Hybrid Digital and Analog Beamforming Design for Large-Scale MIMO Systems

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***Abstract*—Large-scale multiple-input multiple-output (MIMO) systems enable high spectral efficiency by employing large antenna arrays at both the transmitter and the receiver of a wireless communication link. In traditional MIMO systems, full digital beamforming is done at the baseband; one distinct radio-frequency (RF) chain is required for each antenna, which for large-scale MIMO systems can be prohibitive from either cost or power consumption point of view. This paper considers a two-stage hybrid beamforming structure to reduce the number of RF chains for large-scale MIMO systems. The overall beamforming matrix consists of analog RF beamforming implemented using phase shifters and baseband digital beamforming of much smaller dimension. This paper considers precoder and receiver design for maximizing the spectral efficiency when the hybrid structure is used at both the transmitter and the receiver. On the theoretical front, bounds on the minimum number of transmit and receive RF chains that are required to realize the theoretical capacity of the large-scale MIMO system are presented. It is shown that the hybrid structure can achieve the same performance as the fully-digital beamforming scheme if the number of RF chains at each end is greater than or equal to twice the number of data streams. On the practical design front, this paper proposes a heuristic hybrid beamforming design strategy for the critical case where the number of RF chains is equal to the number of data streams, and shows that the performance of the proposed hybrid beamforming design can achieve spectral efficiency close to that of the fully-digital solution.**

I. Introduction

The bandwidth shortage facing the wireless cellular industry has motivated the investigation of the under-utilized millimeter wave (mmWave) frequency spectrum for the future fifth-generation (5G) wireless standard [1]. Due to the shorter wavelength, the antenna arrays at the mmWave frequencies occupy much smaller physical dimension (as compared to the antenna arrays at current 3G or LTE frequencies) [2]. This enables the use of large-scale MIMO (often referred to massive MIMO) systems, for beamforming to combat the higher path-loss and absorption at higher frequencies. However, the use of traditional fully-digital beamforming for large-scale/massive MIMO communications is not practical. This is because traditional beamforming is performed at baseband. This enables both phase control and amplitude signal control, but also requires the use of a dedicated radio frequency (RF) chain for each antenna element. Due to the high cost and power consumption of the RF chains [2], such fully-digital beamforming solution is not viable for implementation for large-scale MIMO systems at mmWave frequencies.

This paper addresses this challenge by considering a hybrid digital and analog beamforming design to reduce the number of required RF chains for beamforming in large-scale MIMO systems. Our main contribution is to show that the number of RF chains only needs to scale as the number of data streams, rather than the number of antenna elements. Specifically, this paper shows theoretically that the fully-digital beamforming performance can be attained, if the number of RF chains at both ends is more than twice the number of data streams. Further, in the critical case where the number of RF chains is equal to the number of data streams, a hybrid structure consisting of analog beamforming using phase shifters and a digital beamformer of much lower dimension can achieve spectral efficiency close to that of the fully-digital solution.

Analog or RF beamforming schemes have been extensively studied in the literature [3], [4], [5]. Analog beamforming is typically implemented using phase shifters. This implies constant modulus constraints on the elements of the beamforming matrix. Analog beamforming is much less complex than digital beamforming; however it also has poorer performance, because it does not control the magnitude of the beamformer elements. To address this issue, [6], [7] consider antenna subset selection scheme using simple analog switches, but such schemes provide limited array gain and still have poor performance in correlated channels [8].

To achieve better performance, hybrid analog and digital beamforming is first suggested in [9] under the term *soft antenna selection*. For the case of diversity transmission, [9] shows that hybrid beamforming can achieve the same performance as optimal fully-digital beamforming, if and only if there are at least two RF chains at each end. The current paper considers a similar hybrid structure, but goes one step further in generalizing the aforementioned result of [9] for the case where multiple data streams are present.

The idea of soft antenna selection is reintroduced under the term *hybrid beamforming* for single-user MIMO systems at mmWave frequencies in [10]. A practical hybrid beamforming algorithm is further proposed in [10] and is shown to have good performance under three scenarios: (i) extremely large number of antennas; (ii) more RF chains than the number of data streams; (iii) extremely correlated channel matrices. But in other cases, there is a significant gap between the achievable rate of fully-digital beamforming scheme and that of the algorithm proposed in [10].

