DEEP LEARNING FOR ROBUST POWER CONTROL FOR WIRELESS NETWORKS

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ABSTRACT

Robust optimization is an important task in wireless communications, because due to fading and feedback delay there is inherent uncertainty in channel state information in a wireless environment. This paper aims to show that a deep learning approach for network utility maximization can produce more robust solutions than the traditional model-based approach. We focus on the classic power control problem for sum-rate maximization in a wireless network with multiple interfering links. By injecting samples of random channel realizations into the unsupervised training process, the neural network is able to learn to adapt to the uncertain channel environment.

***Index Terms*—** Robust optimization, deep learning, wireless communications, power control

1. INTRODUCTION

Traditional network utility maximization typically involves first obtaining network parameters such as the channel state information (CSI), then formulating an network utility objective as a function of the network parameters, and finally optimizing the objective function assuming the fixed parameters. While such an approach simplifies the algorithm design, this *deterministic optimization* framework may not always produce competitive solutions in realistic situations, because it inherently ignores the channel uncertainties, which can significantly affect the optimal solution, especially in a wireless environment. The classical approach for dealing with channel uncertainty is *robust optimization*, which incorporates statistical models of channel uncertainty into the optimization process. Although generalizing towards the reality better than deterministic optimization, the robust optimization approach still relies on the mathematical models of parameter uncertainties, which are often ad-hoc. Further, the parameters of these models are often difficult to estimate. Finally, even if the model and its parameters are known exactly, the resulting optimization problem is often not easy to solve.

This paper leverages a growing literature [1, 2] that show deep neural networks can be an effective tool for network utility maximization, even in situation where CSI is not explicitly known [3], but goes one step further in providing evidence that channel uncertainty can be naturally incorporated into the learning process to produce *robust* solutions. Toward this end, we observe that instead of relying on the statistical models of uncertainty, it is much more practical to obtain samples of uncertain channel realizations, which can be directly injected into the training process using a novel training approach. The main goal of this paper is to provide evidence that being a universal and highly flexible function approximator, a neural network trained under these channel samples can implicitly infer the model uncertainty and produce optimized solutions that are robust against channel variations.

To illustrate the effectiveness of the proposed framework, we focus on the problem of power control for wireless device-to-device (D2D) networks under a robust *sum-rate* maximization objective. The sources of parameter uncertainties are the variations in wireless channels due to shadowing and fast-fading. For this problem, traditional deterministic optimization approaches are all based on non-convex optimization, e.g., [4, 5, 6, 7, 8, 9, 10, 11, 12]. To the best of our knowledge, robust version of this sum-rate optimization has not been formulated in prior literature, but our problem formulation is related to similar scenarios in the cognitive radio network for which optimization under either the ellipsoidal model of channel variations [13] or the probabilistic models [14, 15] based on channel statistics have been considered. In this and other contexts [15, 16], the outage capacity is often used to define the robust rate. This paper also adopts this outage-base notion of robustness in the robust optimization formulation.

As mentioned earlier, machine learning has been used in prior work for power optimization. Although many of these works assume perfect CSI, e.g., [1, 2], numerical evidence already exists that shows neural networks can produce highly generalizable solutions, e.g., as illustrated in [3] where geographical locations (thus only the path-loss instead of the exact CSI) are used as the input to produce competitive solutions. The existing work, however, does not explicit account for the variations of the channels in the training of the neural network. The main point of this paper is that the desired generalizability can be significantly enhanced if samples of the channels are used in addition in the training process.

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2. ROBUST SUM-RATE MAXIMIZATION

2.1. Achievable Rates

Consider a wireless network with N independent D2D links in a two-dimensional region. We use p_i for the maximum power at the ith transmitter; $g_{ij} \in \mathbb{R}$ for the channel from the jth transmitter to the ith receiver; and σ^2 for the background noise power level. We assume full frequency reuse among all the links over bandwidth W. The set of optimization variables for power control is $\{x_i\}_{i\in[1\dots N]}$, where $x_i \in [0,1]$ denotes the proportion of p_i the ith transmitter should transmit for achieving the highest global utility. Given $\{x_i\}$, the achievable rate for the ith link is computed as

$$R_i = W \log\left(1 + \frac{g_{ii}p_i x_i}{\Gamma(\sum_{j\neq i} g_{ij}p_j x_j + \sigma^2)}\right), \qquad (1)$$

where Γ is the SNR gap to the information theoretical channel capacity due to practical coding and modulation for the linear Gaussian channel [17].

