Optimality of Gaussian Fronthaul Compression for Uplink MIMO Cloud Radio Access Networks

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Abstract*—This paper investigates the compress-and-forward scheme for an uplink cloud radio access network (C-RAN) model, where multi-antenna base-stations (BSs) are connected to a cloud-computing based central processor (CP) via capacity-limited fronthaul links. The BSs perform Wyner-Ziv coding to compress and send the received signals to the CP; the CP performs either joint decoding of both the quantization codewords and the user messages at the same time, or the more practical successive decoding of the quantization codewords first, then the user messages. Under this setup, this paper makes progress toward the optimization of the fronthaul compression scheme by proving two results. First, it is shown that if the input distributions are assumed to be Gaussian, then under joint decoding, the optimal Wyner-Ziv quantization scheme for maximizing the achievable rate region is Gaussian. Second, for fixed Gaussian input, under a sum fronthaul capacity constraint and assuming Gaussian quantization, this paper shows that successive decoding and joint decoding achieve the same maximum sum rate. In this case, the optimization of Gaussian quantization noise covariance matrices for maximizing sum rate can be formulated as a convex optimization problem, therefore can be solved efficiently.

I. Introduction

This paper considers the uplink of a cloud radio access network (C-RAN) under finite-capacity fronthaul constraints. The uplink C-RAN model, as shown in Fig. 1, consists of multiple remote users sending independent messages to a cloud-computing based central processor (CP) through multiple multi-antenna base-stations (BSs). The BSs and the CP are connected with digital fronthaul links with per-link capacity C_ℓ. All the user messages are eventually decoded at the CP. This channel model can be thought of as a two-hop relay network, with an interference channel between the users and the BSs, followed by a noiseless multiple-access channel between the BSs and the CP. The C-RAN architecture enables multi-cell processing, which significantly improves the performance of wireless cellular networks by effectively mitigating inter-cell interference.

A key question in the design of the C-RAN architecture is the optimal choice of input distribution at the remote users, the optimal coding strategy at the BSs, and the optimal decoding strategy at the CP. Toward this end, this paper restricts attention to fixed Gaussian input distribution, Wyner-Ziv compress-and-forward relaying strategy at the BSs, and either *successive decoding* of the quantization codewords first, then the user messages, or *joint decoding* of the quantization codewords and user messages together at the CP. Our main results are that, under this assumption, the optimal Wyner-Ziv quantization scheme is Gaussian under joint decoding. Further, if we assume a sum fronthaul constraint, then under Gaussian quantization successive decoding achieves the same sum rate as joint decoding. Moreover, the optimization of Gaussian quantization covariance matrices for maximizing sum rate under the sum fronthaul constraint can be formulated as a convex optimization problem, therefore can be solved efficiently.

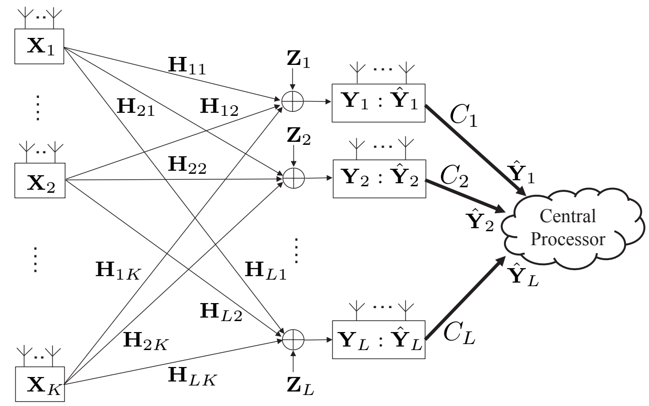

Fig. 1. Uplink of a cloud radio-access network with capacity-limited fronthaul

The achievable rate of compress-and-forward with joint decoding in the uplink C-RAN is first studied in [1] for a single-transmitter model and in [2] for the multi-transmitter case. The uplink C-RAN model is in fact a particular instance of a general relay network with a single destination for which a generalization of compress-and-forward with joint decoding (known as noisy network coding [3], [4]) can be shown to achieve the information theoretical capacity to within a constant gap. The achievable rate region of compress-and-forward with successive decoding (which has lower decoding complexity) has also been studied for the C-RAN model [5]. The recent work [6] further shows that successive decoding can achieve the sum capacity of C-RAN to within constant gap, if the fronthaul links are subjected to a sum capacity constraint. The constant-gap results in the relay literature are typically established assuming Gaussian quantization noise covariance at the relays. This paper goes one step further in proving that

Gaussian quantizer is in fact optimal under joint decoding, if the input distributions are assumed to be Gaussian. The key insight here is a connection between the C-RAN model and the CEO problem in source coding [7], where a source is described to a central unit by remote agents with noisy observations. The solution to the CEO problem is known for the scalar Gaussian case [8] [9], while significant recent progress has been made in the vector case, e.g., [10], [11]. Finding the optimal quantization for the C-RAN model is also related to the mutual information constraint problem [12] [13], which can be solved by the entropy power inequality or the perturbation approach. In this paper, we use techniques for establishing outer bound for the Gaussian vector CEO problem [11] to prove the optimality of Gaussian quantization.

