Hybrid Beamforming with Finite-Resolution Phase Shifters for Large-Scale MIMO Systems

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***Abstract*—In large-scale multiple-input multiple-output (MIMO) systems, high cost and high power consumption of RF chains typically prohibit the use of traditional baseband beamforming which requires one distinct radio-frequency (RF) chain per antenna. One possible architecture to reduce the number of RF chains is hybrid beamforming in which the overall beamformer consists of a concatenation of an analog RF beamformer implemented using phase shifters (PSs) and a low-dimensional baseband digital beamformer. However, conventional hybrid beamforming designs require high-resolution PSs, which are expensive. In this paper, we consider transceiver design for maximizing the spectral efficiency of a large-scale MIMO system with hybrid beamforming architecture where only *finite-resolution* PSs are available at both ends. We propose a heuristic transceiver design for the critical case where the number of RF chains is equal to the number of data streams. We show that the proposed hybrid beamforming design can achieve a rate close to that of optimal exhaustive search. We also suggest how to generalize the algorithm for the setting where the number of RF chains exceeds the number of data streams. We show that the generalized algorithm can use the extra RF chains to significantly improve the system performance in the case of low-resolution PSs.**

I. Introduction

Millimeter wave (mmWave) communication has been considered as a strong candidate for achieving high throughput in future fifth generation (5G) wireless networks [1]. The rather short wavelength of the mmWave systems allows more antenna elements to be packed in the same physical dimension, thus enabling the implementation of large-scale MIMO array. In the conventional beamforming schemes for MIMO systems, beamforming is performed digitally at the baseband which enables both phase control and amplitude control of the signals, but also requires the use of one dedicated RF chain per antenna element. However, the high cost and power consumption of the RF chains [2] can prohibit the implementation of such fully-digital beamforming schemes in large-scale MIMO systems.

To address the challenge of limited transmit/receive RF chains, different schemes have been studied in the literature. Analog or RF beamforming implemented using phase shifters (PSs) is considered in [3], [4], but the difficulty of controlling the signal magnitude makes its performance not as good as the fully-digital beamforming schemes. Antenna subset selection implemented using analog switches is considered in [5], [6]. However, antenna selection provides limited array gain and has relatively poor performance in correlated channels [7].

This paper addresses the challenge of limited transmit/receive RF chains by considering a two-stage hybrid digital and analog beamforming architecture. The hybrid beamforming architecture is extensively studied in the literature [8]–[11]. For the single-user MIMO system, [8]–[10] propose different heuristic hybrid beamforming designs to maximize the spectral efficiency under total power constraint at the transmitter. However, the existing hybrid beamforming designs typically assume the use of infinite-resolution PSs for implementing the analog beamforming part.

This paper proposes novel hybrid beamforming design of a single-user large-scale MIMO system for spectral efficiency maximization, where only *finite-resolution* PSs are available. We first focus on the critical case where the number of RF chains is equal to the number of data streams, and show numerically that the achievable rate of the proposed design is close to the rate of optimal solution obtained by exhaustive search, while being significantly better than the achievable rate of quantized version of the existing algorithms when very-low-resolution PSs are used. We further generalize the algorithm to the setting where the number of RF chains exceeds the number of data streams, and show numerically that extra RF chains can be used to trade off the resolution of PSs.

II. System Model

Consider a single-user MIMO system in which a transmitter with N antennas and N_t^{RF} transmit RF chains sends N_s independent data symbols to a receiver equipped with M antennas and N_r^{RF} receive RF chains. It is assumed that $N_s \leq N_t^{\text{RF}} < N$ and $N_s \leq N_r^{\text{RF}} < M$. To address the challenge of limited RF chains, we consider a hybrid digital and analog beamforming architecture at each end, shown in Fig. 1. To simplify the notation, we assume that the number of RF chains at the transmitter and the receiver is identical; i.e., $N_t^{\text{RF}} = N_r^{\text{RF}} = N^{\text{RF}}$, however the results can be easily applied to the general setting.

