c from itself.

Note that to satisfy $A3$, each BS should cancel interference at $O_{\max} = O = M - K - \zeta + 1$ out-of-cell users, while serving its K scheduled users. To allow this, given $A2$, i.e., assuming each cluster includes exactly $\bar{B}$ BSs, the cluster radius must be $R_c = \sqrt{\bar{B}/\lambda\pi}$, where $\bar{B} = (O + K)/K$.

Under these assumptions, the signal-to-interference-plus-noise ratio (SINR) of user i scheduled by BS b is given by

$$\gamma_{ib} = \frac{\rho\beta_{ib}|\mathbf{h}_{ib}^{\mathsf{H}}\mathbf{w}_{ib}|^2}{\sum_{m\in\Phi_{I_i}}\sum_{k=1}^{K}\rho\beta_{im}|\mathbf{h}_{im}^{\mathsf{H}}\mathbf{w}_{km}|^2 + 1} \quad (1)$$

where $\rho = \frac{P_T}{K\sigma^2}$ indicates the per-user signal-to-noise ratio (SNR). The instantaneous rate of user i denoted by R_{ib} is therefore given by $R_{ib} = \log_2\left(1 + \frac{\gamma_{ib}}{\Gamma}\right)$, where Γ is the SNR gap to capacity.

III. A Stochastic Geometry Analysis of LS-MIMO

In this section, we derive the per-BS ergodic sum rate expression for an LS-MIMO system with user-centric clustering.

Theorem 1. *Under Assumptions $A1$-$A3$, for an LS-MIMO system with user-centric clustering employing ZF beamforming, providing a fixed diversity ζ for each scheduled user, and distributing power equally across the downlink beams, the per-BS ergodic sum rate in nats/sec/Hz is given by*

$$R_{cell} = K\int_0^\infty \frac{e^{-z\Gamma}}{z}\exp\left(2\pi\lambda\left(\Psi_{\mathrm{I}}(z\Gamma) - \Psi_{\mathrm{II}}(z\Gamma)\right)\right)$$
$$\left(1 - \int_0^\infty \frac{\mathrm{d}F_{r_{\min}}}{\mathrm{d}r}\left(1 + z\rho\left(1 + \frac{r}{d_o}\right)^{-\alpha}\right)^{-\zeta}\mathrm{d}r\right)\mathrm{d}z \quad (2)$$

where $F_{r_{\min}}(r) = 1 - \exp\left(-\lambda\pi r^2\right)$ for $r > 0$ is the cumulative distribution function (CDF) of distance between a user and its closest BS [13], and $\Psi_{\mathrm{I}}(s)$ and $\Psi_{\mathrm{II}}(s)$ are given as

$$\Psi_{\mathrm{I}}(s) = \frac{d_o^2}{2}\left(1 + \frac{R_c}{d_o}\right)^2$$
$$\left(1 - {}_2F_1\left(K, -\frac{2}{\alpha}; 1 - \frac{2}{\alpha}; -s\rho\left(1 + \frac{R_c}{d_o}\right)^{-\alpha}\right)\right) \quad (3)$$

$$\Psi_{\mathrm{II}}(s) = d_o^2\left(1 + \frac{R_c}{d_o}\right)$$
$$\left(1 - {}_2F_1\left(K, -\frac{1}{\alpha}; 1 - \frac{1}{\alpha}; -s\rho\left(1 + \frac{R_c}{d_o}\right)^{-\alpha}\right)\right). \quad (4)$$

Proof: Since each BS serves K users, the per-BS ergodic sum rate is given by $R_{cell} = K\mathbb{E}_{\Phi,\mathbf{h}}(R_{ib})$. The proof of Theorem 1 involves expressing the rate function in an integral form using

$$\log(1 + x) = \int_0^\infty \frac{e^{-t}}{t}\left(1 - e^{-xt}\right)\mathrm{d}t$$

and substituting in the Laplace transforms of the interference and signal powers. The proof uses the same approach as that used in [16], [17]. The details are omitted here. ■

Remark 1. Theorem 1 completely characterizes the per-BS ergodic sum rate of an LS-MIMO system with user-centric clustering as a function of important system parameters, such as BS deployment density (λ), number of users scheduled by each BS (K), diversity order per user (ζ), and cluster radius (R_c). Although the expression involves a double integral, using the transformation introduced in Remark 1 of [18], it can be computed efficiently.