This paper makes further progress in observing that the optimization of Gaussian quantization noise covariance matrices can be reformulated as a convex optimization problem under joint decoding, and that successive decoding and joint decoding are equivalent for maximizing the sum rate under a special case of uplink C-RAN model with a sum-capacity fronthaul constraint. The quantization noise covariance optimization problem has been considered in the literature, but only locally convergent algorithms are known previously [14], [15]. The convex formulation proposed in this paper allows globally optimal Gaussian quantization noise covariance to be found efficiently. The assumption of fixed Gaussian input distribution is crucial for the current setup. It is not difficult to come up with examples where non-Gaussian input can outperform Gaussian input [5]. This paper optimizes the quantization noise covariance matrix under the fixed Gaussian input. We remark that the joint optimization of input and quantization covariance matrices remains a difficult problem [16].

Notation: Superscripts $(\cdot)^\dagger$ and $(\cdot)^{-1}$ denote Hermitian transpose, and matrix inverse operators; $\text{cov}(\cdot)$ denotes the covariance operation. For a vector $\mathbf{X}$, $\mathbf{X}_\mathcal{S}$ denotes a vector/matrix with elements whose indices are elements of $\mathcal{S}$. Given matrices $\{\mathbf{X}_1, \dots, \mathbf{X}_L\}$, $\text{diag}\left(\{\mathbf{X}_\ell\}_{\ell=1}^L\right)$ denotes the block diagonal matrix formed with $\mathbf{X}_\ell$ on the diagonal. Denote by $J(\mathbf{X})$ the Fisher information matrix of random vector $\mathbf{X}$. Let $\mathcal{K} = \{1, \cdots, K\}$ and $\mathcal{L} = \{1, \cdots, L\}$.

II. Achievable Rate Regions for Uplink C-RAN

This paper considers the uplink C-RAN, where K mobile users communicate with a CP through L BSs, as shown in Fig. 1. The noiseless fronthaul links connecting the BSs with the CP have per-link capacity C_ℓ. Each user terminal is equipped with M antennas; each BS is equipped with N antennas. Let $\mathbf{X}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_k)$ be the signal transmitted by the kth user. The signal received at the ℓth BS can be expressed as

$$\mathbf{Y}_\ell = \sum_{k=1}^K \mathbf{H}_{\ell,k}\mathbf{X}_k + \mathbf{Z}_\ell, \quad \ell = 1, 2, \dots, L,$$

where $\mathbf{Z}_\ell \sim \mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_\ell)$ are independent noises, and $\mathbf{H}_{\ell,k}$ denotes the complex channel matrix from $\mathbf{X}_k$ to $\mathbf{Y}_\ell$.

Fix Gaussian input distribution as above. We use $\mathcal{R}_{JD}^*$ to denote the achievable rate region of the compress-and-forward relay scheme with joint decoding [1, Proposition IV.1] [4, Theorem 1], i.e., the set of $(R_1, \cdots, R_K)$ for which

$$\sum_{k\in\mathcal{T}} R_k \le \sum_{\ell\in\mathcal{S}} \left[C_\ell - I(\mathbf{Y}_\ell; \hat{\mathbf{Y}}_\ell|\mathbf{X}_\mathcal{K})\right] + I\left(\mathbf{X}_\mathcal{T}; \hat{\mathbf{Y}}_{\mathcal{S}^c}|\mathbf{X}_{\mathcal{T}^c}\right) \tag{1}$$

for all $\mathcal{T} \subseteq \mathcal{K}$ and $\mathcal{S} \subseteq \mathcal{L}$, for some $\prod_{\ell=1}^L p_{\hat{\mathbf{Y}}_\ell|\mathbf{Y}_\ell}$.

