A Coordinated Uplink Scheduling and Power Control Algorithm for Multicell Networks

Kaiming Shen and Wei Yu
Electrical and Computer Engineering Department
University of Toronto, ON M5S 3G4, Canada
Email: {kshen,weiyu}@comm.utoronto.ca

***Abstract*—This paper proposes a coordinated joint uplink user scheduling and power control algorithm across multiple cells in a wireless cellular network. The uplink scheduling and power control problem is more challenging than the downlink, because the uplink interference pattern strongly depends on the schedule decisions of the neighboring cells. This paper considers a weighted sum rate maximization objective in the uplink and formulates the joint scheduling and power control problem as a non-convex mixed-integer problem. The main contribution of the paper is a novel problem reformulation based on sum-of-ratios programming and a distributed iterative algorithm based on a subsequent quadratic transformation. The algorithm optimizes the transmit power and scheduling of each iteration jointly in closed form. The proposed algorithm has provable convergence guarantee, and is shown to significantly outperform existing approaches.**

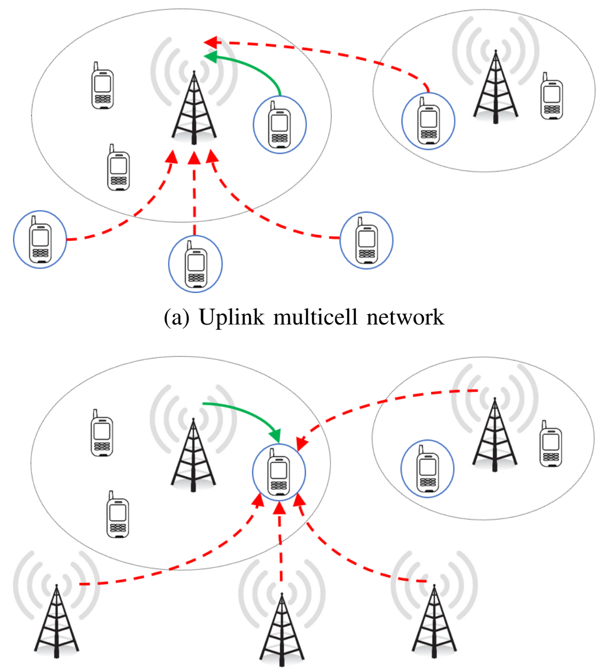
I. Introduction

Multicell cooperation has the potential to significantly improve the performance of wireless cellular networks. This paper explores cooperation at the user scheduling and power control level across the multiple cells in the uplink of a wireless cellular network in order to reduce cross-cell interference and to maximize network utility.

The uplink scheduling and power control problem is a difficult non-convex mixed-integer programming problem, and is numerically more challenging to solve than the corresponding problem in the downlink, because the uplink interference pattern strongly depends on the user scheduling decisions of the neighboring cells, while in the downlink interference pattern does not depend on scheduling, as illustrated in Fig. 1. As consequence, optimization approach that iterates between scheduling and power optimization typically works well in the downlink, but not in the uplink.

This paper formulates the uplink joint user scheduling and power control problem as a mathematical programming problem and proposes several novel transformations that allow efficient numerical algorithms to be developed. We consider a weighted sum rate maximization objective, then reformulate the problem as a sum-of-ratios program, which together with a novel quadratic transform, gives rise to a distributed iterative algorithm that optimizes user scheduling and power control *jointly* in each step. The resulting algorithm significantly outperforms existing approaches that optimize scheduling and power *separately*.

Scheduling has been considered extensively in the wireless networking literature. However, most existing uplink scheduling schemes do not account for interference, i.e., they are either based on channel quality alone or assume worst-case interference [1]. Interference-aware scheduling has been considered from a game theory perspective, e.g., [2], which aims to find a Nash bargaining solution, and [3], which considers the problem in a pseudo-potential game model, but the optimality of these solutions is not easy to establish. Optimization heuristics that can be applied to the uplink scheduling problem also include opportunistic approaches [4], [5] and the greedy method [6]. Further, [7] proposes to simply apply the downlink user schedule to the uplink, but such an approach is in general not optimal. This paper adopts a more rigorous mathematical programming perspective. Our aim is to find efficient algorithm for tackling the joint uplink user scheduling and power optimization problem.