This paper addresses this issue by proposing a heuristic algorithm to design transceiver hybrid beamformers for rate maximization in the case that the number of transmit and receive RF chains is equal to the number of data streams. The proposed algorithm relies on a beamforming strategy proposed in [11] under per-antenna power constraint. The numerical results show that the achievable rate of the proposed algorithm is significantly better than that of the existing algorithms, and is very close to the optimal fully-digital design.

II. System Model

Consider a single-user MIMO system in which the transmitter with N antennas sends N_s data streams to a receiver which is equipped with M antennas. Instead of implementing a fully-digital beamforming, which requires one distinct RF chain for each antenna, this paper considers a two-stage hybrid linear precoding and combining scheme as shown in Fig. 1, where the transmitter is equipped with N_t^{RF} transmit RF chains, with $N_s \le N_t^{\text{RF}} < N$, and the receiver is equipped with N_r^{RF} RF chains, with $N_s \le N_r^{\text{RF}} < M$.

In the hybrid beamforming structure, the overall precoder $\mathbf{V}_t \in \mathbb{C}^{N\times N_s}$ can be written as $\mathbf{V}_t = \mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}$, where $\mathbf{V}_{\text{RF}} \in \mathbb{C}^{N\times N_t^{\text{RF}}}$ is the RF precoder and $\mathbf{V}_{\text{Dig}} \in \mathbb{C}^{N_t^{\text{RF}}\times N_s}$ is the digital precoder. Similarly, the overall hybrid combiner $\mathbf{W}_t \in \mathbb{C}^{M\times N_s}$ at the receiver is $\mathbf{W}_t = \mathbf{W}_{\text{RF}}\mathbf{W}_{\text{Dig}}$, where $\mathbf{W}_{\text{RF}} \in \mathbb{C}^{M\times N_r^{\text{RF}}}$ is the RF combiner and $\mathbf{W}_{\text{Dig}} \in \mathbb{C}^{N_r^{\text{RF}}\times N_s}$ is the digital combiner. Typically, we have $N \gg N_t^{\text{RF}}$ and $M \gg N_r^{\text{RF}}$; i.e., the dimension of the digital beamformers is much smaller than the dimension of the RF beamformers.

Further, as shown in Fig. 1, we assume that the elements of the RF beamformers are implemented using analog phase shifters (with arbitrary phase angles). This implies constant modulus constraints on the elements of the RF beamforming matrices; i.e., $|\mathbf{V}_{\text{RF}}(i,j)|^2 = 1$ and $|\mathbf{W}_{\text{RF}}(i,j)|^2 = 1$, where $\mathbf{V}_{\text{RF}}(i,j)$ and $\mathbf{W}_{\text{RF}}(i,j)$ are the element of i^{th} row and j^{th} column in $\mathbf{V}_{\text{RF}}$ and $\mathbf{W}_{\text{RF}}$, respectively.

Mathematically, the linear transmit precoded signal in the hybrid structure can be written as

$$\mathbf{x} = \mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_s\times 1}$ is the vector of N_s symbols to be sent to the receiver, normalized so that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{N_s}$. Assuming a narrowband block-fading channel with i.i.d additive white Gaussian noise $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_M)$, the received signal is

$$\mathbf{y}_1 = \mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s} + \mathbf{z}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{M\times N}$ is the channel matrix. The receiver uses the combining matrix to obtain the processed signals $\mathbf{y}_2$ and $\mathbf{y}_3$ as shown in Fig. 1. In particular,

$$\mathbf{y}_3 = \mathbf{W}_{\text{Dig}}^H\mathbf{W}_{\text{RF}}^H\mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s} + \mathbf{W}_{\text{Dig}}^H\mathbf{W}_{\text{RF}}^H\mathbf{z}. \tag{3}$$

The problem of interest is to maximize the overall spectral efficiency (rate) under a power budget at the transmitter. Assuming Gaussian signalling, the rate in such a system is

$$R = \log_2\left|\mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{W}_t(\mathbf{W}_t^H\mathbf{W}_t)^{-1}\mathbf{W}_t^H\mathbf{H}\mathbf{V}_t\mathbf{V}_t^H\mathbf{H}^H\right|. \tag{4}$$

