

# Can Interference Alignment Impact Network Utility Maximization?

Gokul Sridharan and Wei Yu

The Edward S. Rogers Sr. Department of Electrical and Computer Engineering  
University of Toronto, Toronto, ON, M5S 3G4, Canada  
Email:gsridharan@comm.utoronto.ca, weiyu@comm.utoronto.ca

**Abstract**—This paper examines whether interference alignment (IA) can be leveraged to improve network utility in a multi-user multi-antenna wireless cellular network. Optimality of IA from a DoF standpoint has the potential to aid conventional network optimization algorithms that typically can only find locally optimal solutions. This paper investigates the usefulness of IA for interference coordination and utility maximization by proposing a two-stage optimization framework for a  $G$ -cell multi-antenna network with  $K$  users/cell, and with full channel state information (CSI) available at all base-stations. The first stage of the proposed framework focuses exclusively on nulling interference from a set of dominant interferers using IA, while the second stage optimizes the transmit and receive beamformers iteratively to maximize a network-wide utility using the IA solution as the initial point. The number of dominant interferers to be nulled in the first stage is guided by a set of new feasibility results for partial IA. This paper focuses on maximizing the specific network utility of minimum rate over all users in the network. Through simulations on two different topologies of cluster of BSs either in isolation or in the presence of other non-cooperating BSs, the proposed framework with IA initialization is observed to outperform straightforward optimization on an isolated cluster of BSs. But, IA loses its impact when there is significant out-of-cluster interference. Thus, in a large-scale dense cellular deployment, the benefit of IA is likely to be limited, even with centralized network optimization and full CSI.

## I. INTRODUCTION

Interference coordination through the joint optimization of the transmission variables has emerged as a promising technique to address inter-cell interference in dense cellular networks. Efforts to develop algorithms for such a joint optimization have largely been divided into two separate domains: that of network utility maximization (NUM) over power, beamforming and frequency allocation and that of interference alignment (IA) for maximizing the degrees of freedom (DoF). But the relationship between the two remains largely unexplored. This paper investigates the role of IA in the context of improving the performance of NUM algorithms in cooperative cellular networks. Specifically, this paper focuses on maximizing the minimum user rate in a  $G$ -cell network having  $K$  users/cell, with  $N$  antennas at each base-station (BS) and  $M$  antennas at each user—a  $(G, K, M \times N)$  network.

Joint optimization in coordinated cellular networks requires solving an optimization problem to maximize a network-wide utility function (e.g., weighted sum-rate, max-min-fairness rate, etc) over transmission parameters such as beamformers and transmit powers [1]–[9]. Although several novel concepts

such as uplink-downlink duality [5], [8] are known in this context, the non-convex nature of these problems makes it challenging to find efficient methods capable of finding solutions that are closer to the global optimum.

In parallel to these developments, significant progress has been made in establishing the DoF of multi-antenna cellular networks [10]–[14]. IA, with and without symbol extensions, has played a key role in establishing these results. Although the capacity of cellular networks is still unknown, crucial insight on the capacity limits of cellular networks at high signal-to-noise ratios (SNRs) can be obtained from DoF perspective. However, due to the asymptotic nature of these results, the value of IA in the context of NUM under realistic channel conditions (e.g., including pathloss) is not yet known.

Since both IA and NUM algorithms have similar channel-state information (CSI) requirements and comparable computational complexity, it is pertinent to assess the value of IA in relation to NUM. However, due to the limited focus of IA on interference suppression while neglecting signal strength, IA cannot be viewed as a substitute for NUM and must instead be considered as a potential augmentation to the optimization process. This paper examines the role of IA in improving the performance of NUM algorithms in this context. Note that the goal of this paper is different from the many existing algorithms that minimize mean-squared-error (MSE) as a proxy for some network utility function [4], [9], [15]–[18]. These algorithms do not explicitly compute aligned beamformers but have been empirically observed to converge to aligned beamformers at high SNRs. Although this observation appears to suggest that such algorithms implicitly account for the value of aligned beamformers, they do not explicitly utilize aligned beamformers at finite SNRs and thus do not shed light on the value of IA at finite SNRs. This paper is precisely trying to fill this void by examining whether NUM algorithms can benefit from explicit IA, even at finite SNRs.