(a) Uplink multicell network

(b) Downlink multicell network

Fig. 1: Interference pattern depends on the user scheduling in the neighboring cells in the uplink, but does not depend on the scheduling decisions of the neighboring cells in the downlink. Here, the solid lines represent the desired signal; the dotted lines represent the interfering signal; the scheduled user terminal in each cell is circled.

II. Problem Statement

Consider the uplink of a wireless cellular network with J cells, one base-station (BS) per cell, and a fixed set of users $\mathcal{U}_i$ associated with each BS i. The BSs and the users are equipped with a single antenna each. In every time slot, one user is scheduled to transmit in each cell; users from different cells create interference for each other.

Let the discrete variable s_i denote the scheduled user at BS i (e.g., $s_5 = 3$ if user 3 is scheduled by BS 5). Let the continuous variable p_{s_i} denote the transmit power of user s_i. Given a set of weights w_{s_i} that reflect the user priorities in each time slot, this paper focuses on the network utility maximization problem of optimizing an objective function of weighted sum rate over all users in the network:

$$f_o(\mathbf{s},\mathbf{p}) = \sum_{i=1}^{J} w_{s_i} \log\left(1+\frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j\neq i}|h_{i,s_j}|^2 p_{s_j}+\sigma^2}\right) \quad (1)$$

where $h_{i,s_j} \in \mathbb{C}$ is the uplink channel from user s_j to BS i, σ^2 denotes the additive white Gaussian background noise power.

The joint uplink scheduling and power control problem is formulated as

$$\begin{aligned} \underset{\mathbf{s},\mathbf{p}}{\text{maximize}} \quad & f_o(\mathbf{s},\mathbf{p}) && (2a)\\ \text{subject to} \quad & 0 \le p_{s_i} \le P && (2b)\\ & s_i \in \mathcal{U}_i, && (2c) \end{aligned}$$

where P is the maximum transmit power level of the user. Constraint (2c) indicates that the scheduled user s_i for BS i must be from the associate set for the BS i.

The optimization problem (2) is not easy to solve, because (i) the scheduling variable s_j is discrete, and (ii) the objective as function of p_{s_j} is nonconvex. This is a non-convex mixed-integer programming problem.

III. Proposed Approach

The primary difficulty in solving the optimization problem (2) is that the scheduling decision and the transmit power level of the scheduled user in each cell interact with its neighboring cells through the interference term in the denominator of rate expression in the objective function (1). A naive way for tackling the problem would be to make scheduling and power allocation decisions on an individual per-cell basis, assuming that the interference is fixed, then iterate between the cells. But such an approach does not work well, because the interference pattern can drastically change when a different user is scheduled; there is no guarantee that the iteration would even converge. The main idea of this paper is to devise a way to enable the individual update of scheduling and power on a per-cell basis, while ensuring convergence. Toward this end, we recast the problem in a sequence of equivalent forms in the following steps.

A. Lagrangian Reformulation

This section reformulates the original problem (2) as a sum-of-ratios programming problem. First, we introduce a new variable γ_i denoting the uplink SINR at BS i, and rewrite (2) as

$$\begin{aligned} \underset{\mathbf{s},\mathbf{p},\boldsymbol{\gamma}}{\text{maximize}} \quad & \sum_{i=1}^{J} w_{s_i}\log(1+\gamma_i) && (3a)\\ \text{subject to} \quad & \text{(2b), (2c)} && (3b)\\ & \gamma_i = \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j\neq i}|h_{i,s_j}|^2 p_{s_j}+\sigma^2}. && (3c) \end{aligned}$$