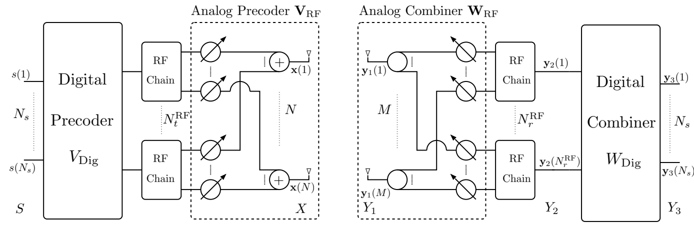

Figure 1: A massive MIMO system with hybrid beamforming structure at the transmitter and the receiver.

The optimal transmitter precoder and the receiver combiner are obtained by solving the following optimization problem,

$$\begin{aligned}
&\underset{\mathbf{V}_{\text{RF}},\mathbf{V}_{\text{Dig}}\mathbf{W}_{\text{RF}},\mathbf{W}_{\text{Dig}}}{\text{maximize}} && R && \text{(5a)}\\
&\text{subject to} && \text{Tr}(\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H) \le P && \text{(5b)}\\
& && |\mathbf{V}_{\text{RF}}(i,j)|^2 = 1,\ \forall i,j && \text{(5c)}\\
& && |\mathbf{W}_{\text{RF}}(i,j)|^2 = 1,\ \forall i,j && \text{(5d)}
\end{aligned}$$

where P is the total transmitter power budget.

III. Minimum Number of RF Chains to Realize Optimal Fully-digital Beamformer

First, we establish theoretical bounds on the minimum number of RF chains that are required to realize the theoretical capacity of the MIMO system. Recall that without the hybrid structure constraint, the optimal linear fully-digital precoder for maximizing the overall rate subject to the total power constraint is given by $\mathbf{V}_{\text{opt}} \in \mathbb{C}^{N\times N_s}$ matching to the set of eigenvectors corresponding to the N_s largest eigenvalues of $\mathbf{H}^H\mathbf{H}$ [12]. Fixing the precoder to $\mathbf{V}_{\text{opt}}$, the optimal linear fully-digital combiner is given by the MMSE receiver. A natural question arises: Fix N_s, is it possible to implement the fully-digital precoder and combiner using a hybrid structure with $N_t^{\text{RF}} < N$ and $N_r^{\text{RF}} < M$? If so, what is the minimum number of RF chains needed? For simplicity of exposition, we focus on the transmitter side.

Lemma 1. *To realize a full-rank precoding matrix, it is necessary that the number of transmit RF chains in hybrid structure exceeds the number of data streams; i.e., $N_t^{RF} \ge N_s$.*

Proof: Note that $\text{rank}(\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}) \le N_t^{\text{RF}}$. Therefore, to implement a rank-N_s $\mathbf{V}_{\text{opt}}$, we need $N_t^{\text{RF}} \ge N_s$. ■

We now address how many RF chains are sufficient for implementing fully-digital $\mathbf{V}_{\text{opt}}$. For the $N_s = 1$ case, it is known that the optimal precoder can be realized using the hybrid structure iff $N_t^{\text{RF}} \ge 2$ [9]. Lemma 2 generalizes this result for spatial multiplexing transmission.

Lemma 2. *For the spatial multiplexing transmission in MIMO systems, i.e., $N_s > 1$, the optimal fully-digital precoder can be realized using the hybrid structure if $N_t^{RF} \ge 2N_s$.*

Proof: Let $N_t^{\text{RF}} = 2N_s$ and denote $\mathbf{V}_{\text{opt}}(i,j) = \beta_{ij}e^{j\phi_{i,j}}$ and $\mathbf{V}_{\text{RF}}(i,j) = e^{j\theta_{i,j}}$. We design the k^{th} column of the digital precoder as $\mathbf{v}_{\text{Dig}}^{(k)} = [\mathbf{0}^T \ \ v_{2k-1} \ \ v_{2k} \ \ \mathbf{0}^T]^T$. With this specific structure, satisfying $\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}} = \mathbf{V}_{\text{opt}}$ is equivalent to

$$v_{2k-1}e^{j\theta_{i,2k-1}} + v_{2k}e^{j\theta_{i,2k}} = \beta_{i,k}e^{j\phi_{i,k}}, \ \ \forall i, k. \tag{6}$$