This paper proposes a two-stage optimization framework that takes advantage of IA in NUM for cooperative cellular networks. The first stage of this framework exclusively focuses on mitigating interference from dominant interferers using IA. The second stage uses this altered interference landscape to optimize the network parameters to maximize a given utility function. Such a framework counters the myopic nature of straightforward NUM algorithms by leveraging IA's ability to comprehensively address interference from the dominant inter-

ferers while subsequently relying on numerical optimization algorithms to account for signal strength and to maximize the network utility. Note that the usefulness of IA solution as the initial starting point of NUM is also mentioned in [19] and is also explored in the context of heterogeneous networks in [20].

Specifically, this paper focuses on max-min fairness and uses the two-stage framework to maximize the minimum rate to the scheduled users subject to per-BS power constraints. We first establish theoretical results on the number of dominant interfering BSs that can be nulled per user in a  $(G, K, M \times N)$  network. We then identify the requisite number of dominant interfering BSs to be nulled in the first stage. After aligning interference from the dominant BSs, we alternately optimize the transmit and receive beamformers to maximize the minimum rate. Simulations on specific topologies of isolated cluster of BSs under realistic channel conditions indicate that (a) aligned beamformers do not naturally emerge from straightforward NUM algorithms even at high signal-to-noise ratios; (b) aligned beamformers provide a significant advantage as initial condition to NUM, especially when BSs are closely spaced; and (c) IA provides insight on the optimal number of users to schedule per cell. It is further seen that IA loses its impact on NUM when out-of-cluster interference is present. In the presence of uncoordinated interference, cancelling interference from just one or two dominant BSs does not sufficiently affect the optimization landscape to yield better solutions. It thus appears that the value of IA in the context of NUM may be limited to small clusters of cooperating BSs with insignificant uncoordinated interference. Interestingly, using tools from stochastic geometry, [21] also draws a similar conclusion on the value of IA in large cellular networks in the absence of any further optimization.

## II. SYSTEM MODEL

Consider the downlink of a cellular network consisting of a cluster of  $G$  interfering cells with  $K$  users per cell. These  $G$  interfering cells could either be isolated or be in the presence of several other interfering cells, resulting in out-of-cluster interference. Each user is assumed to have  $M$  antennas and each BS is assumed to have  $N$  antennas. Let the channel from the  $i$ th BS to the  $k$ th user in the  $g$ th cell be denoted as the  $M \times N$  matrix  $\mathbf{H}_{(i,gk)}$ . We assume that all channels are generic (or equivalently, drawn from a continuous distribution) and known perfectly at a central location. Assuming that each user is served with one data stream, the transmitted signal corresponding to the  $k$ th user in the  $g$ th cell is given by  $\mathbf{v}_{gk}s_{gk}$ , where  $\mathbf{v}_{gk}$  is a  $N \times 1$  linear transmit beamforming vector and  $s_{gk}$  is the symbol to be transmitted. This signal is received at the intended user using a  $M \times 1$  receive beamforming vector  $\mathbf{u}_{gk}$ . The received signal after being processed by the receive beamforming vector can be written as

$$\mathbf{u}_{gk}^H \mathbf{y}_{gk} = \sum_{i=1}^G \sum_{j=1}^K \mathbf{u}_{gk}^H \mathbf{H}_{(i,gk)} \mathbf{v}_{ij} s_{ij} + \mathbf{u}_{gk}^H \mathbf{n}_{gk}, \quad (1)$$

where  $\mathbf{n}_{gk}$  is the  $M \times 1$  vector representing the sum of additive white Gaussian noise and out-of-cluster interference received at the  $(g, k)$ th user. This paper restricts attention to beamforming based IA without symbol extensions in time or frequency. Our aim is to evaluate the role of IA for NUM.

## III. FEASIBILITY OF PARTIAL INTERFERENCE ALIGNMENT

We first investigate feasibility conditions for IA in cellular networks when interference from only a subset of BSs is cancelled at a user. Since interference from only a subset of the interferers is aligned, we call this partial interference alignment (PIA). It is important to establish these results as complete IA may not be feasible in most cases, or sometimes even unnecessary.