In the hybrid beamforming architecture, the overall precoder consists of an $N^{\text{RF}} \times N_s$ digital precoder, $\mathbf{V}_{\text{Dig}}$, followed by an $N \times N^{\text{RF}}$ RF precoder, $\mathbf{V}_{\text{RF}}$. The transmit signal is

$$\mathbf{x} = \mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s}, \tag{1}$$

where $\mathbf{s} \in \mathbb{C}^{N_s \times 1}$ is the vector of intended messages to the receiver, normalized such that $\mathbb{E}\{\mathbf{s}\mathbf{s}^H\} = \mathbf{I}_{N_s}$. Considering a

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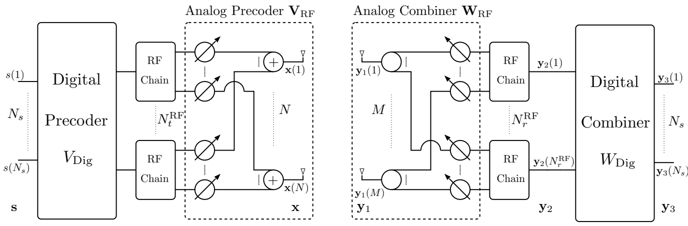

Fig. 1: A large-scale MIMO system with hybrid beamforming architecture at the transmitter and the receiver.

narrowband block-fading channel, the received signal is

$$\mathbf{y}_1 = \mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s} + \mathbf{z}, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{M\times N}$ is the channel matrix and $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I}_M)$ is the additive white Gaussian noise. The receiver uses an $M \times N^{\text{RF}}$ RF combiner, $\mathbf{W}_{\text{RF}}$, to obtain the processed signals as $\mathbf{y}_2 = \mathbf{W}_{\text{RF}}^H\mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s} + \mathbf{W}_{\text{RF}}^H\mathbf{z}$. Finally, a digital combiner, $\mathbf{W}_{\text{Dig}} \in \mathbb{C}^{N^{\text{RF}}\times N_s}$, is used at the receiver to obtain the final digital processed signals as

$$\mathbf{y}_3 = \mathbf{W}_{\text{Dig}}^H\mathbf{W}_{\text{RF}}^H\mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{s} + \mathbf{W}_{\text{Dig}}^H\mathbf{W}_{\text{RF}}^H\mathbf{z}. \tag{3}$$

As shown in Fig. 1, the RF beamformers in the hybrid structure are implemented using analog PSs which implies constant modulus constraints on the elements of the RF beamformers. However, the components required for accurate phase control can be expensive [12], especially since the number of PSs is proportional to the number of antennas. Therefore, in practical implementation, finite-resolution PSs are used, which implies limited choices for the entries of the RF beamformers; i.e., $\mathbf{V}_{\text{RF}}(m,n) \in \mathcal{F}$ and $\mathbf{W}_{\text{RF}}(m,n) \in \mathcal{F}$ where $\mathcal{F} = \{1, \omega, \omega^2, \dots \omega^{n_{\text{PS}}-1}\}$ and $\omega = e^{j\frac{2\pi}{n_{\text{PS}}}}$ and n_{PS} is the number of phases that can be realized. Typically, $n_{\text{PS}} = 2^b$ where b is the number of bits of resolution of PSs.

This paper considers the problem of the hybrid beamforming design to maximize the overall spectral efficiency under a total power constraint at the transmitter as

$$\begin{aligned}
\underset{\mathbf{V}_{\text{RF}},\mathbf{V}_{\text{Dig}}\mathbf{W}_{\text{RF}},\mathbf{W}_{\text{Dig}}}{\text{maximize}} \quad & R && \text{(4a)}\\
\text{subject to} \quad & \text{Tr}(\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H) \le P && \text{(4b)}\\
& \mathbf{V}_{\text{RF}}(m,n) \in \mathcal{F}, \ \ \forall m,n && \text{(4c)}\\
& \mathbf{W}_{\text{RF}}(m,n) \in \mathcal{F}, \ \ \forall m,n && \text{(4d)}
\end{aligned}$$

where P is the transmitter power budget, and R is the overall spectral efficiency assuming Gaussian signalling formulated as

$$R = \log_2\left|\mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{W}_t(\mathbf{W}_t^H\mathbf{W}_t)^{-1}\mathbf{W}_t^H\mathbf{H}\mathbf{V}_t\mathbf{V}_t^H\mathbf{H}^H\right|, \tag{5}$$