Likewise, we use $\mathcal{R}_{SD}^*$ to denote the achievable rate region under fixed Gaussian input distribution of the compress-and-forward scheme with successive decoding [5, Theorem 1], i.e., the set of $(R_1, \cdots, R_K)$ for which

$$\sum_{k\in\mathcal{T}} R_k \le I\left(\mathbf{X}_\mathcal{T}; \hat{\mathbf{Y}}_\mathcal{L}|\mathbf{X}_{\mathcal{T}^c}\right), \quad \forall\, \mathcal{T} \subseteq \mathcal{K}, \tag{2}$$

for some product distribution $\prod_{\ell=1}^L p_{\hat{\mathbf{Y}}_\ell|\mathbf{Y}_\ell}$ that satisfies

$$I\left(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S}|\hat{\mathbf{Y}}_{\mathcal{S}^c}\right) \le \sum_{\ell\in\mathcal{S}} C_\ell, \quad \forall\, \mathcal{S} \subseteq \mathcal{L}. \tag{3}$$

Note that (2) is the multiple-access rate region, (3) represents the Wyner-Ziv decoding constraint, while (1) incorporates the joint decoding of the transmit and quantization codewords.

We now evaluate the joint decoding and successive decoding regions under Gaussian quantization, denoted as $\mathcal{R}_{JD}^G$ and $\mathcal{R}_{SD}^G$, respectively. Set $p_{\hat{\mathbf{Y}}_\ell|\mathbf{Y}_\ell} \sim \mathcal{CN}(\mathbf{Y}_\ell, \mathbf{Q}_\ell)$, where $\mathbf{Q}_\ell$ is the Gaussian quantization noise covariance matrix at the ℓth BS. Instead of parameterizing over $\mathbf{Q}_\ell$, we parameterize over $\mathbf{B}_\ell$ defined as

$$\mathbf{B}_\ell = (\mathbf{\Sigma}_\ell + \mathbf{Q}_\ell)^{-1}. \tag{4}$$

Proposition 1: Fix Gaussian input $\mathbf{X}_\mathcal{K} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_\mathcal{K})$. The joint decoding region $\mathcal{R}_{JD}^G$ for the C-RAN model under Gaussian quantization is the set of $(R_1, \cdots, R_K)$ such that

$$\begin{aligned}\sum_{k\in\mathcal{T}} R_k \le \sum_{\ell\in\mathcal{S}} &\left[C_\ell - \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|}\right] \\ &+ \log\frac{\left|\sum_{\ell\in\mathcal{S}^c} \mathbf{H}_{\ell,\mathcal{T}}^\dagger \mathbf{B}_\ell \mathbf{H}_{\ell,\mathcal{T}} + \mathbf{K}_\mathcal{T}^{-1}\right|}{\left|\mathbf{K}_\mathcal{T}^{-1}\right|}\end{aligned} \tag{5}$$

for all $\mathcal{T} \subseteq \mathcal{K}$ and $\mathcal{S} \subseteq \mathcal{L}$, for some $0 \preceq \mathbf{B}_\ell \preceq \mathbf{\Sigma}_\ell^{-1}$, where $\mathbf{K}_\mathcal{T} = \mathbb{E}[\mathbf{X}_\mathcal{T}\mathbf{X}_\mathcal{T}^\dagger]$ is the covariance matrix of $\mathbf{X}_\mathcal{T}$, and $\mathbf{H}_{\ell,\mathcal{T}}$ denotes the channel matrix from $\mathbf{X}_\mathcal{T}$ to $\mathbf{Y}_\ell$.

Proposition 2: Fix Gaussian input $\mathbf{X}_\mathcal{K} \sim \mathcal{CN}(\mathbf{0}, \mathbf{K}_\mathcal{K})$. The successive decoding region $\mathcal{R}_{SD}^G$ for the C-RAN model under Gaussian quantization is given by the set of $(R_1, \cdots, R_K)$ such that

$$\sum_{k\in\mathcal{T}} R_k \le \log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{T}}^\dagger \mathbf{B}_\ell \mathbf{H}_{\ell,\mathcal{T}} + \mathbf{K}_\mathcal{T}^{-1}\right|}{\left|\mathbf{K}_\mathcal{T}^{-1}\right|}, \qquad \forall\, \mathcal{T} \subseteq \mathcal{K}, \tag{6}$$

for some $0 \preceq \mathbf{B}_\ell \preceq \mathbf{\Sigma}_\ell^{-1}$ satisfying

$$\log \frac{\left|\sum_{\ell=1}^{L} \mathbf{H}_{\ell,\mathcal{K}}^\dagger \mathbf{B}_\ell \mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_\mathcal{K}^{-1}\right|}{\left|\sum_{\ell\in\mathcal{S}^c} \mathbf{H}_{\ell,\mathcal{K}}^\dagger \mathbf{B}_\ell \mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_\mathcal{K}^{-1}\right|} + \sum_{\ell\in\mathcal{S}} \log \frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|} \leq \sum_{\ell\in\mathcal{S}} C_\ell, \qquad \forall\, \mathcal{S} \subseteq \mathcal{L}. \quad (7)$$