For the  $G$ -cell network described above, we construct a list  $\mathcal{I}$  of user-BS pairs where each pair indicates the need to cancel interference from a specific BS to a specific user. Let the double index  $gk$  denote the  $k$ th user in the  $g$ th cell and the single index  $l$  denote the  $l$ th BS. If the pair  $(12, 3) \in \mathcal{I}$ , this implies that the interference from the 3rd BS is to be completely nulled at the 2nd user in the 1st cell. Satisfying this condition requires solving the following  $K$  equations:

$$\mathbf{u}_{12}^H \mathbf{H}_{(3,12)} \mathbf{v}_{3j} = 0, \quad \forall j \in \{1, 2, \dots, K\}. \quad (2)$$

In addition to these conditions, we also require the set of transmit beamformers at any BS to be linearly independent, i.e.,  $\text{rank}([\mathbf{v}_{g1}, \mathbf{v}_{g2}, \dots, \mathbf{v}_{gK}]) = K$ .

Cancelling interference from only a subset of the interfering BSs is analogous to complete IA in partially connected cellular networks where certain cross links are assumed to be absent [22]. When the set  $\mathcal{I}$  consists of all the  $(G-1)GK$  possible pairs (denoted as  $\mathcal{I}_{all}$ ), we get the familiar set of conditions for IA [23], [24]. Each of the  $K$  equations in (2) is quadratic and collectively form a polynomial system of equations. Feasibility of the system of polynomial equations when  $\mathcal{I} = \mathcal{I}_{all}$  is well studied using tools from algebraic geometry and several conditions for feasibility are known [10], [11]. The same set of tools can also be used to establish conditions for feasibility for any given  $\mathcal{I}$ . The following theorem establishes one such result.

*Theorem 3.1:* Consider a  $(G, K, M \times N)$  network where each user is served with one data stream. Let  $\mathbf{v}_{gk}$  and  $\mathbf{u}_{gk}$  denote the transmit and receive beamformer corresponding to the  $(g, k)$ th user where the set of beamformers  $\{\mathbf{v}_{g1}, \mathbf{v}_{g2}, \dots, \mathbf{v}_{gK}\}$  is linearly independent for every  $g$ . Further, let  $\mathcal{I} \subseteq \{(gk, i) : g \neq i, 1 \leq g, i \leq G, 1 \leq k \leq K\}$  be a set of user-BS pairs such that for each  $(gk, i) \in \mathcal{I}$  the interference caused by the  $i$ th BS at the  $(g, k)$ th user is completely nulled, i.e.,

$$\mathbf{u}_{gk}^H \mathbf{H}_{(i,gk)} \mathbf{v}_{ij} = 0, \quad \forall j \in \{1, 2, \dots, K\}. \quad (3)$$

A set of transmit and receive beamformers  $\{\mathbf{v}_{gk}\}$  and  $\{\mathbf{u}_{gk}\}$  that satisfy the polynomial system defined by  $\mathcal{I}$  exist if and only if

$$M \geq 1, \quad N \geq K, \quad (4)$$

and

$$|\mathcal{J}_{users}|(M-1) + |\mathcal{J}_{BS}|(N-K)K \geq |\mathcal{J}|K \quad (5)$$

where  $\mathcal{J}$  is any subset of  $\mathcal{I}$  and  $\mathcal{J}_{users}$  and  $\mathcal{J}_{BS}$  are the set of user and BS indices that appear in  $\mathcal{J}$ .

The proof of this theorem uses the same technique as [10], [11] and is omitted here due to space constraint. Note that intra-cell interference can be subsequently eliminated as the transmit beamformers in each cell are linearly independent. A useful corollary of this theorem is stated below.

*Corollary 3.2:* Consider a  $(G, K, M \times N)$  network where each user is served with one data stream. Suppose that the set  $\mathcal{I}$  is such that each user requires interference from no more than  $q$  BSs to be cancelled, where  $1 \leq q \leq G-1$ , and each BS has no more than  $Kq$  users that require this BS's transmission to be nulled at these users, then a set of sufficient conditions for the feasibility of IA is given by

$$M \geq 1, \quad N \geq K, \quad (6)$$

and

$$M + N \geq K(q+1) + 1. \quad (7)$$

Note that when  $q = G - 1$ , we recover the well-known proper-improper condition for MIMO cellular networks [23]. Fig. 1 illustrates the conditions of Corollary 3.2 imposed on a  $(4, 2, 3 \times 4)$  network for the feasibility of PIA with  $q = 2$ . Each entry in Fig. 1 represents a user-BS pair as identified by its row and column indices. If a certain user-BS pair is in  $\mathcal{I}$ , the corresponding entry is marked with a '×'. Corollary 3.2 requires  $\mathcal{I}$  to be such that each row has no more than  $q$  chosen entries and each column has no more than  $Kq$  chosen entries, where  $q = \lfloor \frac{M+N-1}{K} \rfloor - 1$ .