where $\mathbf{V}_t = \mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}$ and $\mathbf{W}_t = \mathbf{W}_{\text{RF}}\mathbf{W}_{\text{Dig}}$. For fixed RF beamformers, the optimal digital beamformers can be easily found by solving (4) over $\mathbf{V}_{\text{Dig}}$ and $\mathbf{W}_{\text{Dig}}$. However, the optimization over $\mathbf{V}_{\text{RF}}$ and $\mathbf{W}_{\text{RF}}$ is more challenging. In theory, since the possible choices for RF beamformers are finite, the optimal solution of (4) can be found by exhaustively searching over all such choices. However, the number of possible choices for RF beamforming is exponential in the number of antennas and the resolution of the PSs. In this paper we aim to propose a fast heuristic algorithm that achieves a rate close to the optimal rate of the exhaustive search method.

III. Hybrid Beamforming Design for $N^{RF} = N_s$

First, we consider the design of hybrid precoder and combiner for the critical case where $N^{\text{RF}} = N_s$. This case is called critical since in infinite-resolution PS scenario, it can be shown [10] that the hybrid beamforming structure needs at least N_s RF chains at each end to be able to achieve a rate close to the rate of the optimal fully-digital solution.

A. Transmitter Design

The optimization problem (4) involves a joint optimization over the transmitter precoders and receiver combiners. Similar to [9], [10], this paper considers a decoupled design by considering the maximization of the mutual information between the intended messages and the received signal for designing the precoders first, then the maximization of the overall spectral efficiency for designing the combiners. First, the problem of designing the hybrid precoders can be written as

$$\begin{aligned}
\max_{\mathbf{V}_{\text{RF}},\mathbf{V}_{\text{Dig}}} \quad & \log_2\left|\mathbf{I}_M + \frac{1}{\sigma^2}\mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\right| && \text{(6a)}\\
\text{s.t.} \quad & \text{Tr}(\mathbf{V}_{\text{RF}}\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H\mathbf{V}_{\text{RF}}^H) \le P, && \text{(6b)}\\
& \mathbf{V}_{\text{RF}}(m,n) \in \mathcal{F}, \ \ \forall m,n. && \text{(6c)}
\end{aligned}$$

The optimization problem (6) is challenging because it involves a product of $\mathbf{V}_{\text{RF}}$ and $\mathbf{V}_{\text{Dig}}$, and the design of $\mathbf{V}_{\text{RF}}$ is combinatorial. This paper decouples the design of $\mathbf{V}_{\text{Dig}}$ and $\mathbf{V}_{\text{RF}}$ using a procedure adapted in [10], which considers the same problem for the infinite-resolution PSs. First, we show that the optimal $\mathbf{V}_{\text{Dig}}$ for a fixed $\mathbf{V}_{\text{RF}}$ typically satisfies $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H \propto \mathbf{I}$, regardless of the design of $\mathbf{V}_{\text{RF}}$. Then, assuming such a digital precoder, we devise an iterative algorithm to tackle the combinatorial RF precoder design problem. The numerical results show that for low-resolution PSs, the proposed method has a better performance as compared to the straightforward approach of quantizing the solution of [10]. Finally, for designed $\mathbf{V}_{\text{RF}}$, we optimize $\mathbf{V}_{\text{Dig}}$ by solving (6).

1) Digital Precoder Design: Let $\mathbf{Q} = \mathbf{V}_{\text{RF}}^H\mathbf{V}_{\text{RF}}$ and $\mathbf{H}_e = \mathbf{H}\mathbf{V}_{\text{RF}}\mathbf{Q}^{-1/2}$. It is easily verified [10] that the optimal digital precoder for a fixed RF precoder has a well-known solution; i.e., $\mathbf{V}_{\text{Dig}} = \mathbf{Q}^{-1/2}\mathbf{U}_e\mathbf{\Gamma}_e$ where $\mathbf{U}_e$ is the set of eigenvectors corresponding to the N_s largest eigenvalues of $\mathbf{H}_e^H\mathbf{H}_e$ and $\mathbf{\Gamma}_e$ is the diagonal matrix of water-filling power allocation.

