Achievable Rates and Outer Bounds for Full-Duplex Relay Broadcast Channel with Side Message

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***Abstract*—This paper examines the achievable rate region and the converse of a full-duplex relay broadcast channel with three independent messages: from the source to the relay, from the source to the destination, and from the relay to the destination. We are motivated to study this channel, because it models a full-duplex wireless cellular network in which the uplink user also wishes to send an independent device-to-device message to the downlink users. For the discrete memoryless channel case, we incorporate Marton's broadcast coding to obtain a new achievable rate region which is larger than previous rate regions. We further propose a tighter converse than the cut-set bound. For the Gaussian scalar channel case, we show that by using one of two rate-splitting schemes depending on the channel condition, we can already achieve the capacity region of this particular relay broadcast channel to within a constant gap. The proposed scheme outperforms the benchmark methods in terms of the symmetric generalized degree-of-freedom.**

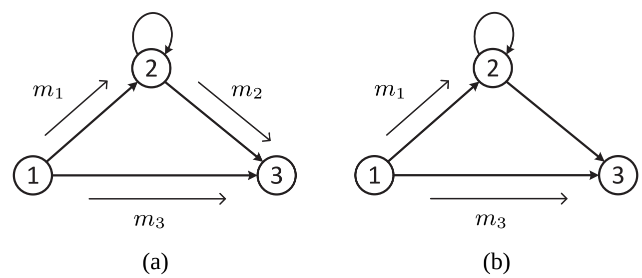

Fig. 1. (a) The relay broadcast channel with side message (or with "private" messages [1]); (b) The relay broadcast channel without side message [2], [3].

I. Introduction

This paper studies a full-duplex relay channel with three independent messages: one message m_1 from the source to the relay, one message m_2 from the relay to the destination, and a message m_3 from the source to the destination. This channel is named the relay channel with "private" messages in [1] as illustrated in Fig. 1(a), and is a generalization of the partially cooperative relay broadcast channel of [2], [3], if the message m_2 is removed, as shown in Fig. 1(b). The main contributions of this paper are two-fold. First, we propose new techniques to enhance the existing achievability and converse results for the discrete memoryless version of this channel model. Second, we provide a constant-gap-to-capacity result for the Gaussian (scalar) case.

We are motivated to study this channel due to its connection to the communication scenario of a full-duplex wireless cellular network (e.g., [4]) in which the base station (BS) has full-duplex self-interference-cancellation capability, but the user terminals are half-duplex. But in addition to the uplink message from the uplink user to the BS and the downlink message from the BS to the downlink user, we further assume that the uplink user wishes to directly send a message to the downlink via a device-to-device (D2D) side-link. The information theoretical model for such a scenario has been considered in our previous work [5], and it corresponds to the relay channel with private message model of Fig. 1(a) [1], in which node 1 plays the role of the uplink user, node 2 the BS, and node 3 the downlink user.

Prior work [1] on this channel model provides two achievable coding methods: a decode-and-forward scheme and a compress-and-forward scheme. The coding scheme proposed in this paper is closely related to decode-and-forward, but we incorporate rate splitting and moreover utilize Marton's broadcast coding scheme [6] to achieve a larger achievable rate region. On the converse, [1] derives an outer bound based on the genie-aided method, but as indicated by the authors, the outer bound of [1] is not computable. This paper develops better use of the auxiliary "genie" variables to improve upon the cut-set bound, and further comes up with a new sum-rate upper bound that would play a key role in characterizing the capacity region for the Gaussian case to within a constant gap. A modified Marton's broadcast coding scheme has already been used in the earlier works [2], [3] for the relay broadcast channel (which is a special case of our channel model). The achievability part of our paper can be thought of as a generalization of [2], [3] in incorporating the transmission of the relay-to-destination message m_2 into the modified Marton's coding.

In terms of characterizing the capacity to within a constant gap, our recent work [5] focuses on a special case without the source-to-destination message m_3, so that Marton's coding is not needed, in which case, successive decoding at the destination suffices to achieve the capacity region of the Gaussian case of the channel to within a constant gap. Although for achieving constant-gap-to-capacity for the scalar Gaussian case, Marton's coding [6] is not needed even with m_3 included, it turns out that successive decoding is no longer sufficient and joint decoding needs to be used to achieve the capacity to within 1 b/s/Hz.