This corollary provides a simpler set of guidelines on choosing the set of user-BS pairs ( $\mathcal{I}$ ) for partial IA than Theorem 3.1 where the number of feasibility constraints grows exponentially with the size of  $\mathcal{I}$ . However, designing  $\mathcal{I}$  according to this corollary rather than Theorem 3.1 comes at the cost of simplifying restrictions on  $\mathcal{I}$  that may otherwise be unnecessary. Note also that a key assumption of the feasibility condition derived in this paper is that each user is served one datastream. Generalization of this condition to the multi-datastream-per-user case is difficult and is in fact still an open problem even for the fully-connected case [10], [11], [25].

Restricting to the single datastream case, a crucial observation from Corollary 3.2 is that there exists a tradeoff between  $K$ , the number of users served in each cell, and  $q$ , the number of interferers each user can cancel interference from. The direct and interfering channel strengths in practical cellular networks can vary significantly. Intuitively, a cellular network should require interference nulling from only the dominant interferers while serving as many users per cell as possible. This necessitates a careful design of the set  $\mathcal{I}$  while ensuring feasibility of partial IA. The condition in the corollary plays an important role in network optimization framework developed in the next section.

	BS1	BS2	BS3	BS4	
U11			×	×	$\leq q$
U12		×	×		$\leq q$
U21	×		×		$\leq q$
U22	×			×	$\leq q$
U31		×		×	$\leq q$
U32	×			×	$\leq q$
U41	×	×			$\leq q$
U42		×	×		$\leq q$

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Fig. 1. Illustration of the sufficient condition for feasibility of PIA in Corollary 3.2.

#### IV. OPTIMIZATION FRAMEWORK

The optimization framework developed in this paper aims to leverage the strength of IA in nulling interference to overcome the limitations imposed by the non-convexity of the NUM problem. In a wireless cellular network, spatial resources can be used in one of three ways: (a) they can be used to serve more users i.e., spatial multiplexing; (b) they can be used to enhance the signal strength (e.g. matched filtering); or (c) they can be used to null interference (zero-forcing/IA). NUM algorithms strive to strike the right balance between these three competing objectives to maximize a certain utility. In dense cellular networks, due to the conflicting nature of these objectives, NUM algorithms may not be able to comprehensively navigate the entire optimization landscape. The main point of this paper is that in certain networks, it may be beneficial to introduce a pre-optimization step to exclusively focus on interference nulling and subsequently use the NUM algorithm to re-balance these priorities to maximize the utility function.

Given a  $(G, K, M \times N)$  network, we propose a two-stage optimization framework where the first stage focuses on nulling interference from the dominant interferers using IA followed by a joint optimization of beamformers and transmit powers to maximize a network utility using the IA solution as the initial condition. Specifically, the optimization objective is to maximize the minimum rate of the scheduled users in the network<sup>1</sup>.

Such a framework is well suited for investigating the benefits of IA in the context of NUM. The difference in performance with and without the first stage of interference cancellation sheds light on the value of IA in enhancing the performance of NUM algorithms. For a given network topology, a significant difference in performance reflects that: (a) IA solutions are valuable from a NUM perspective; and (b) IA solutions (or close-to-IA solutions) do not organically

<sup>1</sup>For this particular utility, spatial multiplexing is handled by an external scheduler thus simplifying the NUM algorithm's task to simply balance signal strength and interference across the network. For utilities such as weighted sum-rate, power control acts as a proxy for controlling the number of users served.

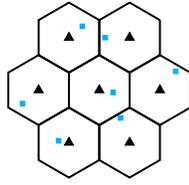
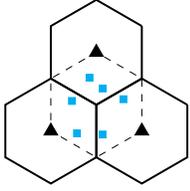


Fig. 2. Network topologies: a three-sector cluster and a 7-cell hexagonal layout.

emerge from NUM algorithms due to the conflicting uses for spatial resources. Details of the proposed optimization framework follow.

### A. Stage I: Partial Interference Alignment

In the first stage, each user identifies  $q$  dominant interferers from whom we attempt to null interference using IA. Note from Corollary 3.2 that for a given  $(G, K, M \times N)$  network, the choice of  $q$  is closely dependent on the number of scheduled users; in fact, it is necessary that  $q \leq \lfloor \frac{M+N-1}{K} \rfloor - 1$ . This suggests that higher the number of scheduled users, fewer the number of interferers that can be nulled and vice versa. Thus, the number of scheduled users  $K$  emerges as a crucial parameter governing the usefulness of IA.