TABLE I
System Design Parameters

BS density	$\lambda = 1/\pi 500^2$ m^{-2}
Total bandwidth	$W = 20$ MHz
BS max. available power	43 dBm
Background noise PSD	$N_o = -174$ dBm/Hz
Noise figure	$N_f = 9$ dB
SNR gap	$\Gamma = 3$ dB
Path-loss exponent	$\alpha = 3.76$
Reference distance	$d_o = 0.3920$ m

IV. Optimization of LS-MIMO Systems

In this section, we use the per-BS ergodic sum rate expression obtained in the preceding section to answer the following central question of this paper: given a fixed number of antennas M at each BS, what is the optimal number of scheduled users K, the optimal diversity order provided to each user ζ, and the optimal number of interference nulling directions O in terms of maximizing the per-BS ergodic sum rate? Note that $K \in [1, \cdots, M]$, $\zeta \in [1, \cdots, M-K+1]$ and $O = M-K-\zeta+1$, i.e., only two of the triple (K, ζ, O) are free variables. The result provided by Theorem 1 allows us to obtain the optimal operating point in an efficient manner. The chosen system parameters are listed in Table I. Numerical results are averaged over network topologies and small-scale channel fading realizations.

Figs. 1 and 2 plot the per-BS ergodic sum rate of an LS-MIMO system with $M = 15$ for various combinations of (K, ζ, O) obtained numerically via simulations and analytically using (2). As the results show, when a substantial number of dimensions are devoted to nulling interference (large O), the numerical and analytic results are in good agreement. However, at low values of O, the two results diverge. The main reason for the discrepancy is that in the simulations, once O is specified, the *exact* number of BSs nulling interference at each user, i.e., its exact cluster size, is known for any given network topology. However, in the analysis, the values of K and O specify the *average* number of BSs in a cluster, i.e., $\bar{B} = (K+O)/K$, as explained in Section II-B. Despite the discrepancy, the analysis does capture the general behavior of the system. When both O and K are small, increasing K improves the per-BS ergodic sum rate, while for large values of O and K, increasing K degrades system performance. Most importantly, the analysis helps identify the region of O with the maximum sum-rate, leading to the main, and surprising, result of this paper.

A remarkable implication of these analytical and simulation results is that the per-BS ergodic sum rate is largely a *decreasing* function of the number of spatial dimensions assigned to null interference. The analytic expression in (2) is, in fact, a strictly decreasing function of O, while the numerical curves are approximately so. It is nearly optimum for each BS to operate independently of the other BSs and *not to spend spatial resources on nulling interference*. More specifically, the sum-rate optimal operating point obtained numerically is $(K^*, \zeta^*, O^*) = (8, 6, 2)$, and the optimal operating point obtained analytically is $(K^*, \zeta^*, O^*) = (10, 6, 0)$. The difference between the achievable sum rates at these two operating points (obtained numerically) is only about 4%. Thus, when K and ζ are properly chosen, an LS-MIMO system without interference nulling, i.e., with each BS operating independently, performs close to optimum.

Fig. 3 plots the CDF of downlink user rates evaluated at various choices of (K, ζ, O). As can be seen, the gap between the CDF of the downlink user rates obtained at $(10, 6, 0)$ and $(8, 6, 2)$ is negligible. Clearly, operating at either of these two points is significantly superior to operating at any of $(1, 15, 0)$, $(15, 1, 0)$ and $(7, 3, 6)$, but there are other choices e.g., $(9, 5, 2)$, that also perform reasonably well. These results indicate that it is important to properly balance K and ζ, and to keep O small.

Finally, Table II lists the optimal (K^*, ζ^*, O^*) obtained

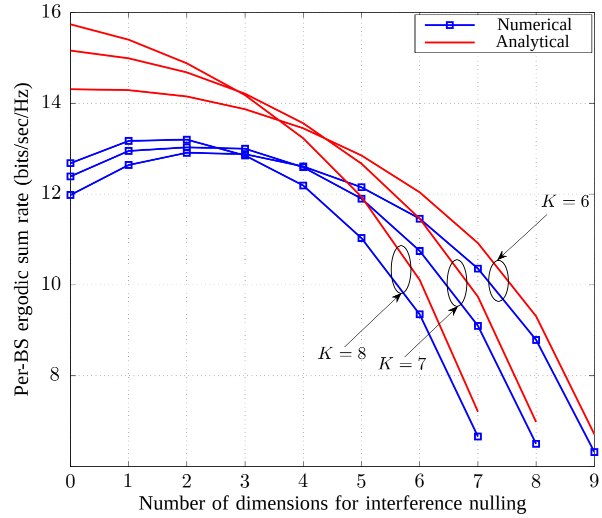

Fig. 1. Per-BS ergodic sum rate of an LS-MIMO system with $M = 15$, $K = 6$, 7, 8, and various choices of ζ and O obtained both numerically and analytically.