The optimal digital precoder when the number of antennas is large approximately satisfies $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H \propto \mathbf{I}$, regardless of the design of RF precoder. This is because the off-diagonal entries of $\mathbf{V}_{\text{RF}}^H\mathbf{V}_{\text{RF}}$ is approximately the inner product of two $N \times 1$ random vectors with elements chosen from $\mathcal{F}$, which is typically much less than N for large N, while the diagonal elements are exactly N. This observation has already been made in [9] for the case of infinite-resolution PSs, but is also true for the finite-resolution case. To find the proportionality constant, we further assume that the effective channel operates in the high SNR regime which implies that equal power allocation is optimal; i.e., $\mathbf{\Gamma}_e \approx \sqrt{P/N_t^{\text{RF}}}\mathbf{I}$. So, optimal digital precoder is $\mathbf{V}_{\text{Dig}} \approx \gamma \mathbf{U}_e$ where $\gamma^2 = P/(NN_t^{\text{RF}})$. Since $\mathbf{U}_e$ is a unitary matrix, we have $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H \approx \gamma^2\mathbf{I}$.

2) RF Precoder Design: We now consider the design of the RF precoder assuming $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H = \gamma^2\mathbf{I}$. With this assumption, the power constraint in (6b) is automatically satisfied, regardless of the design of $\mathbf{V}_{\text{RF}}$. Therefore, the RF precoder design problem is

$$\max_{\mathbf{V}_{\text{RF}}} \quad \log_2\left|\mathbf{I}_{N_s} + \frac{\gamma^2}{\sigma^2}\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\mathbf{H}\mathbf{V}_{\text{RF}}\right| \tag{7a}$$

$$\text{s.t.} \quad \mathbf{V}_{\text{RF}}(m,n) \in \mathcal{F}, \quad \forall m,n. \tag{7b}$$

Since the constraints are completely decoupled in (7), we can use the column update design approach introduced by [13]. Note that the same strategy is used in [10] for the infinite-resolution PSs; however, the subproblem for designing each column of $\mathbf{V}_{\text{RF}}$ now involves an extra quantization step for the finite-resolution case.

Let $\mathbf{F}_1 = \mathbf{H}^H\mathbf{H}$. The contribution of the k^{th} column of the RF precoder to the objective function in (7) can be isolated as

$$\log_2\left|\mathbf{C}_k\right| + \log_2\left(1 + \frac{\gamma^2}{\sigma^2}{\mathbf{v}_{\text{RF}}^{(k)}}^H\mathbf{G}_k\mathbf{v}_{\text{RF}}^{(k)}\right), \tag{8}$$

where $\mathbf{C}_k = \mathbf{I} + \frac{\gamma^2}{\sigma^2}(\bar{\mathbf{V}}_{\text{RF}}^k)^H\mathbf{F}_1\bar{\mathbf{V}}_{\text{RF}}^k$ and $\mathbf{G}_k = \mathbf{F}_1 - \frac{\gamma^2}{\sigma^2}\mathbf{F}_1\bar{\mathbf{V}}_{\text{RF}}^k\mathbf{C}_k^{-1}(\bar{\mathbf{V}}_{\text{RF}}^k)^H\mathbf{F}_1$ is a positive semidefinite matrix and $\bar{\mathbf{V}}_{\text{RF}}^k$ is the sub-matrix of $\mathbf{V}_{\text{RF}}$ with k^{th} column removed. The algorithm seeks to increase (8) iteratively over each column of $\mathbf{V}_{\text{RF}}$ assuming that the other columns are fixed. Since $\mathbf{C}_k$ is independent of $\mathbf{v}_{\text{RF}}^{(k)}$ and $\log(.)$ is an increasing function, the subproblem for updating the k^{th} column is just

$$\max_{\mathbf{v}_{\text{RF}}^{(k)}} \quad {\mathbf{v}_{\text{RF}}^{(k)}}^H\mathbf{G}_k\mathbf{v}_{\text{RF}}^{(k)} \tag{9a}$$

$$\text{s.t.} \quad \mathbf{v}_{\text{RF}}^{(k)}(i) \in \mathcal{F}, \quad \forall i = 1,\ldots,N, \tag{9b}$$

where $\mathbf{v}_{\text{RF}}^{(k)}(i)$ is the i^{th} element of the vector $\mathbf{v}_{\text{RF}}^{(k)}$.