Using a similar procedure as in [9], it can be easily verified that one possible set of solution to this problem is

$$v_{2k-1} = \frac{\beta_{\max}^{(k)} + \beta_{\min}^{(k)}}{2}, \quad v_{2k} = \frac{\beta_{\max}^{(k)} - \beta_{\min}^{(k)}}{2},$$

$$\theta_{i,2k-1} = \phi_{i,k} - \cos^{-1}\left(\frac{\beta_{i,k}^2 + \beta_{\max}^{(k)}\beta_{\min}^{(k)}}{\beta_{i,k}(\beta_{\max}^{(k)} + \beta_{\min}^{(k)})}\right),$$

$$\theta_{i,2k} = \phi_{i,k} + \cos^{-1}\left(\frac{\beta_{i,k}^2 - \beta_{\max}^{(k)}\beta_{\min}^{(k)}}{\beta_{i,k}(\beta_{\max}^{(k)} - \beta_{\min}^{(k)})}\right). \tag{7}$$

where $\beta_{\max}^{(k)} = \max_i\{\beta_{i,k}\}$, and $\beta_{\min}^{(k)} = \min_i\{\beta_{i,k}\}$. ■

IV. Transceiver Design for the case $N_t^{\text{RF}} = N_r^{\text{RF}} = N_s$

We now consider the design of hybrid precoder and combiner for the critical case where $N_t^{\text{RF}} = N_r^{\text{RF}} = N_s$. The digital beamformers in this case have dimension N_s by N_s. We aim to show that such a structure is able to approximate a fully-digital beamformer supporting N_s data streams.

The optimal beamforming design to solve the problem in (5) involves a joint optimization over $\mathbf{V}_t$ and $\mathbf{W}_t$. In this section, we decouple the design of precoding matrix from the design of combining matrix by considering the maximization of the mutual information between S and Y_1 (see Fig. 1) for designing the precoder first, then subsequently the design of the receiver combiner for a fixed precoder.

By the above simplification and assuming Gaussian signalling, the precoder design problem can be written as

$$\max_{\mathbf{V}_{\text{RF}}, \mathbf{V}_{\text{Dig}}} \quad \log_2\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\right| \tag{8a}$$

$$\text{s.t.} \quad \text{Tr}(\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H) \leq P, \tag{8b}$$

$$|\mathbf{V}_{\text{RF}}(i,j)|^2 = 1, \quad \forall i, j. \tag{8c}$$

The optimization problem (8) is not convex. This paper proposes the following heuristic strategy for obtaining a good solution to (8). First, we assume a digital precoder of the form $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H = \gamma^2\mathbf{I}$ where γ is a constant. Under this assumption, the remaining problem for designing the RF precoder happens to have the form of a beamforming problem with per-antenna power constraint considered in [11]. We design the RF precoder according to the algorithm in [11]. Finally, the digital precoder is set to be the global maximizer of the problem (8) given that RF precoder.

With the precoder already designed, we next find the RF combiner and the digital combiner that maximize the achievable rate. We show that for large-MIMO system the RF combiner design problem has the same form as the RF precoder design problem. Therefore, the RF combiner can also be obtained using the algorithm in [11]. Finally, the digital combiner is set to be the MMSE receiver. The design procedure is explained in more detail below.

A. RF Precoder Design Given $\mathbf{V}_{Dig}\mathbf{V}_{Dig}^H = \gamma^2\mathbf{I}$

First, we seek to design the RF precoder assuming that the digital precoder is such that $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H = \gamma^2\mathbf{I}$. The motivation behind this assumption is that by setting γ to $\sqrt{\frac{P}{NN_t^{\text{RF}}}}$, the power constraint in (8b) is automatically satisfied, regardless of the value of the entries of the RF precoding matrix. Therefore, the RF precoder design problem is reduced to

$$\max_{\mathbf{V}_{\text{RF}}} \quad \log_2\left|\mathbf{I} + \frac{\gamma^2}{\sigma^2}\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\mathbf{H}\mathbf{V}_{\text{RF}}\right| \tag{9a}$$

$$\text{s.t.} \quad |\mathbf{V}_{\text{RF}}(i,j)|^2 = 1, \quad \forall i, j. \tag{9b}$$