To derive the above expressions, first we use $\mathbf{B}_\ell = (\mathbf{\Sigma}_\ell + \mathbf{Q}_\ell)^{-1}$ to evaluate

$$I(\mathbf{Y}_\ell; \hat{\mathbf{Y}}_\ell | \mathbf{X}_\mathcal{K}) = \log \frac{|\mathbf{\Sigma}_\ell + \mathbf{Q}_\ell|}{|\mathbf{Q}_\ell|} = \log \frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|}. \quad (8)$$

Further, in deriving (5), $I\left(\mathbf{X}_\mathcal{T}; \hat{\mathbf{Y}}_{\mathcal{S}^c} | \mathbf{X}_{\mathcal{T}^c}\right)$ is evaluated as

$$\log \frac{\left|\mathbf{H}_{\mathcal{S}^c,\mathcal{T}} \mathbf{K}_\mathcal{T} \mathbf{H}_{\mathcal{S}^c,\mathcal{T}}^\dagger + \mathrm{diag}\left(\{\mathbf{\Sigma}_\ell + \mathbf{Q}_\ell\}_{\ell=1}^L\right)\right|}{\left|\mathrm{diag}\left(\{\mathbf{\Sigma}_\ell + \mathbf{Q}_\ell\}_{\ell=1}^L\right)\right|}. \quad (9)$$

Substituting (8) and (9) into (1) gives (5). Likewise a similar expression holds for $I\left(\mathbf{X}_\mathcal{T}; \hat{\mathbf{Y}}_\mathcal{T} | \mathbf{X}_{\mathcal{T}^c}\right)$, together giving (6) and (7).

In deriving (7), we start with the chain rule on mutual information

$$\begin{aligned} & I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_\mathcal{S} | \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) + I\left(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S} | \mathbf{X}_\mathcal{K} \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) \\ &= I\left(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S} | \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) + I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_\mathcal{S} | \mathbf{Y}_\mathcal{S} \hat{\mathbf{Y}}_{\mathcal{S}^c}\right), \quad (10) \end{aligned}$$

and make use of the Markov chain

$$\hat{\mathbf{Y}}_i \leftrightarrow \mathbf{Y}_i \leftrightarrow \mathbf{X}_\mathcal{K} \leftrightarrow \mathbf{Y}_j \leftrightarrow \hat{\mathbf{Y}}_j, \quad \forall\, i \neq j$$

to note that $I(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S} | \mathbf{X}_\mathcal{K} \hat{\mathbf{Y}}_{\mathcal{S}^c}) = I(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S} | \mathbf{X}_\mathcal{K})$, and $I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_\mathcal{S} | \mathbf{Y}_\mathcal{S} \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) = 0$. Hence,

$$\begin{aligned} I\left(\mathbf{Y}_\mathcal{S}; \hat{\mathbf{Y}}_\mathcal{S} | \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) &= I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_\mathcal{S} | \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) + \sum_{\ell\in\mathcal{S}} I(\mathbf{Y}_\ell; \hat{\mathbf{Y}}_\ell | \mathbf{X}_\mathcal{K}) \\ &= I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_\mathcal{L}\right) - I\left(\mathbf{X}_\mathcal{K}; \hat{\mathbf{Y}}_{\mathcal{S}^c}\right) + \sum_{\ell\in\mathcal{S}} I(\mathbf{Y}_\ell; \hat{\mathbf{Y}}_\ell | \mathbf{X}_\mathcal{K}). \end{aligned}$$

Then, (7) can be derived by evaluating the above mutual information expressions assuming Gaussian input and Gaussian quantization test channel.

Clearly, in general we have $\mathcal{R}_{SD}^G \subseteq \mathcal{R}_{SD}^*$, $\mathcal{R}_{JD}^G \subseteq \mathcal{R}_{JD}^*$, and $\mathcal{R}_{SD}^G \subseteq \mathcal{R}_{JD}^G$. However, Gaussian quantization is desirable, because it leads to achievable rate regions that can be easily evaluated. Further, successive decoding is more desirable than joint decoding, since it has much lower complexity. Therefore, this paper focuses on $\mathcal{R}_{JD}^G$ and $\mathcal{R}_{SD}^G$. We go toward establishing the optimality of Gaussian quantization and the efficient optimization of the rate region by showing that under fixed Gaussian input: (1) Gaussian quantization is optimal for joint decoding, i.e., $\mathcal{R}_{JD}^G = \mathcal{R}_{JD}^*$; (2) If we assume a sum fronthaul capacity constraint, the maximum sum rate achieved by successive decoding and joint decoding under Gaussian quantization are identical, and the optimization of Gaussian quantization noise covariance matrix for maximizing sum rate can be formulated as a convex optimization problem, which can be solved efficiently.