For a fixed  $K$ , set  $q = \lfloor \frac{M+N-1}{K} \rfloor - 1$ . The  $q$  dominant interferers are identified by their interference strength with the transmit and receive beamformers set to certain predetermined values. In our simulations we set all beamformers to be equal to the all-ones vector.

Once the dominant interferers are identified, we then ensure that the chosen set of user-BS pairs, denoted as  $\mathcal{I}$ , conforms to the condition for feasibility of PIA as stated in Corollary 3.2. Constructing a matrix analogous to that shown in Fig. 1, it is easy to see that while the rows of this matrix have no more than  $q$  chosen entries by construction, the columns may have more than  $Kq$  chosen entries. To eliminate such cases, if any column has more than  $Kq$  chosen cells, we sort the chosen cells of this column in the descending order of their interference strengths and prune this sorted list, from the bottom, until no more than  $Kq$  cells are left. The set of user-BS pairs that result at the end of this process (denoted as  $\tilde{\mathcal{I}}$ ), satisfies the conditions imposed by Corollary 3.2, ensuring the feasibility of PIA. As a result of the pruning, not all users have interference from all their  $q$  dominant interferers nulled. This is however unavoidable to ensure feasibility of PIA. Note that for the case  $q = G - 1$ , no such pruning is necessary.

Once  $\tilde{\mathcal{I}}$  is obtained, aligned beamformers satisfying the conditions for PIA can be designed using any algorithm developed for IA such as interference leakage minimization [15], [22], [23], iterative matrix norm minimization [26], etc.

### B. Stage II: Utility Maximization

This stage focuses on maximizing a given network utility function using the aligned beamformers obtained in the previous stage as the initialization. As stated before, this paper focuses on maximizing the minimum rate for the scheduled

TABLE I  
SIMULATION PARAMETERS

Network	3-Sector	$(3, K, 3 \times 4)$
	Hexagonal Layout	$(7, K, 4 \times 4)$
BS-to-BS distance	600m to 1800m	
Transmit power PSD	-35dBm/Hz	
Thermal noise PSD	-169dBm/Hz	
Antenna gain	10dBi	
SINR gap	6dB	
Distance dependent pathloss	$128.1 + 37\log_{10}(d)$	
Shadowing	Log-normal, 8dB SD	
Fading	Rayleigh	

users subject to per-BS power constraints. In mathematical terms, we solve the following optimization problem:

$$\begin{aligned}
 & \underset{\mathbf{v}_{gk}, \mathbf{u}_{gk}}{\text{maximize}} && t \\
 & \text{subject to} && \frac{|\mathbf{u}_{gk}^H \mathbf{H}_{(g,gk)} \mathbf{v}_{gk}|^2}{\sigma^2 + \sum_{(i,j) \neq (g,k)} |\mathbf{u}_{gk}^H \mathbf{H}_{(i,gk)} \mathbf{v}_{ij}|^2} \geq t, \quad \forall (g, k), \\
 & && \sum_{k=1}^K |\mathbf{v}_{gk}|^2 \leq P_{max}, \quad \forall g, \\
 & && |\mathbf{u}_{gk}|^2 = 1, \quad \forall (g, k), \tag{8}
 \end{aligned}$$

where  $\mathbf{v}_{gk}$ ,  $\mathbf{u}_{gk}$  are the variables for optimization and  $P_{max}$  is the maximum transmit power permitted at any BS. This problem is non-convex in its current form and no convex reformulation is known except when the users have a single antenna. Several techniques for finding a local optimum of this problem have been proposed [7]–[9]. We solve (8) by alternately optimizing the transmit and receive beamformers, leveraging the convex reformulation that emerges when users have a single antenna [27]. Fixing the receive beamformers to be the aligned beamformers obtained from the first stage, we use a bisection search over  $t$  to find a maximal min-rate as proposed in [27]. Fixing the transmit beamformers to those obtained at the end of this bisection search, the optimal receive beamformers are given by the MMSE beamformers. Once the receive beamformers are updated, we proceed to re-optimize the transmit beamformers and this procedure is repeated for a fixed number of iterations.