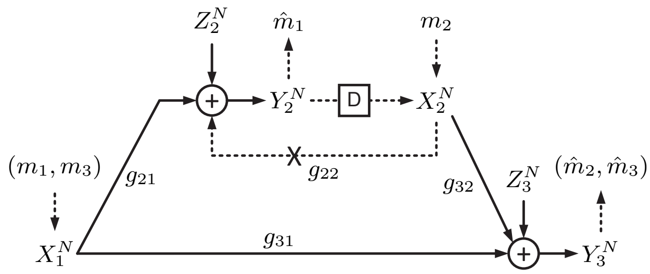

Fig. 2. Gaussian full-duplex relay broadcast channel with side message m_2. Here, the block "D" refers to a one-epoch delay.

Notation: $[1:N]$ is used to denote $\{1,2,\ldots,N\}$, $\mathsf{C}(x)$ the function $\log_2(1+x)$, $\mathbf{X}^N = (X_1, X_2, \ldots, X_N)$, $\mathbb{C}$ the set of complex numbers, and $\mathcal{CN}$ the complex Gaussian distribution.

II. Channel Model

The relay broadcast channel with side message consists of three nodes, as shown in Fig 2. Let $X_{in} \in \mathcal{X}_i$ be the transmitted signal of the node i and $Y_{jn} \in \mathcal{Y}_j$ be the received signal at node j, at time n, over the finite alphabet sets $(\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_2, \mathcal{Y}_3)$. The discrete memoryless version of the channel model is defined by the channel transition probability $p(y_{2n}, y_{3n}|x_{1n}, x_{2n})$. Over the N channel uses, node 1 wishes to send $m_1 \in [1:2^{NR_1}]$ to node 2, and to send $m_3 \in [1:2^{NR_3}]$ to node 3, while node 2 wishes to send $m_2 \in [1:2^{NR_2}]$ to node 3. The messages m_1 and m_3 are encoded at X_1^n. The message m_2 is encoded at X_2^n, and since the transmitter of X_{2n} and the receiver of Y_{2n} are co-located at node 2, the encoding of X_{2n} can depend on the past received signal $\mathbf{Y}_2^{n-1}$:

$$X_{1n} = \mathcal{E}_1(m_1, m_3, n) \text{ and } X_{2n} = \mathcal{E}_2(m_2, \mathbf{Y}_2^{n-1}, n). \quad (1)$$

After N channel uses, node 3 decodes (m_2, m_3) based on $\mathbf{Y}_3^N$. Because node 2 is both the uplink receiver and the downlink transmitter, it can make use of both $\mathbf{X}_2^N$ and $\mathbf{Y}_2^N$ in decoding m_1, i.e.,

$$\hat{m}_1 = \mathcal{D}_2(\mathbf{Y}_2^N, \mathbf{X}_2^N) \text{ and } (\hat{m}_2, \hat{m}_3) = \mathcal{D}_3(\mathbf{Y}_3^N). \quad (2)$$

A rate triple (R_1, R_2, R_3) is said to be achievable if there exists a set of deterministic functions $(\mathcal{E}_1, \mathcal{E}_2, \mathcal{D}_2, \mathcal{D}_3)$ such that the probability of error, $\Pr\{(\hat{m}_1, \hat{m}_2, \hat{m}_3) \neq (m_1, m_2, m_3)\}$, tends to zero as $N \to +\infty$.

The above discrete memoryless channel model can be specialized to the Gaussian case by letting $X_{in}, Y_{jn} \in \mathbb{C}$, and by imposing power constraints P_i on X_{in}, $i \in \{1,2\}$, i.e., $\sum_{n=1}^N |X_{in}|^2 \leq NP_i$. As illustrated in Fig. 2, we have

$$Y_{2n} = g_{21}X_{1n} + Z_{2n}, \quad (3)$$
$$Y_{3n} = g_{31}X_{1n} + g_{32}X_{2n} + Z_{3n}, \quad (4)$$

where $g_{ji} \in \mathbb{C}$ is the channel gain from node i to node j, and $Z_{jn} \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise at node j in the nth channel use. Note that due to the fact the relay (i.e., the BS) operates in a full-duplex mode, the self-interference at the relay has been removed implicitly.