We then form the Lagrangian function of (3) with respect to the constraint (3c)

$$L(\mathbf{s},\mathbf{p},\boldsymbol{\gamma},\boldsymbol{\lambda}) = \sum_{i=1}^{J} w_{s_i}\log(1+\gamma_i) - \sum_{i=1}^{J}\lambda_i\left(\gamma_i - \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j\neq i}|h_{i,s_j}|^2 p_{s_j}+\sigma^2}\right), \quad (4)$$

where λ_i is the dual variable. Since $\partial L/\partial\gamma_i = 0$ at the optimum, we have

$$\gamma_i = \frac{w_{s_i}}{\lambda_i} - 1. \quad (5)$$

Combining the above equation with (3c), we arrive at a relationship between the optimal dual variable λ_i and the primal variables $\mathbf{s}$ and $\mathbf{p}$:

$$\lambda_i = w_{s_i} - \frac{w_{s_i}|h_{i,s_i}|^2 p_{s_i}}{\sum_j |h_{i,s_j}|^2 p_{s_j}+\sigma^2}. \quad (6)$$

Now, the original optimization problem (2) can be thought of as an optimization of the Lagrangian (4) with appropriate $\boldsymbol{\lambda}$. But since the optimal $\boldsymbol{\lambda}$ is related to the primal variables through (6), we can substitute the above optimal λ_i expression (6) in $L(\mathbf{s},\mathbf{p},\boldsymbol{\gamma},\boldsymbol{\lambda})$ to arrive at a new form of the objective function, denoted below as f_r,

$$f_r(\mathbf{s},\mathbf{p},\boldsymbol{\gamma}) = \sum_{i=1}^{J} w_{s_i}\log(1+\gamma_i) - \sum_{i=1}^{J} w_{s_i}\gamma_i + \sum_{i=1}^{J}\frac{w_{s_i}(\gamma_i+1)|h_{i,s_i}|^2 p_{s_i}}{\sum_j |h_{i,s_j}|^2 p_{s_j}+\sigma^2}. \quad (7)$$

In essence, the original problem (2) is now equivalently reformulated as

$$\begin{aligned} \underset{\mathbf{s},\mathbf{p},\boldsymbol{\gamma}}{\text{maximize}} \quad & f_r(\mathbf{s},\mathbf{p},\boldsymbol{\gamma}) && (8a)\\ \text{subject to} \quad & \text{(2b), (2c).} && (8b) \end{aligned}$$

The above reformulation is easier to tackle than the original problem in the sense that the optimizing variables $\mathbf{s}$ and $\mathbf{p}$ are now outside of the logarithm function. Further, the last term in $f_r(\mathbf{s},\mathbf{p},\boldsymbol{\gamma})$ takes the form of a sum-of-ratios programming problem. The optimization of sum-of-ratios is a well-known problem in fractional programming literature. However, the problem is still non-convex; the state-of-arts algorithm can only handle problems with limited number of ratios [8]. In the next subsection, we propose a new way of dealing with these ratios.

B. Quadratic Transform

We propose to reformulate the sum-of-ratios problem using a novel quadratic transform, based on the following fact.

Proposition 1. Given a constraint set $\mathcal{X}$ and two scalar valued functions $A(\mathbf{x})$ and $B(\mathbf{x})$ that both output positive values for any $\mathbf{x} \in \mathcal{X}$, the optimization problem for the ratio

$$\underset{\mathbf{x}}{\text{maximize}} \quad \frac{A(\mathbf{x})}{B(\mathbf{x})} \tag{9a}$$

$$\text{subject to} \quad \mathbf{x} \in \mathcal{X} \tag{9b}$$

is equivalent to

$$\underset{\mathbf{x},y}{\text{maximize}} \quad 2y\sqrt{A(\mathbf{x})} - y^2 B(\mathbf{x}) \tag{10a}$$

$$\text{subject to} \quad \mathbf{x} \in \mathcal{X}. \tag{10b}$$

Proof. Construct a quadratic function $h(y) = 2y\sqrt{A(\mathbf{x})} - y^2B(\mathbf{x})$. It can be easily verified that $\max_y\{h(y)\} = \frac{A(\mathbf{x})}{B(\mathbf{x})}$. Therefore, (9a) is equivalent to $\max_{\mathbf{x}} \max_y\{h(y)\}$. □

The main advantage of the above reformulation is that the optimization of ratio now becomes an optimization of two terms involving the numerator and the denominator separately. When applied to the objective function $f_r(\mathbf{s}, \mathbf{p}, \boldsymbol{\gamma})$, Proposition 1 allows us to "flatten" each ratio in the last term, essentially decoupling (s_i, p_{s_i}) over i.