Although the objective function of the problem (9) is still not concave in $\mathbf{V}_{\text{RF}}$, the constraints are completely decoupled in this formulation. This allows us to design each column of the $\mathbf{V}_{\text{RF}}$ matrix separately. This design approach is first proposed in [11] for solving the problem of transmitter precoder design with per-antenna power constraint. A brief explanation of the algorithm is stated below.

Let $\mathbf{F}_1 = \mathbf{H}^H\mathbf{H}$. We can isolate the contribution of the k^{th} column of the beamformer to the objective function in (9) as

$$\log_2\left|\mathbf{I} + \frac{\gamma^2}{\sigma^2}\mathbf{V}_{\text{RF}}^H\mathbf{F}_1\mathbf{V}_{\text{RF}}\right| = \tag{10}$$

$$\log_2\left|\mathbf{C}_k\right| + \log_2\left(1 + \frac{\gamma^2}{\sigma^2}\mathbf{v}_{\text{RF}}^{(k)^H}\mathbf{G}_k\mathbf{v}_{\text{RF}}^{(k)}\right),$$

where $\mathbf{C}_k = \mathbf{I} + \frac{\gamma^2}{\sigma^2}(\bar{\mathbf{V}}_{\text{RF}}^k)^H\mathbf{F}_1\bar{\mathbf{V}}_{\text{RF}}^k$ and $\mathbf{G}_k = \mathbf{F}_1 - \frac{\gamma^2}{\sigma^2}\mathbf{F}_1\bar{\mathbf{V}}_{\text{RF}}^k\mathbf{C}_k^{-1}(\bar{\mathbf{V}}_{\text{RF}}^k)^H\mathbf{F}_1$ is a positive semidefinite matrix and $\bar{\mathbf{V}}_{\text{RF}}^k$ is the sub-matrix of $\mathbf{V}_{\text{RF}}$ with k^{th} column removed. The algorithm in [11] is based on the iterative maximization over the columns of the beamformer to reach to a local optimal point. At the k^{th} step of each iteration, the algorithm seeks to find the optimal k^{th} column of the RF precoder; i.e., $\mathbf{v}_{\text{RF}}^{(k)}$, assuming that the other columns are fixed. Since $\mathbf{C}_k$ is independent of $\mathbf{v}_{\text{RF}}^{(k)}$, the optimization problem at the k^{th} step of each iteration is

$$\max_{\mathbf{v}_{\text{RF}}^{(k)}} \quad \log_2\left(1 + \frac{\gamma^2}{\sigma^2}\mathbf{v}_{\text{RF}}^{(k)^H}\mathbf{G}_k\mathbf{v}_{\text{RF}}^{(k)}\right) \tag{11a}$$

$$\text{s.t.} \quad |\mathbf{v}_{\text{RF}}^{(k)}(i)|^2 = 1, \quad \forall i = 1, \ldots, N. \tag{11b}$$

It can be shown that any local optimal solution of (11) satisfies

$$\mathbf{v}_{\text{RF}}^{(k)}(i) = \psi\Big(\sum_{\ell \neq i} g_{i\ell}^k\mathbf{v}_{\text{RF}}^{(k)}(\ell)\Big), \quad \forall i = 1, \ldots, N, \tag{12}$$

where for the complex variable w,

$$\psi(w) = \begin{cases} 1, & \text{if } w = 0 \\ \frac{w}{|w|}, & \text{otherwise} \end{cases}, \tag{13}$$

and $g_{i\ell}^k$ is the element of $\mathbf{G}_k$ at the i^{th} row and ℓ^{th} column. Therefore, one way to find a local optimal solution to (11) is to iteratively update the elements of $\mathbf{v}_{\text{RF}}^{(k)}$ according to (12). The convergence of the overall algorithm for solving (9) is proved in [11], although only local optimality can be guaranteed.