III. Achievable Rate Region and Converse for the Discrete Memoryless Case

A. Achievability

We use the existing works [2], [3] on the relay broadcast channel as a starting point. The works [2], [3] propose to modify the classic Marton's coding [6] for the broadcast channel to the case where one receiver further helps the other receiver via a relay link. The channel model considered in this paper is a further generalization in which the extra side message m_2 is carried in this relay link. The coding strategy proposed below incorporates m_2 in Marton's coding.

The coding strategy of [2], [3] splits each message (i.e., m_1 and m_3) into the private and common parts which are dealt with differently. The common part is decoded by both node 2 and node 3; node 2 further acts as a relay to assist node 3 in decoding the common message. In contrast, the private parts are decoded only by the intended node through the broadcast channel without using node 2 as relay, so Marton's coding can be applied. This paper makes two modifications to this strategy in order to enable an extra transmission of m_2. First, we let X_2 be encoded based on both m_1 and m_2. Second, we let node 3 decode the original common and private message jointly with the new message m_2. The resulting achievable rate region is stated below.

Theorem 1: A rate triple (R_1, R_2, R_3) of the discrete memoryless relay broadcast channel with side message is achievable if it is in the convex hull of

$$R_1 \leq \pi_3, \quad (5a)$$
$$R_2 \leq \min\{\pi_5, \pi_2 + \pi_6 - \pi_1\}, \quad (5b)$$
$$R_1 + R_3 \leq \pi_3 + \pi_4 - \pi_1, \quad (5c)$$
$$R_2 + R_3 \leq \pi_7, \quad (5d)$$
$$R_1 + R_2 + R_3 \leq \min\{\pi_2 + \pi_7 - \pi_1, \pi_3 + \pi_6 - \pi_1\} \quad (5e)$$

for some $p(u)p(v, w_1, w_3, x_1|u)p(x_2|u)$ under the constraint that $\pi_1 \leq \pi_2 + \pi_4$, where

$$\pi_1 = I(W_1; W_3|U, V), \quad (6a)$$
$$\pi_2 = I(W_1; Y_2|U, V, X_2), \quad (6b)$$
$$\pi_3 = I(V, W_1; Y_2|U, X_2), \quad (6c)$$
$$\pi_4 = I(W_3; Y_3|U, V, X_2), \quad (6d)$$
$$\pi_5 = I(X_2; Y_3|U, V, W_3), \quad (6e)$$
$$\pi_6 = I(W_3, X_2; Y_3|U, V), \quad (6f)$$
$$\pi_7 = I(U, V, W_3, X_2; Y_3). \quad (6g)$$

Proof: Split m_i into the common-private message pair $(m_{i0}, m_{ii}) \in [1:2^{nR_{i0}}] \times [1:2^{nR_{ii}}]$ for $i \in \{1,3\}$. Introduce a total of T blocks for block-Markov coding. For each block $t \in [1:T]$, in an i.i.d. manner according to their respective distributions, generate a common codebook $\mathbf{u}^N(m_{10}, m_{30})$, a relay codebook $\mathbf{v}^N(m_{10}, m_{30}|m_{10}, m_{30})$, a private codebook $\mathbf{x}_2^N(m_2|m_{10}, m_{30})$, a binning codebook $(\mathbf{w}_1^N(\ell_{11}), \mathbf{w}_3^N(\ell_{33}))$ for $(\ell_{11}, \ell_{33}) \in [1:2^{NR'_{11}}] \times [1:2^{NR'_{11}}]$ where $R'_{ii} \geq R_{ii}$, with each ℓ_{ii} uniformly mapped to the bin of m_{ii}, i.e., $m_{ii} =$

$\mathcal{B}_i(\ell_{ii})$, and another private codebook $\mathbf{x}_1^N(\ell_{11},\ell_{33}|m_{10},m_{30})$.