Introducing auxiliary variables $y_1, y_2, \cdots, y_J$ for each ratio in the last term in $f_r(\mathbf{s}, \mathbf{p}, \boldsymbol{\gamma})$ and applying the quadratic transform to each ratio, and after some algebra, the problem (8) can now be recast equivalently as

$$\underset{\mathbf{s},\mathbf{p},\boldsymbol{\gamma},\mathbf{y}}{\text{maximize}} \quad f_q(\mathbf{s}, \mathbf{p}, \boldsymbol{\gamma}, \mathbf{y}) \tag{11a}$$

$$\text{subject to} \quad \text{(2b), (2c)} \tag{11b}$$

where

$$f_q(\mathbf{s}, \mathbf{p}, \boldsymbol{\gamma}, \mathbf{y}) = \sum_{i=1}^{J} \Bigg(w_{s_i} \log(1+\gamma_i) - w_{s_i}\gamma_i - y_i^2\sigma^2 + 2y_i\sqrt{w_{s_i}(\gamma_i+1)\left|h_{i,s_i}\right|^2 p_{s_i}} - \sum_{j=1}^{J} y_j^2 \left|h_{j,s_i}\right|^2 p_{s_i} \Bigg). \tag{12}$$

The above new objective function is derived by grouping all the terms related to the same s_i together. The key observation is that the scheduling and power variables $(\mathbf{s}, \mathbf{p})$ are now *decoupled* in this new formulation (11). Specifically, the scheduling and power optimization in each cell, i.e., (s_i, p_{s_i}), can be done independently in each cell, as long as $\boldsymbol{\gamma}$ and $\boldsymbol{y}$ are fixed. This motivates an iterative approach for solving (11).

C. Proposed Algorithm

We propose to maximize f_q over variables $\boldsymbol{\gamma}$, $\mathbf{y}$, $\mathbf{s}$ and $\mathbf{p}$ in an iterative manner as follows. When all the other variables are fixed, the optimal $\mathbf{y}$ can be obtained by setting $\nabla_{\mathbf{y}} f_q$ to zero, i.e.,

$$y_i^* = \frac{\sqrt{w_{s_i}(1+\gamma_i)|h_{i,s_i}|^2 p_{s_i}}}{\sum_j |h_{i,s_j}|^2 p_{s_j} + \sigma^2}. \tag{13}$$

We substitute the above optimal $\mathbf{y}$ expression in f_q, then find the optimal $\boldsymbol{\gamma}$ by setting $\nabla_{\boldsymbol{\gamma}} f_q$ to zero to get

$$\gamma_i^* = \frac{|h_{i,s_i}|^2 p_{s_i}}{\sum_{j\neq i} |h_{i,s_j}|^2 p_{s_j} + \sigma^2}. \tag{14}$$

Fixing $\mathbf{y}$ and $\boldsymbol{\gamma}$, if user s is to be scheduled by its associated BS i, we can derive its optimal transmit power level p_s by setting $\partial f_q / \partial p_s$ to zero. This power optimization problem is concave, subject to simple boundary constraints, so it has an analytic solution:

$$p_s^* = \min\left\{ \frac{w_s(1+\gamma_i)\left|h_{i,s}\right|^2 y_i^2}{\left(\sum_j \left|h_{j,s}\right|^2 y_j^2\right)^2}, P \right\}. \tag{15}$$