Lemma 1. *Any local optimal solution of* (9) *satisfies*

$$\mathbf{v}_{RF}^{(k)}(i) = \mathcal{Q}\Big(\psi\Big(\sum_{\ell\neq i} g_{i\ell}^k\mathbf{v}_{RF}^{(k)}(\ell)\Big)\Big), \quad \forall i = 1,\ldots,N, \tag{10}$$

where for a non-zero complex variable a, $\psi(a) = \frac{a}{|a|}$ and for $a = 0$, $\psi(a) = 1$, and $g_{i\ell}^k$ is the element of $\mathbf{G}_k$ at the i^{th} row and ℓ^{th} column. The function $\mathcal{Q}(c)$ quantize a complex unit-norm variable c to the nearest point in the set $\mathcal{F}$.

Proof. Considering the fact that $|\mathbf{v}_{\text{RF}}^{(k)}(i)|^2 = 1$, the objective function of (9) can be rewritten as

$${\mathbf{v}_{\text{RF}}^{(k)}}^H\mathbf{G}_k\mathbf{v}_{\text{RF}}^{(k)} = \zeta_i^{(k)} + 2\,\text{Re}\{{\mathbf{v}_{\text{RF}}^{(k)}(i)}^H\eta_i^{(k)}\}, \tag{11}$$

where $\zeta_i^{(k)} = g_{ii}^k + 2\,\text{Re}\{\sum_{m\neq i}\sum_{n\neq i}{\mathbf{v}_{\text{RF}}^{(k)}(m)}^H g_{mn}^k\mathbf{v}_{\text{RF}}^{(k)}(n)\}$ and $\eta_i^{(k)} = \sum_{\ell\neq i} g_{i\ell}^k\mathbf{v}_{\text{RF}}^{(k)}(\ell)$ are independent of the i^{th} element of the vector $\mathbf{v}_{\text{RF}}^{(k)}$. Assuming all of the elements of the vector $\mathbf{v}_{\text{RF}}^{(k)}$ are fixed except the i^{th} element, we need to maximize $\text{Re}\{{\mathbf{v}_{\text{RF}}^{(k)}(i)}^H\eta_i^{(k)}\}$ which is equivalent to minimizing the angle between $\mathbf{v}_{\text{RF}}^{(k)}(i)$ and $\eta_i^{(k)}$ on the complex plane. Since $\mathbf{v}_{\text{RF}}^{(k)}(i)$ is constrained to be chosen from the set $\mathcal{F}$, we should have

$$\mathbf{v}_{\text{RF}}^{(k)}(i) = \mathcal{Q}\Big(\psi\Big(\sum_{\ell\neq i} g_{i\ell}^k\mathbf{v}_{\text{RF}}^{(k)}(\ell)\Big)\Big),$$

which completes the proof. □

Algorithm 1

1: Set $\mathbf{V}_{\text{RF}} = \mathbf{1}_{N\times N^{\text{RF}}}$.
2: **for** $k = 1 \to N^{\text{RF}}$ **do**
3: Construct $\mathbf{G}_k$ as defined following (8).
4: Find $\mathbf{V}_k^*$ by solving the convex problem (12).
5: Find initial $\mathbf{v}_{\text{RF}}^{(k)}$ by randomization procedure of [14].
6: **end for**
7: **for** $k = 1 \to N^{\text{RF}}$ **do**
8: Construct $\mathbf{G}_k$ as defined following (8).
9: **while** (10) is not satisfied **do**
10: **for** $i = 1 \to N$ **do**
11: $\mathbf{v}_{\text{RF}}^{(k)}(i) = \mathcal{Q}\Big(\psi\Big(\sum_{\ell\neq i} g_{i\ell}^k\mathbf{v}_{\text{RF}}^{(k)}(\ell)\Big)\Big)$.
12: **end for**
13: **end while**
14: **end for**
15: Check convergence. If yes, stop; if not go to Step 7.