III. Optimality of Gaussian Quantization under Joint Decoding

Theorem 1: For the uplink C-RAN under fixed Gaussian input distribution and assuming joint decoding, Gaussian quantization is optimal, i.e. $\mathcal{R}_{JD}^G = \mathcal{R}_{JD}^*$.

Proof: Recall that the achievable rate region of the compress-and-forward scheme under joint decoding is given by the set of $(R_1, \cdots, R_K)$ given by (1) under joint distribution

$$p_{\mathbf{X}_\mathcal{K}, \mathbf{Y}_\mathcal{L}, \hat{\mathbf{Y}}_\mathcal{L}} = \prod_{k=1}^K p_{\mathbf{X}_k} \prod_{\ell=1}^L p_{\mathbf{Y}_\ell | \mathbf{X}_\mathcal{K}} \prod_{\ell=1}^L p_{\hat{\mathbf{Y}}_\ell | \mathbf{Y}_\ell}. \quad (11)$$

Fix $p_{\hat{\mathbf{Y}}_\ell | \mathbf{Y}_\ell}$, choose $\mathbf{B}_\ell$ with $\mathbf{0} \preceq \mathbf{B}_\ell \preceq \mathbf{\Sigma}_\ell^{-1}$ such that

$$\mathrm{cov}(\mathbf{Y}_\ell | \mathbf{X}_\mathcal{K}, \hat{\mathbf{Y}}_\ell) = \mathbf{\Sigma}_\ell - \mathbf{\Sigma}_\ell \mathbf{B}_\ell \mathbf{\Sigma}_\ell, \quad \ell = 1, \cdots, L.$$

We proceed to show that the achievable rate region as given by (5) with a Gaussian $p_{\hat{\mathbf{Y}}_\ell | \mathbf{Y}_\ell} \sim \mathcal{CN}(\mathbf{Y}_\ell, \mathbf{Q}_\ell)$, where $\mathbf{Q}_\ell = \mathbf{B}_\ell^{-1} - \mathbf{\Sigma}_\ell$, is as large as that of (1).

First, note that

$$\begin{aligned} I(\mathbf{Y}_\ell; \hat{\mathbf{Y}}_\ell | \mathbf{X}_\mathcal{K}) &= \log |(\pi e) \mathbf{\Sigma}_\ell| - h(\mathbf{Y}_\ell | \mathbf{X}_\mathcal{K}, \hat{\mathbf{Y}}_\ell) \\ &\geq \log |(\pi e) \mathbf{\Sigma}_\ell| - \log |(\pi e) \,\mathrm{cov}(\mathbf{Y}_\ell | \mathbf{X}_\mathcal{K}, \hat{\mathbf{Y}}_\ell)| \\ &= \log \frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|}, \quad \ell = 1, \cdots, L, \quad (12) \end{aligned}$$

where we use the fact that Gaussian distribution maximizes differential entropy.

Moreover, we have

$$\begin{aligned} I(\mathbf{X}_\mathcal{T}; \hat{\mathbf{Y}}_{\mathcal{S}^c} | \mathbf{X}_{\mathcal{T}^c}) &= h(\mathbf{X}_\mathcal{T}) - h(\mathbf{X}_\mathcal{T} | \mathbf{X}_{\mathcal{T}^c}, \hat{\mathbf{Y}}_{\mathcal{S}^c}) \\ &\leq \log |\mathbf{K}_\mathcal{T}| - \log |J^{-1}(\mathbf{X}_\mathcal{T} | \mathbf{X}_{\mathcal{T}^c}, \hat{\mathbf{Y}}_{\mathcal{S}^c})|, \end{aligned}$$

where the inequality is due to [11, Lemma 2] [17]. Since

$$\mathbf{Y}_{\mathcal{S}^c} = \mathbf{H}_{\mathcal{S}^c,\mathcal{T}} \mathbf{X}_\mathcal{T} + \mathbf{H}_{\mathcal{S}^c,\mathcal{T}^c} \mathbf{X}_{\mathcal{T}^c} + \mathbf{N}_{\mathcal{S}^c},$$

it follows that

$$\begin{aligned} \mathbf{X}_\mathcal{T} &= \mathbb{E}[\mathbf{X}_\mathcal{T} | \mathbf{X}_{\mathcal{T}^c}, \mathbf{Y}_{\mathcal{S}^c}] + \mathbf{N}_{\mathcal{T},\mathcal{S}^c} \\ &= \sum_{\ell\in\mathcal{S}^c} \mathbf{G}_{\mathcal{T},\ell} (\mathbf{Y}_\ell - \mathbf{H}_{\ell,\mathcal{T}^c} \mathbf{X}_{\mathcal{T}^c}) + \mathbf{N}_{\mathcal{T},\mathcal{S}^c}, \end{aligned}$$