## V. SIMULATION RESULTS

The value of IA is best illustrated in a dense cluster of isolated BSs where interference mitigation plays an increasingly important role as the distance between BSs decreases. Towards this end, we consider two different network topologies as shown in Fig. 2. The first network is a 3-sector cluster with 3 cooperating BSs and the second is a 7-cell hexagonal cluster with 7 cooperating BSs. The same pathloss, shadowing and fading models are used for both networks. Users are assumed to be uniformly distributed in each cell, and are served with one data stream each. Table I lists the antenna configuration for each of the networks, along with other parameter settings.

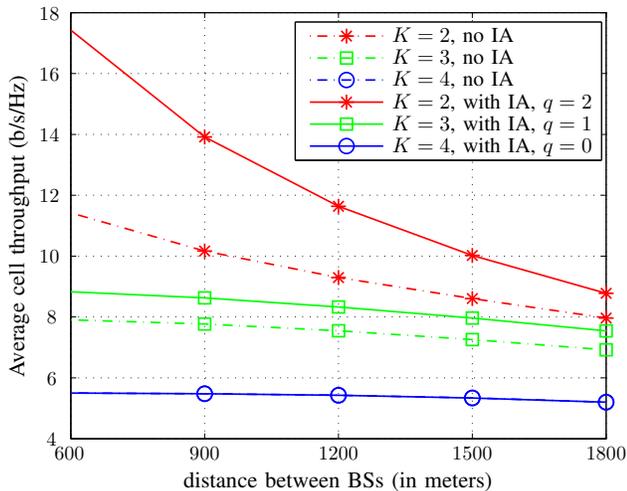


Fig. 3. Average per-cell throughput in a  $(3, K, 3 \times 4)$  three-sector network for maximizing minimum user rate under per-BS power constraints.

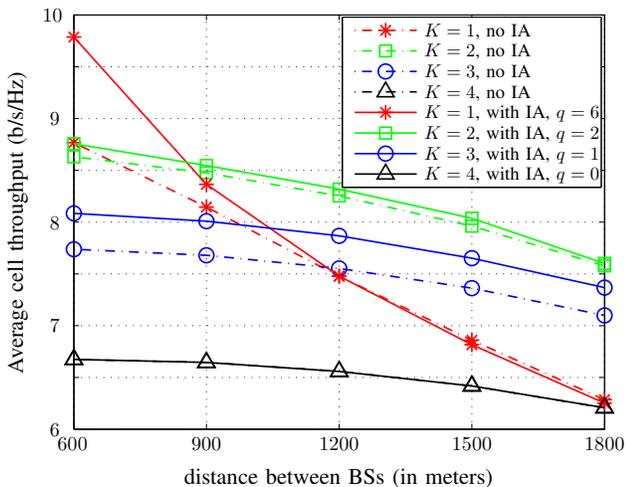


Fig. 4. Average per-cell throughput in a  $(7, K, 4 \times 4)$  hexagonal layout for maximizing minimum user rate under per-BS power constraints.

For each network, the number of scheduled users per cell,  $K$ , is varied from  $\lfloor \frac{M+N-1}{G} \rfloor$  to  $N$ . Note that as  $K$  increases, the number of dominant BSs that can be cancelled in the first stage decreases. When  $K > \frac{M+N-1}{2}$ , no dominant interferers can be nulled and the beamformers are chosen to only cancel intra-cell interference.

For a given set of scheduled users, the proposed optimization framework is used to maximize the minimum user rate. For each user, interference from at most  $q = \lfloor \frac{M+N-1}{K} \rfloor - 1$  interferers is nulled using the interference leakage minimization algorithm [15]. Using these aligned beamformers as initialization, the optimization problem presented in (8) is solved by alternately optimizing the transmit and receive beamformers for a fixed number of iterations. The convex optimization problem arising from (8) for a fixed set of receive beamformers is solved using CVX, a package for specifying and solving convex programs [28]. The performance of the