Therefore, a local maximizer of (9) can be found by iteratively updating the elements of $\mathbf{v}_{\text{RF}}^{(k)}$ according to (10). Note that, in each such update, the objective function in (9) always increases. The algorithm terminates when (10) is satisfied for all elements of $\mathbf{v}_{\text{RF}}^{(k)}$. According to Lemma 1 this point is a local optimal solution to (9). The algorithm always terminates properly since the set $\mathcal{F}$ is finite.

The natural question arises: How good is the local optimal solution of each subproblem (9) given by the proposed iterative algorithm? In the following, we modify the algorithm to provide a performance lower bound on that.

A lower bound on the expected performance of the solution of (9) can be found by using SDP relaxation method in [14]. Consider the following complex SDP relaxation of (9)

$$\max_{\mathbf{V}_k\in\mathbb{C}^{N\times N}} \quad \text{Tr}(\mathbf{V}_k\mathbf{G}_k) \tag{12a}$$

$$\text{s.t.} \quad \mathbf{V}_k \succeq \mathbf{0} \tag{12b}$$

$$\mathbf{V}_k(i,i) = 1, \quad \forall i = 1,\ldots,N. \tag{12c}$$

Since this problem is convex, the optimal solution can be found efficiently. Suppose $\mathbf{V}_k^*$ is an optimal solution of the problem (12). One conventional way to generate a feasible $\mathbf{v}_{\text{RF}}^{(k)}$ from the solution of the (12) is to draw a random vector $\boldsymbol{\zeta} \sim \mathcal{CN}(\mathbf{0},\mathbf{V}_k^*)$, then to quantize each element of $\boldsymbol{\zeta}$ to the nearest point in $\mathcal{F}$. In [14], it is shown that the expectation of the objective function of the solution obtained using the above procedure is greater than $\alpha_{n_{\text{PS}}}\,\text{Tr}(\mathbf{V}_k^*\mathbf{G}_k)$, where

$$\alpha_{n_{\text{PS}}} = \begin{cases} \frac{2}{\pi}, & \text{if } n_{\text{PS}} = 2 \\ \frac{n_{\text{PS}}^2(1-\cos\frac{2\pi}{n_{\text{PS}}})}{8\pi}, & \text{if } n_{\text{PS}} \geq 3. \end{cases} \tag{13}$$

Therefore, if we first obtain $\mathbf{v}_{\text{RF}}^{(k)}$ using the solution of the SDP problem in (12), then use it as an initial point for the proposed iterative algorithm, the overall algorithm ends up with a local maximizer with the above lower bound. Although, this approach provides a theoretical tool for having a lower bound on the performance of the algorithm, it imposes extra computational burden to the algorithm. Further, our simulations show that adding the SDP actually does not improve the overall performance of the algorithm significantly. But it may be useful to include SDP in early stages to reduce number of the iterations. In this paper, we propose to use SDP only in the first iteration. The proposed algorithm for solving the RF beamformer in (7) is summarized in Algorithm 1.

B. Receiver Design

Finally, we seek to design the hybrid combiners that maximize the achievable rate for the designed precoder. For $N^{RF} = N_s$, the design of $\mathbf{W}_{\text{RF}}$ and $\mathbf{W}_{\text{Dig}}$ can be decoupled without loss of optimality by designing $\mathbf{W}_{\text{RF}}$ first then $\mathbf{W}_{\text{Dig}}$ [10]. Moreover, assuming $\mathbf{W}_{\text{RF}}^H\mathbf{W}_{\text{RF}} \approx M\mathbf{I}$, it is shown in [10] that the RF combiner design problem can be formulated in the same form as the RF precoder design problem (7). Therefore, the RF combiner can also be obtained using Algorithm 1. Finally, the optimal digital combiner is the MMSE receiver.

IV. Hybrid Beamforming Design for $N^{RF} > N_s$

In the previous section, a heuristic algorithm is proposed for the case that $N^{\text{RF}} = N_s$. When $N^{RF} > N_s$, the transmitter design problem can still be formulated as in (6). For any given RF precoder, the optimal digital precoder can still be found analogously to the case $N^{RF} = N_s$, however now it satisfies $\mathbf{V}_{\text{Dig}}\mathbf{V}_{\text{Dig}}^H \approx \gamma^2[\mathbf{I}_{N_s}\ \mathbf{0}]$. Assuming that digital precoder, the objective function of the (6) can be written as