The most important part is the optimization of the scheduling variable $\mathbf{s}$. As stated previously, the objective function f_q has the desirable property that the optimization of $\mathbf{s}$ is decoupled on a per-cell basis, i.e., the optimization of s_i does not depend on the other s_j's for $j \neq i$, when $\boldsymbol{\gamma}$ and $\mathbf{y}$ are fixed. Now, since the optimal transmit power level p_s^* is already determined by (15) if user s is scheduled, we can substitute the optimized power p_s^* into f_q and make optimal scheduling decision through a simple search to find the user that maximizes f_q in each cell. Moreover, we can rewrite f_q in the form of difference between two positive functions, and formally state the scheduling decision as follows:

$$s_i^* = \arg\max_{s\in\mathcal{U}_i} \left(G_i(s) - \sum_{j\neq i} D_j(s) \right) \tag{16}$$

where $G_i(s)$ and $D_j(s)$ are defined as

$$G_i(s) = w_s \log(1+\gamma_i) - w_s\gamma_i + 2y_i\sqrt{w_s(1+\gamma_i)\left|h_{i,s}\right|^2 p_s^*} - p_s^* y_i^2 \left|h_{i,s}\right|^2 \tag{17}$$

and

$$D_j(s) = y_j^2 \left|h_{j,s}\right|^2 p_s^*. \tag{18}$$

We interpret $G_i(s)$ and $D_j(s)$ as the utility and penalty functions, respectively, so that the scheduling decision has an intuitive utility-minus-price structure. More precisely, $G_i(s)$ is the utility gain of scheduling user s at BS i and $D_j(s)$ is the penalty for interfering a neighboring cell j by scheduling user s. The best user to schedule is the one that balances these two effects. Note that the scheduling and power control are done on a per-cell basis. This enables distributed implementation.

We summarize the proposed algorithm in the following:

Algorithm 1 Joint uplink scheduling and power control

Initialization: Initialize $\mathbf{s}$ and $\mathbf{p}$.
repeat
1) Update $\boldsymbol{\gamma}$ by (14);
2) Update $\mathbf{y}$ by (13);
3) Update $(\mathbf{s}, \mathbf{p})$ by (15) and (16);
until Convergence

It is easy to verify that Algorithm 1 is guaranteed to converge (although not necessarily to a global optimum), because f_q is monotonically nondecreasing with each iteration. Further, we can show that the original objective function, i.e., the weighted sum rate, is also monotonically nondecreasing with each iteration, as shown below.

Lemma 1. When $\boldsymbol{\gamma}$ satisfies (14), $f_o(\mathbf{s},\mathbf{p}) = f_r(\mathbf{s},\mathbf{p},\boldsymbol{\gamma})$.

Lemma 2. When $\mathbf{y}$ satisfies (13), $f_r(\mathbf{s},\mathbf{p},\boldsymbol{\gamma}) = f_q(\mathbf{s},\mathbf{p},\boldsymbol{\gamma},\mathbf{y})$.

Proposition 2. Algorithm 1 is guaranteed to converge. The weighted sum rate f_o is monotonically nondecreasing with each iteration.

Proof. Let superscript t be the iteration number. Let $\boldsymbol{\gamma}(\mathbf{s},\mathbf{p})$ be the function defined in (14); let $\mathbf{y}(\mathbf{s},\mathbf{p},\boldsymbol{\gamma})$ be the function defined in (13). It can be seen that