B. Digital Precoder Design Given $\mathbf{V}_{RF}$

Next, we consider the design of the digital precoder assuming that the RF precoder is fixed. Toward this end, we find the closed-form solution for the optimal digital precoder that maximizes (8a) given the RF precoder. If $\mathbf{V}_{\mathrm{RF}}$ is fixed, $\mathbf{H}_{\mathrm{eff}} = \mathbf{H}\mathbf{V}_{\mathrm{RF}}$ can be seen as an effective channel. Therefore, the optimal digital precoder can be found by solving

$$\max_{\mathbf{V}_{\mathrm{Dig}}} \quad \log_2 \left|\mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{H}_{\mathrm{eff}}\mathbf{V}_{\mathrm{Dig}}\mathbf{V}_{\mathrm{Dig}}^H\mathbf{H}_{\mathrm{eff}}^H\right| \tag{14a}$$

$$\text{s.t.} \quad \mathrm{Tr}(\mathbf{Q}\mathbf{V}_{\mathrm{Dig}}\mathbf{V}_{\mathrm{Dig}}^H) \leq P, \tag{14b}$$

where $\mathbf{Q} = \mathbf{V}_{\mathrm{RF}}^H\mathbf{V}_{\mathrm{RF}}$. If we denote $\mathbf{H}_e = \mathbf{H}_{\mathrm{eff}}\mathbf{Q}^{-1/2}$, the problem in (14) has the well-known solution; i.e., $\mathbf{V}_{\mathrm{Dig}} = \mathbf{Q}^{-1/2}\mathbf{U}_e\mathbf{\Gamma}_e$ where $\mathbf{U}_e$ is the set of eigenvectors corresponding to the N_s largest eigenvalues of $\mathbf{H}_e^H\mathbf{H}_e$ and $\mathbf{\Gamma}_e$ is the diagonal matrix of powers allocated by water-filling.

C. Receiver Design Given the Precoder

Finally, we seek to design the hybrid combiner that maximizes the achievable rate assuming that the precoder is fixed. Since there is no constraint on the entries of the digital combiner, it is possible to design the RF combiner first such that it maximizes the mutual information between S and Y_2 (see Fig. 1), then to set the digital combiner to be a MMSE receiver. The RF combiner is designed by solving

$$\max_{\mathbf{W}_{\mathrm{RF}}} \quad \log_2 \left|\mathbf{I} + \frac{1}{\sigma^2}(\mathbf{W}_{\mathrm{RF}}^H\mathbf{W}_{\mathrm{RF}})^{-1}\mathbf{W}_{\mathrm{RF}}^H\mathbf{F}_2\mathbf{W}_{\mathrm{RF}}\right| \tag{15a}$$

$$\text{s.t.} \quad |\mathbf{W}_{\mathrm{RF}}(i,j)|^2 = 1, \quad \forall i,j, \tag{15b}$$

where $\mathbf{F}_2 = \mathbf{H}\mathbf{V}_t\mathbf{V}_t^H\mathbf{H}^H$. It can be shown that when the number of receive antennas is large, we have $\mathbf{W}_{\mathrm{RF}}^H\mathbf{W}_{\mathrm{RF}} \approx M\mathbf{I}$ with high probability [10]. Under this assumption, the problem (15) has the same form as the problem in (9). Therefore, the problem (15) can be solved with the same algorithm as in Section IV-A, and it can be verified that the resulting $\mathbf{W}_{\mathrm{RF}}$ indeed satisfies $\mathbf{W}_{\mathrm{RF}}^H\mathbf{W}_{\mathrm{RF}} \approx M\mathbf{I}$. Finally, assuming the precoder and the RF combiner already designed, the MMSE digital combiner is $\mathbf{W}_{\mathrm{Dig}} = \mathbf{J}^{-1}\mathbf{W}_{\mathrm{RF}}^H\mathbf{H}\mathbf{V}_t$, where $\mathbf{J} = \mathbf{W}_{\mathrm{RF}}^H\mathbf{H}\mathbf{V}_{\mathrm{t}}\mathbf{V}_{\mathrm{t}}^H\mathbf{H}^H\mathbf{W}_{\mathrm{RF}} + \sigma^2\mathbf{W}_{\mathrm{RF}}^H\mathbf{W}_{\mathrm{RF}}$.