where

$$\mathbf{G}_{\mathcal{T},\ell} = (\mathbf{K}_\mathcal{T}^{-1} + \sum_{j\in\mathcal{S}^c} \mathbf{H}_{j,\mathcal{T}}^\dagger \mathbf{\Sigma}_j^{-1} \mathbf{H}_{j,\mathcal{T}})^{-1} \mathbf{H}_{\ell,\mathcal{T}}^\dagger \mathbf{\Sigma}_\ell^{-1}, \quad (13)$$

and

$$\mathbf{N}_{\mathcal{T},\mathcal{S}^c} \sim \mathcal{CN}\left(\mathbf{0}, \Lambda_\mathbf{N}\right) \quad (14)$$

with $\Lambda_\mathbf{N} = \left(\mathbf{K}_\mathcal{T}^{-1} + \sum_{\ell\in\mathcal{S}^c} \mathbf{H}_{\ell,\mathcal{T}}^\dagger \mathbf{\Sigma}_\ell^{-1} \mathbf{H}_{\ell,\mathcal{T}}\right)^{-1}$. By the matrix complementary identity between Fisher information matrix and MMSE [11, Lemma 3] [18], we have

$$
\begin{aligned}
&J(\mathbf{X}_{\mathcal{T}}|\mathbf{X}_{\mathcal{T}^c}, U_{\mathcal{S}^c}) \\
&= \Lambda_{\mathbf{N}}^{-1} - \Lambda_{\mathbf{N}}^{-1} \\
&\qquad \mathrm{cov}\left(\sum_{\ell\in\mathcal{S}^c} \mathbf{G}_{\mathcal{T},\ell}(\mathbf{Y}_\ell - \mathbf{H}_{\ell,\mathcal{T}^c}\mathbf{X}_{\mathcal{T}^c})|\mathbf{X}_{\mathcal{K}}, \hat{\mathbf{Y}}_{\mathcal{S}^c}\right)\Lambda_{\mathbf{N}}^{-1} \\
&= \Lambda_{\mathbf{N}}^{-1} - \Lambda_{\mathbf{N}}^{-1}\,\mathrm{cov}\left(\sum_{\ell\in\mathcal{S}^c} \mathbf{G}_{\mathcal{T},\ell}\mathbf{Y}_\ell|\mathbf{X}_{\mathcal{K}}, \hat{\mathbf{Y}}_{\mathcal{S}^c}\right)\Lambda_{\mathbf{N}}^{-1} \\
&= \Lambda_{\mathbf{N}}^{-1} - \Lambda_{\mathbf{N}}^{-1}\left[\sum_{\ell\in\mathcal{S}^c} \mathbf{G}_{\mathcal{T},\ell}\,\mathrm{cov}(\mathbf{Y}_\ell|\mathbf{X}_{\mathcal{K}}, \hat{\mathbf{Y}}_\ell)\mathbf{G}_{\mathcal{T},\ell}^\dagger\right]\Lambda_{\mathbf{N}}^{-1} \\
&= \Lambda_{\mathbf{N}}^{-1} - \sum_{\ell\in\mathcal{S}^c} \mathbf{H}_{\ell,\mathcal{T}}^\dagger\left(\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell\right)\mathbf{H}_{\ell,\mathcal{T}} \\
&= \mathbf{K}_{\mathcal{T}}^{-1} + \sum_{\ell\in\mathcal{S}^c} \mathbf{H}_{\ell,\mathcal{T}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{T}}.
\end{aligned}
$$

Therefore,

$$
I(\mathbf{X}_{\mathcal{T}}; \hat{\mathbf{Y}}_{\mathcal{S}^c}|\mathbf{X}_{\mathcal{T}^c}) \le \log\frac{|\mathbf{K}_{\mathcal{T}}^{-1} + \sum_{\ell\in\mathcal{S}^c}\mathbf{H}_{\ell,\mathcal{T}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{T}}|}{|\mathbf{K}_{\mathcal{T}}^{-1}|} \tag{15}
$$

for all $\mathcal{T} \subseteq \mathcal{K}$ and $\mathcal{S} \subseteq \mathcal{L}$. Combining (12) and (15), we conclude that $\mathcal{R}_{JD}^G$ as given by (5) is as large as $\mathcal{R}_{JD}^*$ as given by (1). Therefore, $\mathcal{R}_{JD}^G = \mathcal{R}_{JD}^*$. ■