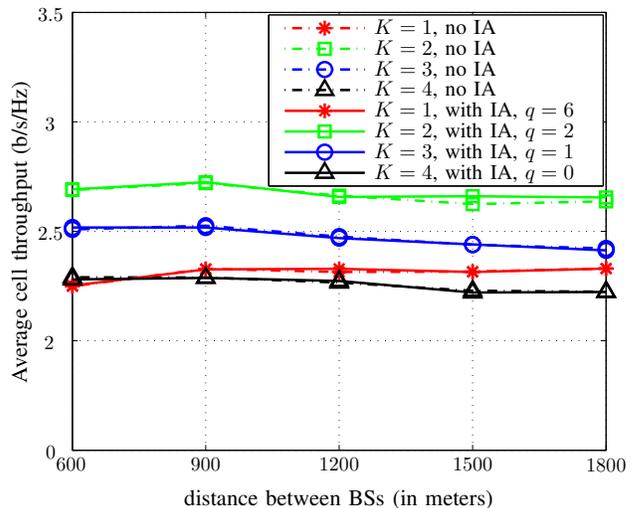


Fig. 5. Average per-cell throughput in a  $(7, K, 4 \times 4)$  network forming a 7-cell hexagonal cluster in a 49-cell hexagonal network for maximizing the minimum user rate under per-BS power constraints.

proposed framework is compared to the setup where the first stage is omitted, i.e., the dominant interferers are not nulled using IA (marked as ‘no IA’). The results of the optimization are averaged over 100 user locations.

Figs. 3 and 4 plot the results of maximizing the minimum rate for each of the three networks as a function of BS-to-BS distance and the number of scheduled users. Average cell throughput—measured as the max-min rate times the number of scheduled users ( $K$ ), averaged over user locations and channel fading—is used as the performance metric for comparison. It is seen that IA solutions provide an altered interference landscape that is otherwise non-trivial to find; this altered landscape enhances the performance of subsequent NUM algorithms. Focusing on Fig. 3, it is clear that IA has a significant impact on optimization, especially when BSs are closely spaced. The gain of IA depends on the number of users scheduled. In particular, when 2 users/cell are scheduled, it is possible to achieve 1 DoF/user as interference can be completely nulled in the network ( $q = G - 1 = 2$ ). In this case, IA provides 4-6 b/s/Hz improvement at small BS-to-BS distances. When 3 users/cell are scheduled, IA can cancel interference from up to one interferer for each user. Such IA solutions are seen to enhance the cell throughput by about 1 b/s/Hz. However, when 4 users/cell are scheduled, only intra-cell interference can be nulled, and IA has no impact on the optimization. Note also that because it is possible to completely null inter-cell interference only when  $K = 2$  (or equivalently,  $q = 2$ ), this is the only scenario where throughput does not saturate as the BS-to-BS distance decreases. Finally, we observe that for a broad range of BS-to-BS distances, scheduling 2 users/cell appears to be optimal.

Fig. 4 considers the 7-cell network where it can be seen that with increasing cluster size, scheduling  $K = \lfloor \frac{M+N-1}{G} \rfloor$  users (in this case,  $K = 1, q = 6$ ), is a good strategy only at small BS-to-BS distances. At these small distances, IA does

provide benefit; but the rate gain due to IA is not consistent in other cases. The simulation does provide insight on the optimal number of users to schedule. It appears that the number of scheduled users should be such that nulling interference from one or two of the dominant interferers for each user is feasible.

To test the effectiveness of IA in an environment where the given cluster of cooperating BSs is surrounded by other non-cooperating BSs, we simulate a hexagonal 49-cell network with the central 7 cells forming a cluster. Applying the proposed framework in such an environment while treating out-of-cluster interference as noise, it is seen from Fig. 5 that (a) density has little impact on the overall throughput, and (b) aligned beamformers carry little significance. It is clear from the spectral efficiencies achieved that such environments are significantly limited by out-of-cluster interference. Nulling interference from a few dominant interferers does not impact the final outcome of the optimization. These results suggest that when investigating beamformer design in practical cellular environments, where out-of-cluster interference is unavoidable, focusing exclusively on the design of aligned beamformers does not warrant sufficient importance.

## VI. CONCLUSION

This paper investigates the role of IA in NUM. In order to leverage the strengths of IA and to overcome the shortcoming of conventional NUM algorithms, a two-stage optimization framework is proposed. This framework is used to evaluate the value of IA in practical cellular network optimization. Through simulations on different network topologies for maximizing the minimum rate achieved in a given network, it is established that IA is valuable in network topologies with a small number of BSs and without significant uncoordinated interference. In networks with significant out-of-cluster interference, nulling interference from a few dominant BSs does not appear to make an impact on the performance of NUM algorithms. Thus in dense cellular networks, IA is likely to play a limited role even with centralized network optimization and full CSI.

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