$$\begin{aligned}
& f_o(\mathbf{s}^{t+1},\mathbf{p}^{t+1}) \\
& \overset{(a)}{=} f_r(\mathbf{s}^{t+1},\mathbf{p}^{t+1},\boldsymbol{\gamma}(\mathbf{s}^{t+1},\mathbf{p}^{t+1})) \\
& \overset{(b)}{\geq} f_r(\mathbf{s}^{t+1},\mathbf{p}^{t+1},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t})) \\
& \overset{(c)}{=} f_q(\mathbf{s}^{t+1},\mathbf{p}^{t+1},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}),\mathbf{y}(\mathbf{s}^{t+1},\mathbf{p}^{t+1},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}))) \\
& \overset{(d)}{\geq} f_q(\mathbf{s}^{t+1},\mathbf{p}^{t+1},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}),\mathbf{y}(\mathbf{s}^{t},\mathbf{p}^{t},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}))) \\
& \overset{(e)}{\geq} f_q(\mathbf{s}^{t},\mathbf{p}^{t},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}),\mathbf{y}(\mathbf{s}^{t},\mathbf{p}^{t},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t}))) \\
& \overset{(f)}{=} f_r(\mathbf{s}^{t},\mathbf{p}^{t},\boldsymbol{\gamma}(\mathbf{s}^{t},\mathbf{p}^{t})) \\
& \overset{(g)}{=} f_o(\mathbf{s}^{t},\mathbf{p}^{t})
\end{aligned}$$

where (a) follows by Lemma 1, (b) follows since the update (14) maximizes f_r when all the other variables are fixed, (c) follows by Lemma 2, (d) follows since the update (13) maximizes f_q when all the other variables are fixed, (e) follows since the updates (16) and (15) maximize f_q when all the other variables are fixed, (f) follows by Lemma 2, (g) follows by Lemma 1.

Therefore, f_o is monotonically nondecreasing with each iteration. Together with the fact that f_o is bounded, this shows that the iterative algorithm must converge. □

IV. Connection with Fixed-Point Method for Power Control

The proposed iterative algorithm optimizes scheduling and power control in each step. In this section, we seek a further understanding of the algorithm by focusing on the power control aspect of the algorithm alone. We show that in this case the proposed algorithm becomes a fixed-point method for solving the power optimization problem.

Consider a scenario with fixed user schedule with user transmit powers as the optimization variables. In this case, the original objective function $f_o(\mathbf{s},\mathbf{p})$ is reduced to $f_o(\mathbf{p})$. Power optimization reduces to finding solution to the first-order condition

$$\nabla_{\mathbf{p}} f_o(\mathbf{p}) = 0, \tag{19}$$

which can be written as

$$\frac{1}{p_{s_i}} \cdot \underbrace{\frac{w_{s_i}\gamma_i(\mathbf{p})}{1+\gamma_i(\mathbf{p})}}_{Q_1(\mathbf{p})} - \underbrace{\sum\nolimits_{j\neq i} \frac{w_{s_j}\gamma_j^2(\mathbf{p})|h_{j,s_i}|^2}{(1+\gamma_j(\mathbf{p}))|h_{j,s_j}|^2 p_{s_j}}}_{Q_2(\mathbf{p})} = 0 \tag{20}$$

where $\gamma_i(\mathbf{p})$ denotes the SINR expression in $\mathbf{p}$ as in (14). To find a set of powers that satisfy the above condition, one strategy is to isolate p_{s_i} at one side of the equation—this automatically results in an update equation for power, which, if converges, would achieve at least a local optimum solution to the power control problem.

However, it is in general not easy to decide which part of the condition $\nabla_{\mathbf{p}} f_o(\mathbf{p}) = 0$ should be fixed in order to ensure the convergence of fixed-point iteration. For instance, [9] proposes to fix $Q_1(\boldsymbol{p})$ and $Q_2(\boldsymbol{p})$ as shown in (20) and arrives at the following fixed-point method for power control

$$p_{s_i}^{t+1} = \frac{Q_1(\mathbf{p}^t)}{Q_2(\mathbf{p}^t)} \tag{21}$$

where the superscript t indicates the iteration number. However, this fixed-point iteration does not necessarily converge. (In fact, [9] proves that this iteration is guaranteed to converge when the resulting SINR values are all sufficiently high.)