In block t, node 1 finds $(\ell_{11}^t,\ell_{33}^t)$ such that $m_{ii}^t=\mathcal{B}_i(\ell_i^t)$ for $i\in\{1,3\}$ and that $(\mathbf{w}_1^N(\ell_{11}^t),\mathbf{w}_3^N(\ell_{33}^t))$ is *strongly typical*, then transmits $\mathbf{x}_1^N(\ell_{11}^t,\ell_{33}^t|m_{10}^{t-1},m_{30}^{t-1})$. This encoding is guaranteed to be successful provided that

$$R'_{11}+R'_{33}-R_{11}-R_{33}\ge I(W_1;W_3|U,V). \tag{7}$$

In block t, after obtaining $(\hat{m}_{10}^{t-1},\tilde{m}_{30}^{t-1})$ from the previous block $t-1$, node 2 transmits $\mathbf{x}_2^N(m_2^t|\hat{m}_{10}^{t-1},\tilde{m}_{30}^{t-1})$, and recovers $(\hat{m}_{10}^t,\tilde{m}_{30}^t)$ jointly from the received signal $\mathbf{y}_2^N$; this decoding is successful if

$$R'_{11}\le I(W_1;Y_2|U,V,X_2), \tag{8}$$

$$R_{10}+R_{30}+R'_{11}\le I(V,W_1;Y_2|U,X_2). \tag{9}$$

Node 3 decodes the blocks in a backward direction (unlike the sliding window decoding of [3]), i.e., block $t-1$ prior to block t. In block t, after obtaining $(\tilde{m}_{10}^t,\hat{m}_{30}^t)$ from the previous block $t+1$, node 1 recovers $(\tilde{m}_{10}^t,\hat{m}_{30}^t,\hat{m}_{33}^t,\hat{m}_2^t)$ jointly; the following conditions guarantee successful decoding:

$$R'_{33}\le I(W_3;Y_3|U,V,X_2), \tag{10}$$

$$R_2\le I(X_2;Y_3|U,V,W_3), \tag{11}$$

$$R'_{33}+R_2\le I(W_3,X_2;Y_3|U,V), \tag{12}$$

$$R_{10}+R_{30}+R'_{33}+R_2\le I(U,V,W_3,X_2;Y_3). \tag{13}$$

Combining (7)–(13) with $R_{11}\le R'_{11}$, $R_{33}\le R'_{33}$, $R_1=R_{10}+R_{11}$, $R_3=R_{30}+R_{33}$, and a nonnegative constraint on all the rate variables, and letting $T\to+\infty$, we establish the proposed inner bound, including the constraint $\pi_1\le\pi_2+\pi_4$, via the Fourier-Motzkin elimination. ■

In Theorem 1, the term π_1 is due to Marton's coding [6], reflecting the extent to which the encodings of the private messages m_{11} and m_{33} are coordinated. The following proposition further shows that the constraint $\pi_1\le\pi_2+\pi_4$ must be satisfied automatically if $p(u)p(v,w_1,w_3,x_1|u)p(x_2|u)$ is optimally chosen for maximizing the rate region (5).

Proposition 1: The achievable rate region of Theorem 1 remains the same if the constraint $\pi_1\le\pi_2+\pi_4$ is removed.

Proof: Let $\mathcal{A}_1$ be the achievable rate region of Theorem 1, and let $\mathcal{A}_2$ be the version without the constraint $\pi_1\le\pi_2+\pi_4$. Clearly, $\mathcal{A}_1\subseteq\mathcal{A}_2$, so it suffices to prove $\mathcal{A}_2\subseteq\mathcal{A}_1$. Consider some $p(u)p(v,w_1,w_3,x_1|u)p(x_2|u)$ such that $\pi_1>\pi_2+\pi_4$. Under this probability mass function, it can be shown that $\mathcal{A}_2\subseteq\mathcal{A}'_2$ where $\mathcal{A}'_2$ is

$$R_2\le\min\{\pi_5,I(X_2;Y_3|U,V)\}, \tag{14a}$$

$$R_1+R_3\le I(V;Y_2|U,X_2), \tag{14b}$$

$$R_1+R_2+R_3\le I(U,V,X_2;Y_3). \tag{14c}$$

In the meanwhile, $\mathcal{A}'_2$ can be attained by setting $W_1=\varnothing$ in Theorem 1. Thus, $\mathcal{A}_2\subseteq\mathcal{A}_1$. ■

Remark 1: Theorem 1 encompasses the following existing achievability results. It reduces to the inner bound of [2], [3] for the relay broadcast channel when $U=X_2$, and reduces to a decode-forward inner bound of [1] for the same channel when $W_1=W_3=\varnothing$.