We observe further that the optimization of Gaussian quantization noise covariance matrices under joint decoding is the following convex optimization problem over $(R_k, \mathbf{B}_\ell)$:

$$
\begin{aligned}
\max_{R_k, \mathbf{0}\preceq\mathbf{B}_\ell\preceq\mathbf{\Sigma}_\ell^{-1}} \quad & \sum_{k=1}^K \mu_k R_k & (16)\\
\text{s.t.} \quad & \sum_{k\in\mathcal{T}} R_k \le \sum_{\ell\in\mathcal{S}}\left[C_\ell - \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|}\right] \\
& \qquad + \log\frac{\left|\sum_{\ell\in\mathcal{S}^c}\mathbf{H}_{\ell,\mathcal{T}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{T}} + \mathbf{K}_{\mathcal{T}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{T}}^{-1}\right|}
\end{aligned}
$$

where the set of constraints is over all $\mathcal{T} \subseteq \mathcal{K}$ and $\mathcal{S} \subseteq \mathcal{L}$. Note that the number of constraints grows exponentially in the size of the network. Because of this, the above optimization problem can only be solved for small networks in practice.

IV. Optimization of Successive Decoding Region under Sum Fronthaul Constraint

Successive decoding is more practical than joint decoding because of its lower complexity, but can also give lower rate [5]. In this section, we show that in the special case where the fronthaul links are subject to a sum capacity constraint, successive decoding actually achieves the same maximum sum rate as joint decoding, assuming Gaussian input and Gaussian quantization. Combining with the result on the optimality of Gaussian quantization for joint decoding, this implies that for maximizing the sum rate under sum fronthaul constraint, Gaussian quantization with successive decoding is optimal.

More specifically, the sum fronthaul constraint is modeled as $\sum_{\ell=1}^L C_\ell \le C$, justifiable in certain situations where the fronthaul are implemented in shared medium, as has been considered in [6], [14]. Assuming Gaussian quantization, the joint decoding rate $R_{JD,s}$ under sum fronthaul constraint is

$$
\begin{aligned}
R_{JD,s} \le \min\Bigg\{ & C - \sum_{\ell=1}^L \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|}\,, \\
& \log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{K}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_{\mathcal{K}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{K}}^{-1}\right|}\Bigg\}
\end{aligned} \tag{17}
$$

for some $0 \preceq \mathbf{B}_\ell \preceq \mathbf{\Sigma}_\ell^{-1}$, which can be derived from (5) by noting that only $\mathcal{T} = \mathcal{K}$ and only the constraints corresponding to $\mathcal{S} = \emptyset$ and $\mathcal{S} = \mathcal{L}$ are relevant under the sum fronthaul constraint. Likewise, based on (6) and (7), the sum rate for successive decoding $R_{SD,s}$ is given by

$$
R_{SD,s} \le \log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{K}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_{\mathcal{K}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{K}}^{-1}\right|} \tag{18}
$$

for some $0 \preceq \mathbf{B}_\ell \preceq \mathbf{\Sigma}_\ell^{-1}$ that satisfies

$$
\log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{K}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_{\mathcal{K}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{K}}^{-1}\right|} + \sum_{\ell=1}^L \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|} \le C. \tag{19}
$$

Let $R_{JD,s}^*$ and $R_{SD,s}^*$ be the maximum sum rates under (17) and (18)-(19), respectively. Clearly, $R_{JD,s}^* \ge R_{SD,s}^*$, since any $\mathbf{B}_\ell$ that satisfies the constraint (19) also gives $R_{JD,s} = R_{SD,s}$. To show that $R_{JD,s}^* \le R_{SD,s}^*$, observe that if this is not the case, then the optimal $\mathbf{B}_\ell$ that attains $R_{JD,s}^*$ must satisfy

$$
C - \sum_{\ell=1}^L \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|} < \log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{K}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_{\mathcal{K}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{K}}^{-1}\right|}. \tag{20}
$$

But then, we can scale $\mathbf{B}_\ell' = \gamma\mathbf{B}_\ell$ with $0 < \gamma \le 1$ to increase the left-hand side and to decrease the right-hand side in the above, leading to a higher $R_{JD,s}$. This contradicts the optimality of $\mathbf{B}_\ell$. Thus, we have proved:

Theorem 2: Under sum fronthaul constraint and with fixed Gaussian input, the maximum sum rates achieved by successive decoding and joint decoding over Gaussian quantization are the same, i.e., $R_{SD,s}^* = R_{JD,s}^*$.