$$\log_2 \prod_{i=1}^{N_s}(1+\frac{\gamma^2}{\sigma^2}\lambda_i), \tag{14}$$

where λ_i is the i^{th} largest eigenvalues of $\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\mathbf{H}\mathbf{V}_{\text{RF}}$. Since maximizing a function of subset of eigenvalues is difficult, we instead maximize an expression including all of the eigenvalues, i.e., $\log_2 \prod_{i=1}^{N^{\text{RF}}}(1+\frac{\gamma^2}{\sigma^2}\lambda_i)$, or equivalently

$$\log_2\left|\mathbf{I}_{N^{\text{RF}}} + \frac{\gamma^2}{\sigma^2}\mathbf{V}_{\text{RF}}^H\mathbf{H}^H\mathbf{H}\mathbf{V}_{\text{RF}}\right|. \tag{15}$$

This approximation is reasonable for the practical settings where N^{RF} is not too much larger than N_s. The approximated RF precoder design problem is now formulated similar to (7). So, Algorithm 1 can be used to design the RF precoder. In other words, we suggest to first find the RF precoder assuming the number of data streams is equal to the number of RF chains, then for given RF precoder, to design the digital precoder for the actual number of data streams. At the receiver side, we still suggest to design the RF combiner first, then set the digital combiner to be the MMSE solution.

V. Simulations

In this section, simulation results are presented to show the performance of the proposed algorithm. The propagation environment of the large-scale MIMO systems is modeled as a geometric channel with L paths between the transmitter and the receiver [9]. Assume an antenna configuration with uniform linear array, the channel matrix can be expressed as

$$\mathbf{H} = \sqrt{\frac{NM}{L}}\sum_{\ell=1}^{L}\alpha_\ell\mathbf{a}_r(\phi_r^\ell)\mathbf{a}_t(\phi_t^\ell)^H, \tag{16}$$

where $\alpha_\ell \sim \mathcal{CN}(0,1)$ is the complex gain of the ℓ^{th} path, and $\phi_r^\ell \in [0,\ 2\pi)$, $\phi_t^\ell \in [0,\ 2\pi)$, and $\mathbf{a}_r(.)$ and $\mathbf{a}_t(.)$ are the antenna array response vectors at the receiver and the transmitter, respectively. The antenna array response vector in a uniform linear array configuration with N antenna elements is modeled as $\mathbf{a}(\phi) = \frac{1}{\sqrt{N}}[1, e^{jkd\sin(\phi)}, \ldots, e^{jkd(N-1)\sin(\phi)}]^T$, where $k = \frac{2\pi}{\lambda}$, λ is the wavelength and d is the antenna spacing.

In the following simulations, we consider an environment with $L = 15$ scatterers with uniformly random angles of arrival and departure and $d = \frac{\lambda}{2}$. For each simulation, the average spectral efficiency is plotted versus signal-to-noise-ratio (SNR $= \frac{P}{\sigma^2}$) over 100 channel realizations.

In the first simulation, we consider a 10×10 MIMO system with hybrid structure in which 1-bit PSs are available and $N^{\text{RF}} = N_s = 2$. We consider a relatively small MIMO system with low-resolution PSs in order to be able to compare the performance of the proposed algorithm with that of the exhaustive search method. We also show the performance of the quantized version of the algorithms in [9], [10] as a baseline, where the RF beamformers are first designed under the assumption of infinite-resolution PSs, then each element of the RF beamformers is quantized to the nearest point of the set $\mathcal{F}$. Finally, for the designed RF beamformers, the digital beamformers are optimized in each case. Fig. 2 shows that the performance of the proposed design for the hybrid structure with 1-bit resolution PSs is very close to that of the exhaustive search while there is a significant gap between the performance of the proposed method with the quantized version of the algorithms in [9], [10].