If we focus on the power control part of Algorithm 1 (with scheduling variable $\mathbf{s}$ fixed), we can see that the power update formula (15) for $\mathbf{p}$, with (14) and (13) substituted in, is just a fixed-point iteration of the first-order condition, exactly like (20) except that different components $\tilde{Q}_1(\mathbf{p})$ and $\tilde{Q}_2(\mathbf{p})$, shown below, are fixed

$$\frac{1}{\sqrt{p_{s_i}}} \cdot \underbrace{\frac{w_{s_i}\gamma_i(\mathbf{p})}{\sqrt{p_{s_i}}}}_{\tilde{Q}_1(\mathbf{p})} - \underbrace{\sum\nolimits_{j} \frac{w_{s_j}\gamma_j^2(\mathbf{p})|h_{j,s_i}|^2}{(1+\gamma_j(\mathbf{p}))|h_{j,s_j}|^2 p_j}}_{\tilde{Q}_2(\mathbf{p})} = 0. \tag{22}$$

In this case, the transmit power variable $\mathbf{p}$ update becomes

$$p_{s_i}^{t+1} = \left(\frac{\tilde{Q}_1(\mathbf{p}^t)}{\tilde{Q}_2(\mathbf{p}^t)}\right)^2, \tag{23}$$

which, along with an additional projection step onto the constraint set, can be seen to be (15) after some algebra. Thus, the power control part of Algorithm 1 is just a fixed-point iteration, but with a crucial advantage that it is guaranteed to converge, in contrast to the update (21) proposed in [9].

V. Simulation Results

Numerical simulation is performed in a 7-cell wrapped-around topology with a total of 84 users uniformly placed in the network. The BS-to-BS distance is 800m. Each user is associated with the strongest BS. The maximum transmit power spectrum density (PSD) of the users is -47dBm/Hz; the background noise PSD is set to be -171dBm/Hz over 10MHz bandwidth. The wireless channel model includes a distance-dependent pathloss component at $128.1 + 37.6\log_{10}(d)$dB (where the distance d is in km) and a log-normal shadowing component with 8dB standard deviation.

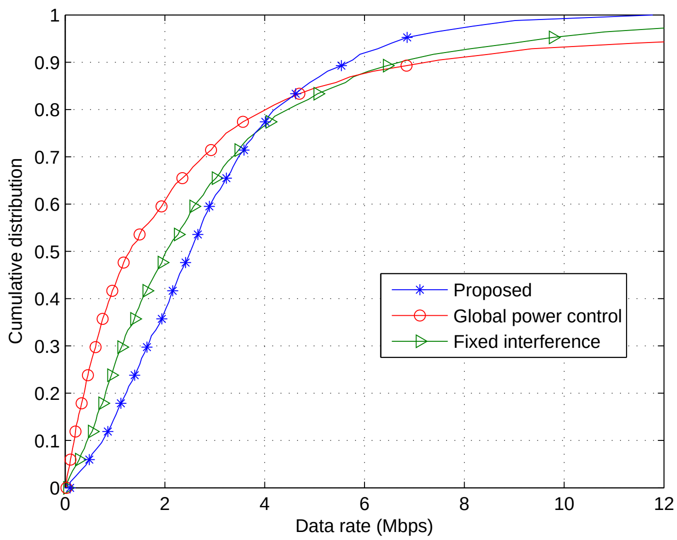


Fig. 2: Comparison of the proposed coordinated uplink user scheduling and power control method with two baseline methods in term of cummulative distribution function of user rates.

In the simulation, the joint user scheduling and power control problem is solved across the multiple cells in each time slot with the user priority weights updated as the reciprocals of long-term average user rates over the time, in order to ensure proportional fairness across the users. Over time, this setting of the weights maximizes the log-utility, $\sum_s \log(\bar{R}_s)$, over all users in the network, where $\bar{R}_s$ is the long-term average rate of user s, expressed in Mbps in the numerical results below.