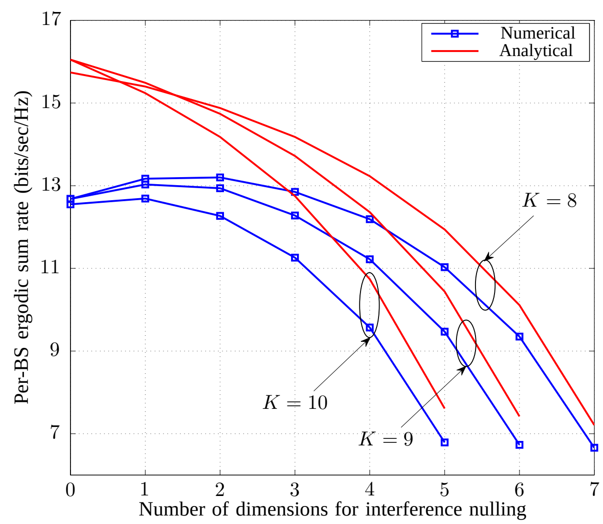

Fig. 2. Per-BS ergodic sum rate of an LS-MIMO system with $M = 15$, $K = 8$, 9, 10, and various choices of ζ and O obtained both numerically and analytically.

both numerically and analytically for various values of M. The table also lists the performance gap between these operating points (evaluated numerically). In all the scenarios, the analysis suggests that it is optimal to ignore intercell interference and retain all spatial dimensions for multiplexing and diversity. The simulation results, in good agreement with the analysis, suggest that only a very few of the spatial dimensions should be used for interference cancellation. Moreover, the simulation results indicate that the performance gap between these points for various values of M is negligible.

An interesting observation from the table is that the optimal loading factor $\eta = (K+O)/M$ appears to be always close to 0.6. This implies that, at a close-to-optimum operating point, a MIMO BS should devote about 60% of its resources to multiplexing ($K/M = 0.6$) and the remaining to providing diversity to each user ($\zeta = M-K+1, O = 0$).

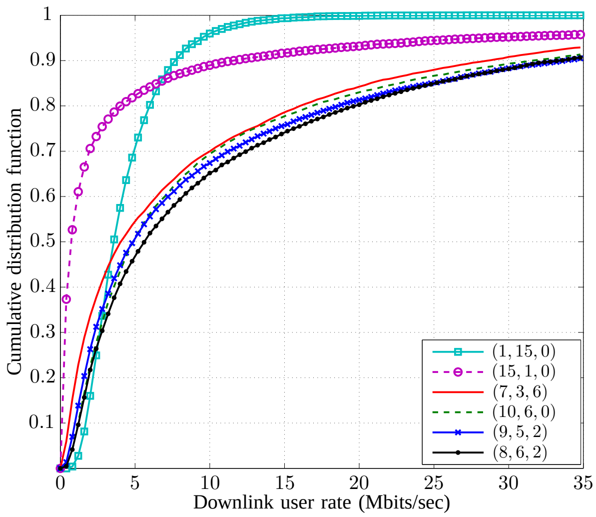

Fig. 3. CDF of the downlink user rates evaluated at different (K, ζ, O).

TABLE II
OPTIMAL OPERATING POINTS FOR MAXIMIZING PER-BS ERGODIC SUM RATE OF LS-MIMO SYSTEMS

M	analytical	numerical	performance gap	η^*
10	$(6, 5, 0)$	$(5, 5, 1)$	4%	0.6
15	$(10, 6, 0)$	$(8, 6, 2)$	4%	0.66
40	$(25, 16, 0)$	$(24, 15, 2)$	3%	0.62

It is worth noting that these results are obtained without accounting for the overhead needed for channel training. To implement interference nulling as suggested by the optimal solutions obtained numerically, CSI acquisition overhead would increase. As an example, with $M = 40$, at the optimal operating point obtained numerically, in order to serve 24 scheduled users and null interference at 2 out-of-cell users, 26 channels are needed to be estimated, whereas at the optimal point suggested by analysis, i.e., with no interference nulling, 25 channels should be estimated at each BS. Furthermore, with user-centric clustering, clusters may overlap. Therefore, pilot allocation for CSI acquisition must be organized by some central entity across the entire network. This, in fact, represents another level of overhead. As a result, in practical scenarios, enabling interference nulling may diminish the 3-4% apparent gain of operating under the optimal point obtained numerically. Consequently, it is highly likely that in cellular LS-MIMO systems, ignoring interference nulling, while properly selecting K and ζ, is, in fact, the best operating condition.

V. CONCLUSION

This paper considers an LS-MIMO system with user-centric clustering operating under ZF beamforming and equal power assignment. For such a system, we derive computationally efficient expression for the per-BS ergodic sum rate as a function of average cluster size using tools from stochastic geometry. We then optimize the per-BS ergodic sum rate of the system as a function of the number of users to schedule, the diversity order of each user and the number of interference directions to cancel. This paper shows that at the optimal operating point, only a very few of spatial dimensions should be reserved for interference nulling. In fact, our analysis shows that it is near optimal to allocate none of the spatial resources for interference nulling, while using 60% of spatial dimensions for achieving multiplexing and the rest for providing diversity. Reducing multiplexing and diversity dimensions in order to perform interference nulling does not appear to bring much benefit to the overall system sum rate.

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