B. Converse

The existing works [2], [3] on the relay broadcast channel use auxiliary "genie" variables to improve the cut-set bound. Similarly, with the aid of genie, [1] enhances the cut-set bound for the case with side message. As compared to [1], we provide two improvements. First, we further tighten the genie-aided bound by using more suitable auxiliary variables. Second, we propose a new upper bound on $R_1+R_2+R_3$ that improves the cut-set bound. Our converse is specified in the following.

Theorem 2: Any achievable rate triple (R_1,R_2,R_3) of the discrete memoryless relay broadcast channel with side message must be in the convex hull of

$$R_1\le I(U;Y_2|X_2), \tag{16a}$$

$$R_1\le I(X_1;Y_2,Y_3|V,X_2), \tag{16b}$$

$$R_2\le I(X_2;Y_3|X_1), \tag{16c}$$

$$R_3\le I(X_1;Y_2,Y_3|U,X_2), \tag{16d}$$

$$R_3\le I(V;Y_2,Y_3|X_2), \tag{16e}$$

$$R_1+R_3\le I(X_1;Y_2,Y_3|X_2), \tag{16f}$$

$$R_2+R_3\le I(X_1,X_2;Y_3), \tag{16g}$$

$$R_1+R_2+R_3\le I(X_1;Y_2,Y_3|X_2)+I(X_2;Y_3), \tag{16h}$$

for some $p(u,v,x_1,x_2)$.

Proof: Observe that (16c), (16f) and (16g) are directly from the cut-set bound. The rest of the bound except (16h) is based on the auxiliary variables U and V. The existing work [1] assumes a genie that provides $U_n=(\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1})$ and $V_n=M_3$ to node 1 and node 2. In contrast, by letting $U_n=(M_1,M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1})$ and $V_n=(M_2,M_3,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1})$, we propose a different genie that provides U_n to node 1 and node 3, and provides V_n to node 1 and node 2. This new use of genie yields a tighter outer bound.

Regarding (16h), the main idea is to relax both encoding and decoding. Considering node 2 and node 3 as two receivers, we follow Sato's approach in [7] and assume that they could fully coordinate in their decoding. Considering node 2 as the transmitter of m_2, we introduce a genie that provides feedback $\mathbf{Y}_3^{n-1}$ to it to improve encoding. The converse is then established by letting $N\to+\infty$. Specifically,

$$\begin{aligned}
&N(R_1+R_2+R_3-\epsilon_N)\\
&\le I(M_1;\mathbf{X}_2^N,\mathbf{Y}_2^N)+I(M_2;\mathbf{Y}_3^N)+I(M_3;\mathbf{Y}_3^N)\\
&\overset{(a)}{\le} I(M_1,M_3;\mathbf{Y}_2^N,\mathbf{Y}_3^N,\mathbf{X}_2^N|M_2)+I(M_2;\mathbf{Y}_3^N)\\
&\overset{(b)}{=} I(M_1,M_3;\mathbf{Y}_2^N,\mathbf{Y}_3^N|M_2)+I(M_2;\mathbf{Y}_3^N)\\
&\overset{(c)}{\le}\sum_{n=1}^N\Big[I(M_1,M_3;Y_{2n},Y_{3n}|M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1})\\
&\qquad+I(M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1};Y_{3n})\Big]\\
&\overset{(d)}{=}\sum_{n=1}^N\Big[I(M_1,M_3;Y_{2n},Y_{3n}|M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1},X_{2n})\\
&\qquad+I(M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1},X_{2n};Y_{3n})\Big]
\end{aligned}$$

$$R_1 \le \min\left\{\mathsf{C}\left(\frac{(1-\rho^2)|g_{21}|^2P_1}{\sigma^2+\alpha(1-\rho^2)|g_{21}|^2P_1}\right), \mathsf{C}\left(\frac{\beta(1-\rho^2)|g_{21}|^2P_1}{\sigma^2}\right)\right\}, \tag{15a}$$