2.2. Robust Power Control based on Path-Losses

Widely accepted wireless channel models typically include three components: path-loss, shadowing, and small-scale fading. Among them, the path-loss is the most stable and accurately measurable, especially in environments with direct line-of-sight paths (e.g. rural areas), while shadowing and fast fading can vary over time in faster timescales. For this reason, this paper assumes that a central controller has access to only the path-loss component of the channels, and seeks to find *robust* power allocations that work well over different realizations of the shadowing and fast fading components.

2.3. Maximization of Robust Sum Rate

Under a fixed power allocation, the statistical variations of the channel result in varying achievable rates of each link. Since the controller only has access to the path-loss information, the actual transmission rate can only be a function of the path-loss. In other words, we must accept a certain probability of outage. This leads to the notion of outage capacity, defined to be the maximum rates $\{\hat{R}_i\}$ such that

$$\Pr[R_i < \hat{R}_i] \leq \gamma, \ \forall i \qquad (2)$$

where γ is the fixed outage probability, for example, at 5%.

In this work, we formulate the robust network utility maximization problem as that of maximizing a utility function of the outage capacities of multiple links in the network as defined above. Specifically, if we take a sum-rate maximization formulation, then the problem becomes that of maximizing the sum of the γ-percentile rates of all the links, which we refer to as the *robust sum-rate maximization* problem

$$\begin{aligned} \underset{\mathbf{x}}{\text{maximize}} \quad & \sum_{i=1}^{N} \hat{R}_i & \text{(3a)} \\ \text{subject to} \quad & \Pr[R_i < \hat{R}_i] \leq \gamma, \ \forall i & \text{(3b)} \\ & 0 \leq x_i \leq 1, \ \forall i. & \text{(3c)} \end{aligned}$$

where $\hat{R}_i$ serves as the target rate for the ith link, with acceptable outage probability set under γ. The corresponding solution enjoys a notion of robustness, under which the failure probability of each individual link is less than γ.

Note that one may be tempted to define an alternative notion of robust sum rate as the sum rate $\hat{R}_{\text{sum}}$ at which

$$\Pr\left[\sum_i R_i < \hat{R}_{\text{sum}}\right] \leq \gamma. \qquad (4)$$

This would not have been the correct definition unless coding across the links is possible, which is not the case for the D2D network with independent links.

We also remark that the sum-rate objective does not necessarily provide fairness across users. To ensure fairness, the sum-rate objective function in (3) may be replaced by a different utility function, e.g., the minimum rate across all users.

3. DEEP LEARNING BASED ROBUST OPTIMIZATION

Although the fast-fading realizations of the channels are not known for the real-time operation of power control, it is often possible to obtain or to generate samples of these channels for offline use. Traditional robust optimization relies on building statistical models of the channel uncertainty then performing robust optimization based on these models. In this paper, we propose an approach of using a neural network to solve (3). The neural network takes only the path-loss values as inputs, but during training we inject samples of uncertain channels in addition and use unsupervised learning to adjust the neural network weights for robust sum-rate maximization. During testing, the neural network performs power control as function of the path-loss values and computes solutions that are robust to unseen uncertain channel realizations.

We emphasize that our method of training with uncertainty realizations injection is different from the idea of data augmentation, in which various transformations or noises are applied to the training data before feeding to the model. In contrast, our proposed strategy uses injected uncertain channel realizations to compute an alternative objective (i.e. the robust sum rate, instead of the nominal sum rate based on the path loss inputs), and sequentially optimize our neural network for this objective. This allows the neural network to learn the underlying uncertainty distributions to achieve robustness, which is impossible to do under data augmentation.