V. Simulations

In this section, simulation results are presented to show the performance of the proposed algorithm in comparison with the algorithm in [10] and the optimal fully-digital beamforming scheme. In order to model the propagation environment, we consider a geometric channel model with L paths between the transmitter and receiver. Furthermore, we consider an antenna configuration with a uniform linear array. Under these assumptions, the channel matrix can be expressed as [10],

$$\mathbf{H} = \sqrt{\frac{NM}{L}}\sum_{\ell=1}^{L}\alpha_\ell\mathbf{a}_r(\phi_r^\ell)\mathbf{a}_t(\phi_t^\ell)^H, \tag{16}$$

where $\alpha_\ell \sim \mathcal{CN}(0,1)$ is the complex gain of the ℓ^{th} path, and $\phi_r^\ell \in [0,\ 2\pi)$, $\phi_t^\ell \in [0,\ 2\pi)$. Moreover, $\mathbf{a}_r(.)$ and $\mathbf{a}_t(.)$ are the antenna array response vectors at the

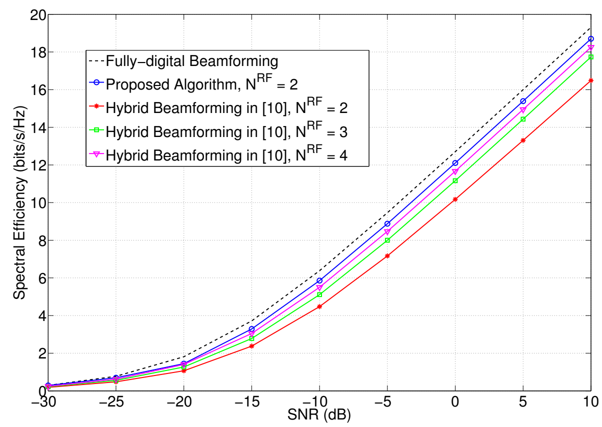

Figure 2: Spectral efficiencies achieved by different methods in a 64×8 MIMO system where $N_s = 2$ and $L = 20$.

receiver and the transmitter, respectively. The antenna array response vector in a uniform linear array configuration with N antenna elements is modeled as $\mathbf{a}(\phi) = \frac{1}{\sqrt{N}}[1, \exp^{jkd\sin(\phi)}, \ldots, \exp^{(N-1)jkd\sin(\phi)}]^T$ where $k = \frac{2\pi}{\lambda}$, λ is the transmission wavelength and d is the antenna spacing. In the simulation, we assume a 64×8 MIMO system in an environment with $L = 20$ scatterers with uniformly random angles of arrival and departure. The antenna spacing is set to be half of the wavelength. The number of data streams is set to be $N_s = 2$. For hybrid beamforming schemes, we assume that the number of RF chains at the transmitter and the receiver is identical; i.e., $N_t^{\mathrm{RF}} = N_r^{\mathrm{RF}} = N^{\mathrm{RF}}$. Fig. 2 plots the average spectral efficiency versus signal-to-noise-ratio (SNR $= \frac{P}{\sigma^2}$) over 100 channel realizations for different beamforming methods. It is shown that the proposed algorithm with 2 RF chains already has a better performance as compared to algorithm of [10], with 2, 3 or 4 RF chains. It is also shown that with the same number of RF chains ($N^{\mathrm{RF}} = 2$), the performance improvement of the proposed algorithm as compared to the algorithm in [10] is about 4 dB at high SNR regime which is significant. Moreover, the performance of the proposed algorithm with 2 RF chains is already very close to the upper bound given by the rate of optimal fully-digital beamforming scheme, indicating that the proposed algorithm is near optimal.

VI. Conclusion

This paper considers single-user hybrid beamforming for a large-scale MIMO system with limited number of RF chains at both ends. We show that the hybrid beamforming structure can achieve the same performance as the fully-digital beamforming scheme, if the number of RF chains at each end is greater than twice the number of data streams. For the case where the number of RF chains at both ends is equal to the number of data streams, we propose a heuristic algorithm which has better performance as compared to existing hybrid beamforming algorithms and in fact achieves a rate very close to the capacity limit with optimal fully-digital beamforming.

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