$$R_2 \le \mathsf{C}\left(\frac{(1-\rho^2)|g_{32}|^2P_2}{\sigma^2}\right), \tag{15b}$$

$$R_3 \le \min\left\{\mathsf{C}\left(\frac{\alpha(1-\rho^2)(|g_{21}|^2+|g_{31}|^2)P_1}{\sigma^2}\right), \mathsf{C}\left(\frac{(1-\rho^2)(|g_{21}|^2+|g_{31}|^2)P_1}{\sigma^2+\beta(1-\rho^2)(|g_{21}|^2+|g_{31}|^2)P_1}\right)\right\}, \tag{15c}$$

$$R_1+R_3 \le \mathsf{C}\left(\frac{(1-\rho^2)(|g_{21}|^2+|g_{31}|^2)P_1}{\sigma^2}\right), \tag{15d}$$

$$R_2+R_3 \le \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+J\rho}{\sigma^2}\right), \tag{15e}$$

$$R_1+R_2+R_3 \le \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+J\rho}{\sigma^2}\right)+\mathsf{C}\left(\frac{(1-\rho^2)g_{21}^2P_1}{\sigma^2+(1-\rho^2)g_{31}^2P_1}\right), \text{where } J=2|g_{31}g_{32}|\sqrt{P_1P_2}. \tag{15f}$$

$$\begin{aligned}
&= \sum_{n=1}^{N}\Big[I(M_1,M_3;Y_{2n},Y_{3n}|M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1},X_{2n}) \\
&\quad + I(M_2,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1};Y_{3n}|X_{2n})+I(X_{2n};Y_{3n})\Big] \\
&\le \sum_{n=1}^{N}\Big[I(M_1,M_2,M_3,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1};Y_{2n},Y_{3n}|X_{2n}) \\
&\quad + I(X_{2n};Y_{3n})\Big] \\
&\overset{(e)}{=} \sum_{n=1}^{N}\Big[I(X_{1n};Y_{2n},Y_{3n}|X_{2n})+I(X_{2n};Y_{3n})\Big] \\
&\le NI(X_1;Y_2,Y_3|X_2)+NI(X_2;Y_3)
\end{aligned} \tag{17}$$

where (a) follows by letting node 1 and node 3 fully coordinate, both (b) and (d) follow as X_{2n} is a function of $(M_2,\mathbf{Y}_2^{n-1})$, (c) introduces a genie that provides the past $\mathbf{Y}_3^{n-1}$ to the encoder of X_{2n}, and (e) follows since $(M_1,M_2,M_3,\mathbf{Y}_2^{n-1},\mathbf{Y}_3^{n-1}) \to X_{1n} \to (Y_{2n},Y_{3n})$ form a Markov chain given X_{2n}. ■

Remark 2: As compared to the converse in [1], the converse of Theorem 2 has extra inequalities (16a), (16d) and (16h). We remark that the sum-rate bound (16h) is new; it is crucial for proving the approximate capacity result for the Gaussian case as shown in the next section.

IV. Capacity to Within Constant Gap for the Gaussian Case

We now characterize the capacity region of the Gaussian relay broadcast channel with side message to within one bit.

A. Achievability and Converse

First, we specialize the converse of Theorem 2 to the Gaussian case. The following outer bound is an evaluation of (16), but the evaluation relies on the entropy power inequality and is nontrivial. We omit the detailed proof here.

Proposition 2: Any achievable rate triple (R_1,R_2,R_3) of the Gaussian relay broadcast channel with side message is in the convex hull of (15), which is displayed at the top of the page, for some parameters $0 \le \alpha,\beta,\rho \le 1$.

For achievability, instead of evaluating the full mutual information bounds of Theorem 1, we propose two simpler schemes, corresponding to rate splitting of either m_1 or m_3, that turn out to be sufficient for proving the constant-gap result.