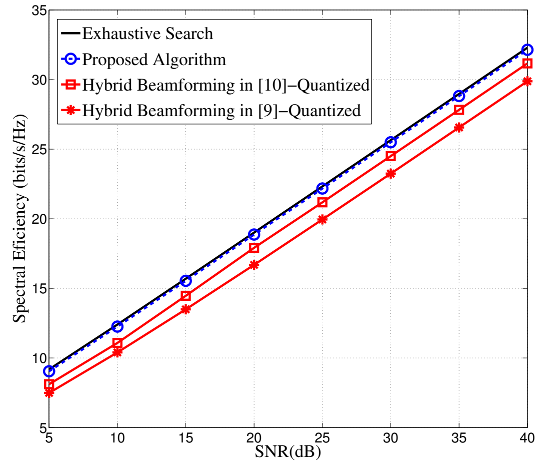

Fig. 2: Spectral efficiencies versus SNR for different methods in a 10×10 system where $L = 15$, $N^{\text{RF}} = N_s = 2$ and $b = 1$.

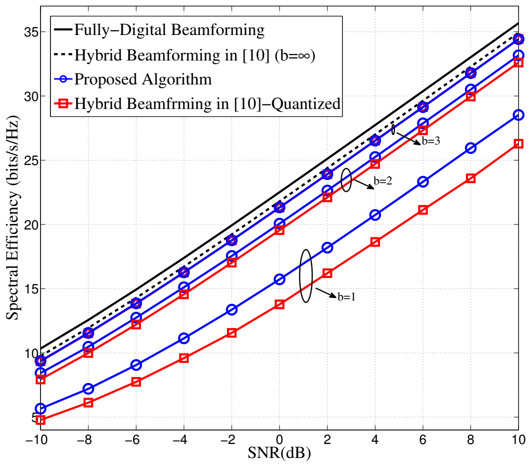

Fig. 3: Spectral efficiencies versus SNR for different methods in a 64×16 system where $L = 15$, $N^{\mathrm{RF}} = N_s = 4$.

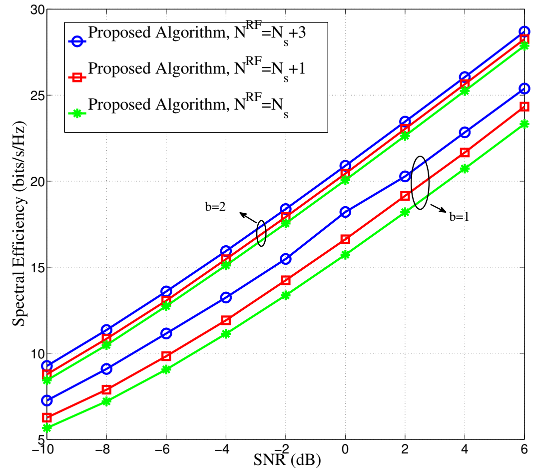

Fig. 4: Spectral efficiencies versus SNR for different methods in a 64×16 system where $L = 15$, $N_s = 4$.

The second simulation considers a 64×16 MIMO system with $N^{\mathrm{RF}} = N_s = 4$. Fig. 3 shows that the performance gain of the proposed algorithm in comparison with the quantized version of [10] is larger when lower resolution PSs are used. This suggests that designing the hybrid beamformers for finite-resolution PSs is crucial when very-low-resolution PSs are used; i.e., here when $b = 1$ or 2. Moreover, Fig. 3 indicates that the performance of 3-bit PSs is very close to that of hybrid beamforming with infinite-resolution PSs.

Finally, we study the effect of the number of RF chains on the performance of the proposed algorithm for a 64×16 MIMO system in which $N_s = 4$. Fig. 4 verifies that by using the proposed strategy in Section IV, the spectral efficiency of the overall system can be improved by utilizing more RF chains than the number of data streams. Fig. 4 also shows that adding extra RF chains is more effective when very-low-resolution PSs are used; i.e., b=1. Thus, the extra RF chains can be used to trade off the resolution of PSs.

VI. Conclusion

This paper considers hybrid beamforming for single-user large-scale MIMO systems with limited number of RF chains and finite-resolution PSs. A fast heuristic algorithm is proposed for the case where the number of RF chains is equal to the number of data streams. The simulation results verify that the performance of the proposed algorithm is close to that of the exhaustive search method. We show that this approach has significantly better performance as compared to the quantized version of existing hybrid beamforming algorithms when very-low-resolution PSs are used. Finally, we generalize our algorithm for the setting where the number of RF chains exceeds the number of data streams, and show that the achievable rate can be improved significantly by adding extra RF chains in low-resolution PS case.

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