The following two baseline uplink scheduling strategies are also simulated for comparison purpose:

- *Scheduling Assuming Fixed Interference*: In this method, uplink scheduling and power control are performed iteratively. Each user is initialized at some power level. In the scheduling stage, the user that maximizes the weighted rate in each cell is scheduled, assuming fixed interference pattern from the previous iteration. In the power control stage, the powers of the scheduled users are updated by solving a weighted sum rate maximization problem. We iterate between the two steps until convergence or a fixed number of iterations is reached.
- *Scheduling by Global Power Control*: The uplink scheduling and power control problem can also be thought of as a global power control problem, in which users not being scheduled are assigned zero power. Thus, we can run power control for all the users in the network at the same time. Most users will be assigned zero power; users assigned positive transmit power levels (typically at most one per cell) are the ones scheduled. This global power control problem is highly non-convex. We use Newton's method [7] or fixed-point iteration (23) to arrive at a local optimum.

Fig. 2 shows the cumulative distribution of the rates for all users in the network and Table I shows the resulting utility achieved using the proposed method as compared to the two baselines. We see that global power control performs very poorly, mainly because of the inability of the power control algorithm to find a global optimal solution of the non-convex problem. The fixed-interference method is also not capable of arriving at a desirable solution. In contrast, the proposed algorithm performs much better in terms of utility. For instance, the 10th-percentile user rate of the proposed algorithm is almost double of that of the fixed-interference method. Since these low-rate users are typically located at the cell-edge where cross-cell interference is the strongest, this shows that the proposed algorithm is effective in alleviating interference by coordinating uplink scheduling and power control.

TABLE I: Sum log-utilities of the proposed coordinated uplink scheduling and power control method as compared to the two baseline schemes

Algorithm	Sum log-utility
Proposed Method	67.6
Global Power Control	22.9
Fixed Interference	52.8

VI. Conclusion

This paper proposes a multicell coordinated joint uplink scheduling and power control algorithm with provable convergence guarantee using a novel problem formulation based on sum-of-ratios programming. The proposed method decouples the scheduling and power control problem on a per-cell basis, therefore allowing distributed implementation across the multiple cells. Simulation results show that the proposed algorithm significantly improves the overall network performance through interference-aware scheduling and power control.

References

[1] H. Safa and K. Tohme, "LTE uplink scheduling algorithms: Performance and challenges," in *19th Int. Conf. Telecommun. (ICT)*, Apr. 2012, pp. 1–6.

[2] E. Yaacoub and Z. Dawy, "A game theoretical formulation for proportional fairness in LTE uplink scheduling," in *IEEE Wireless Commun. and Networking Conf. (WCNC)*, Apr. 2009, pp. 1–5.

[3] T. Heikkinen, "A potential game approach to distributed power control and scheduling," *Elsevier Computer Networks*, vol. 50, no. 13, pp. 2295–2311, Sept. 2006.

[4] G. Y. Li, J. Niu, D. Lee, and J. Fan, "Multi-cell coordinated scheduling and MIMO in LTE," *IEEE Commun. Surveys Tutorials*, vol. 16, no. 2, pp. 761–775, Mar. 2014.

[5] H. Lee, T. Kwon, and D.-H. Cho, "An enhanced uplink scheduling algorithm based on voice activity for VoIP services in IEEE 802.16d/e system," *IEEE Commun. Lett.*, vol. 9, no. 8, pp. 691–693, Aug. 2005.

[6] X. Wu, S. Tavildar, S. Shakkottai, and T. Richardson, "FlashLinQ: A synchronous distributed scheduler for peer-to-peer ad hoc networks," *IEEE/ACM Trans. Netw.*, vol. 21, no. 4, pp. 1215–1228, June 2013.

[7] W. Yu, T. Kwon, and C. Shin, "Multicell coordination via joint scheduling, beamforming and power spectrum adaptation," *IEEE Trans. Wireless Commun.*, vol. 12, no. 7, pp. 1–14, June 2013.

[8] S. Schaible and J. Shi, "Fractional programming: The sum-of-ratios case," *Optimization Methods and Software*, vol. 18, no. 4, pp. 219–229, 2003.

[9] H. Dahrouj, W. Yu, and T. Tang, "Power spectrum optimization for interference mitigation via iterative function evaluation," *EURASIP J. Wireless Commun. Netw.*, Aug. 2012.