Proposition 3 (D2D Message Rate Splitting): A rate triple (R_1,R_2,R_3) of the Gaussian relay broadcast channel with side message is achievable if it is in the convex hull of

$$R_1 \le \mathsf{C}\left(\frac{b|g_{21}|^2P_1}{\sigma^2+c|g_{21}|^2P_1}\right), \tag{18a}$$

$$R_2 \le \mathsf{C}\left(\frac{e|g_{32}|^2P_2}{\sigma^2}\right), \tag{18b}$$

$$R_1+R_3 \le \mathsf{C}\left(\frac{b|g_{21}|^2P_1}{\sigma^2+c|g_{21}|^2P_1}\right)+\mathsf{C}\left(\frac{c|g_{31}|^2P_1}{\sigma^2}\right), \tag{18c}$$

$$R_1+R_2+R_3 \le \mathsf{C}\left(\frac{b|g_{21}|^2P_1}{\sigma^2+c|g_{21}|^2P_1}\right)+ \mathsf{C}\left(\frac{c|g_{31}|^2P_1+e|g_{32}|^2P_2}{\sigma^2}\right), \tag{18d}$$

$$R_1+R_2+R_3 \le \mathsf{C}\left(\frac{|g_{31}|^2P_1+|g_{32}|^2P_2+J\sqrt{ad}}{\sigma^2}\right), \tag{18e}$$

for some $a,b,c,d,e \ge 0$ with $a+b+c=1$ and $d+e=1$.

Proof: Splitting only m_3 into (m_{30},m_{33}), we treat (m_1,m_{30}) as the common part to be decoded at both node 2 and node 3. The codebooks $\mathbf{w}_2^N(m_2)$, $\mathbf{w}_3^N(m_{33})$, $\check{\mathbf{v}}^N(m_1,m_{30})$, and $\mathbf{u}^N(m_1,m_{30})$ are generated randomly and independently according to $\mathcal{CN}(0,1)$. In block $t \in [1:T]$, node 1 transmits

$$\mathbf{x}_1^N(t) = \mathbf{v}^N(t)+\sqrt{cP_1}\mathbf{w}_3^N(m_{33}^t) \tag{19}$$

where

$$\mathbf{v}^N(t) = \sqrt{aP_1}\mathbf{u}^N(m_1^{t-1},m_{30}^{t-1})+\sqrt{bP_1}\check{\mathbf{v}}^N(m_1^t,m_{30}^t). \tag{20}$$

In block t, with $(\hat{m}_1^{t-1},\tilde{m}_{30}^{t-1})$ obtained from the previous block $t-1$, node 2 transmits

$$\mathbf{x}_2^N(t) = \sqrt{dP_2}\mathbf{u}^N(\hat{m}_1^{t-1},\tilde{m}_{30}^{t-1})+\sqrt{eP_2}\mathbf{w}_2^N(m_2^t). \tag{21}$$

Using the decoding strategy of Theorem 1 establishes the proposed achievability result. ■

Alternatively, we can split m_1 to obtain the following inner bound. Full proof is omitted here.

Proposition 4 (Uplink Rate Splitting): A rate triple (R_1, R_2, R_3) of the Gaussian relay broadcast channel with side message is achievable if it is in the convex hull of

$$R_2 \le \mathsf{C}\left(\frac{e|g_{32}|^2 P_2}{\sigma^2 + c|g_{31}|^2 P_1}\right), \tag{22a}$$

$$R_1 + R_3 \le \mathsf{C}\left(\frac{(b+c)|g_{21}|^2 P_1}{\sigma^2}\right), \tag{22b}$$

$$R_2 + R_3 \le \mathsf{C}\left(\frac{(a+b)|g_{31}|^2 P_1 + |g_{32}|^2 P_2 + J\sqrt{ad}}{\sigma^2 + c|g_{31}|^2 P_1}\right), \tag{22c}$$

$$R_1 + R_2 + R_3 \le \mathsf{C}\left(\frac{c|g_{21}|^2 P_1}{\sigma^2}\right) + \mathsf{C}\left(\frac{(a+b)|g_{31}|^2 P_1 + |g_{32}|^2 P_2 + J\sqrt{ad}}{\sigma^2 + c|g_{31}|^2 P_1}\right). \tag{22d}$$

for some $a, b, c, d, e \ge 0$ with $a + b + c = 1$ and $d + e = 1$.