A consequence of this result is that under the sum fronthaul constraint, the optimization of the Gaussian quantization noise covariance for maximizing the sum rate under successive decoding can be formulated as a convex optimization problem. More precisely, the sum rate maximization problem can be formulated as:

$$
\begin{aligned}
\max_{R, 0\preceq\mathbf{B}_\ell\preceq\mathbf{\Sigma}_\ell^{-1}} \quad & R & (21)\\
\text{s.t.} \quad & R \le \log\frac{\left|\sum_{\ell=1}^L \mathbf{H}_{\ell,\mathcal{K}}^\dagger\mathbf{B}_\ell\mathbf{H}_{\ell,\mathcal{K}} + \mathbf{K}_{\mathcal{K}}^{-1}\right|}{\left|\mathbf{K}_{\mathcal{K}}^{-1}\right|}, \\
& R + \sum_{\ell=1}^L \log\frac{|\mathbf{\Sigma}_\ell^{-1}|}{|\mathbf{\Sigma}_\ell^{-1} - \mathbf{B}_\ell|} \le C.
\end{aligned}
$$

It can be verified that the above problem is convex in $(R, \mathbf{B}_\ell)$, so it can solved efficiently. Convexity is a key advantage of this reformulation of the problem as compared to previous approaches in the literature (e.g. [6], [14], [15]) that parameterize the optimization problem over the quantization noise covariance $\mathbf{Q}_\ell$, which leads to a nonconvex formulation.

V. Numerical Example

This section presents a numerical example of a wireless cellular network with three cells forming a cooperating cluster. A total of $K = 6$ users are randomly located within the cluster. Both the users and the BSs are equipped with $M = N = 2$ antennas each. The noise power spectral density is set to be -124.6 dBm/Hz; the user's transmit power is set to be 23 dBm over 10 MHz; a distance-dependent path-loss model is used with $L = 128.1 + 37.6 \cdot \log_{10}(d)$ (where d is in km) and with 8dB log normal shadowing and a Rayleigh component. The distance between neighboring BSs is set to be 0.5 km.

The achievable sum rates of the compress-and-forward schemes are plotted in Fig. 2. In the simulation, the sum rates of joint decoding and successive decoding with optimal Gaussian quantization are obtained under the individual fronthaul capacity constraint and the sum fronthaul capacity constraint using the convex formulations (16) and (21), respectively. As performance comparison, the sum rate achieved by joint decoding with the quantizers suggested by noisy network coding [4], i.e., $\mathbf{Q}_\ell = \mathbf{\Sigma}_\ell$, which achieves capacity to within constant gap, is also provided. This is referred to as constant-gap quantizer. The cut-set like sum-capacity upper bound [1]

$$\bar{C} = \min\left\{ \log \frac{\left|\sum_{k=1}^{K} \mathbf{H}_{\mathcal{L},k}\mathbf{K}_k\mathbf{H}_{\mathcal{L},k}^\dagger + \mathbf{\Omega}\right|}{|\mathbf{\Omega}|}, \sum_{\ell=1}^{L} C_\ell \right\} \quad (22)$$

where $\mathbf{\Omega} = \operatorname{diag}\left(\{\mathbf{\Sigma}_\ell\}_{\ell=1}^L\right)$ is also plotted. It is observed that the optimal quantizer significantly outperforms the constant-gap qunatizer in achievable sum rate for the C-RAN model.

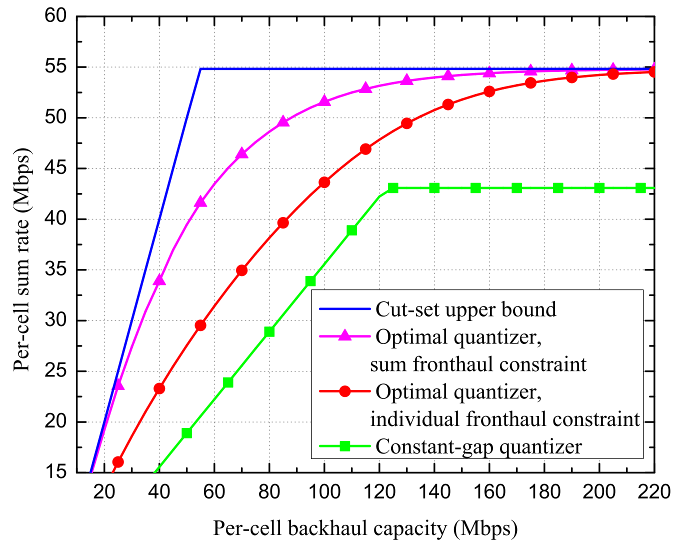

Fig. 2. Achievable sum rate of a C-RAN with different quantizers.

VI. Conclusion

This paper studies the compress-and-forward scheme for an uplink C-RAN model where the BSs are connected to a CP through noiseless fronthaul links of limited capacities. We show the optimality of Gaussian quantization under certain condition, and show the equivalence of joint decoding and successive decoding for maximum sum rate under sum fronthaul constraint. Further, we show that the optimization of Gaussian quantizer for maximizing the sum rate under successive decoding can be cast as a convex optimization problem, which facilitates its efficient numerical solution.

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