B. Constant Gap Optimality

We now have two achievable rate regions based on two different rate-splitting strategies. Suppose that the source-to-destination channel g_{31} is much stronger than the uplink channel g_{21}, we would let node 3 decode the entire m_1 for interference cancellation, so m_1 ought not to be split in this situation. Likewise, we would not split m_3 if g_{21} is much stronger. Hence, we propose to apply the side-message rate splitting strategy if $|g_{31}| \ge |g_{21}|$, and the uplink rate splitting strategy otherwise. This approach turns out to be *near-optimal* as stated in the following main result of this paper.

Theorem 3: For the Gaussian relay broadcast channel with side message, the outer bound of Proposition 2 is at most 1 b/s/Hz from the inner bound of Proposition 3 if $|g_{31}| \ge |g_{21}|$, and at most 1 b/s/Hz from the inner bound of Proposition 4 if $|g_{31}| < |g_{21}|$. Thus, the achievability result of Theorem 1 and the converse result of Theorem 2 are within 1 b/s/Hz from each other for the Gaussian case. This constant gap result carries over to the Gaussian relay broadcast channel of [2], [3].

Proof: We use the power splitting strategy of [8] to set $a = d = 0$, $b = 1 - c$, and $e = 1$, but to set c differently for the two strategies. We choose $c = \min\{1, |g_{21}|^2 P_1/\sigma^2\}$ in Proposition 3, and choose $c = \min\{1, |g_{31}|^2 P_1/\sigma^2\}$ in Proposition 4. The gap is established after some algebra. ■

Remark 3: To split rate differently depending on the channel condition is crucial in the above result; using either of the two strategies alone does give us a bounded gap.

Finally, we give a numerical example to illustrate the performance of the proposed scheme. Let $\frac{|g_{21}|^2 P_1}{\sigma^2} = \frac{|g_{32}|^2 P_2}{\sigma^2} = \mathsf{SNR}$ and $\frac{|g_{31}|^2 P_1}{\sigma^2} = \mathsf{INR}$, we compute the symmetric generalized degree-of-freedom (GDoF) defined as $d_{\text{sym}} = \lim_{\mathsf{SNR}\to+\infty} \frac{\min_i\{R_i\}}{\log \mathsf{SNR}}$ for the different $\alpha = \frac{\log \mathsf{INR}}{\log \mathsf{SNR}}$. Note that when $|g_{31}| \ge |g_{21}|$, we use the rate-splitting strategy of Proposition 3; when $|g_{31}| < |g_{21}|$, we use the strategy of Proposition 4. In addition to the decode-and-forward scheme of [1], we also plot the following benchmarks: (i) treating-interference-as-noise; (ii) rate splitting; (iii) half-duplex, all with a separate frequency band for side message, and with the portionalities of the frequency band optimally determined by the exhaustive search. Fig. 3 shows that the proposed scheme outperforms all other scheme in terms of GDoF.

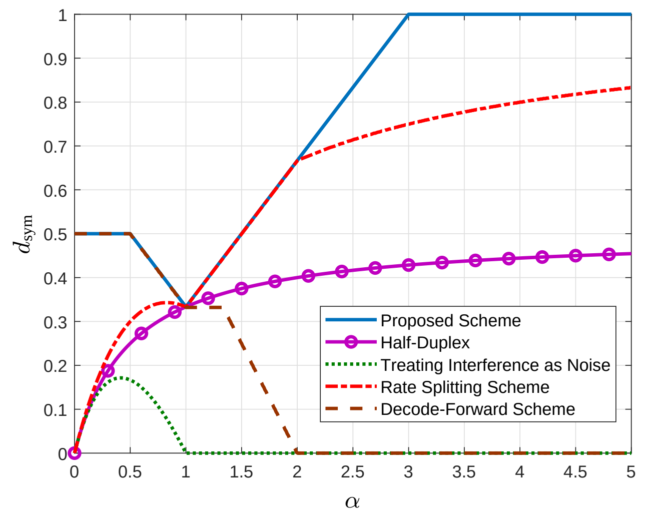

Fig. 3. Symmetric GDoF d_{sym} vs. $\alpha = \frac{\log \mathsf{INR}}{\log \mathsf{SNR}}$ for the Gaussian relay broadcast channel with side message.

V. Conclusion

The paper models the full-duplex wireless cellular network with D2D link as a (partially cooperative) relay broadcast channel with side message. We provide novel achievability and converse results and prove its capacity to within 1 b/s/Hz for the Gaussian case.

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