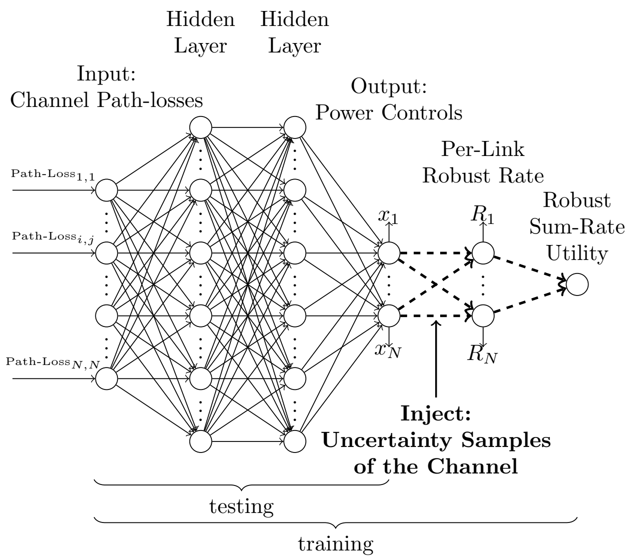

Fig. 1. Neural Network Architecture

3.1. Neural Network Architecture

To explore the full potential of deep learning, we design the neural network based on the most general architecture: the fully connected neural networks, in which the path-loss values of the links are mapped to the power allocations $\{x_i\}$ via fully connected layers.

The neural network is trained in an unsupervised fashion in order to allow for training with channel uncertainties injection. Unsupervised training on sum-rate maximization has been explored in [3] and [2]. During training, the CSI values are used for computing the sum-rate. Through computing gradient ascent on the robust sum-rate, which is a function of neural network parameters, the neural network parameters are updated in the direction of improving the robust objective.

The overall neural network structure is shown in Fig. 1. During training, the computation flows all the way towards the end for the sum-rate computation; while during testing or for applications, the computation stops at the power control outputs, without needing the CSI values.

3.2. Training for Robustness via Uncertainty Injection

We generate a large number of samples of channel realizations and inject these channel uncertainty to the neural network in the training process. More specifically, the fixed path-loss components are used as the input to the neural network, while the varying shadowing and fast-fading components are used to generate the instantaneous achievable rates. During training, with the same path-loss inputs, the randomly generated channel realizations are used to compute the individual link rates under these realizations. Through this sampling and computing process, we obtain empirical distributions of the individual rate of each link over the channel realizations for this wireless network. Note that this empirical distribution depends on not only the variation of the direct channel of each link, but also that of all the interfering links. Based on these distributions, we can then find an empirical approximation of the robust sum-rate objective, then subsequently train the neural network to optimize this robust objective.

Denote the collective neural network parameters as W. Both the power control outputs and the robust sum-rates are differentiable functions on the model parameters W. The neural network can be optimized with stochastic gradient descent during uncertainty injection training.

To compute the gradients, we make the following crucial observation: Let $\{g_i\}$ be the set of CSI (among the many injected channel realizations) that corresponds to the outage rate R_i^γ at the γ-percentile point of the empirical distribution under the current power control strategy. The gradient of the robust sum-rate with respect to the neural network parameters W can be obtained by computing the contribution from each link $\frac{\partial R_i^\gamma}{\partial W}$ and with $\{g_i\}$ fixed as the above constant. This is because the CSI at the γ-percentile outage point is the only relevant channel parameter for the robust rate of each link. More specifically, the gradient of the robust sum-rate for the entire wireless network, $\sum_{i=1}^N R_i^\gamma$, is just $\sum_{i=1}^N \frac{\partial R_i^\gamma}{\partial W}$, where different sets of $\{g_i\}$'s are involved in different links. The detailed expressions for these gradients depend on the neural network structure, and are readily computed by any deep learning frameworks such as Tensorflow[18].

4. EXPERIMENTAL VALIDATION

4.1. Wireless Environment

We consider a number of D2D links randomly deployed within a region, with the transceiver distances following uniform distributions. We impose a minimum of 0.5-meter distance between any transmitter and any receiver. For the path-loss, we follow the short-range outdoor model ITU-1411, with 5MHz bandwidth at the carrier frequency of 2.4GHz. The antennas are set with 2.5dBi antenna gain, located at 1.5-meter height. We assume the maximum transmit power at 40dBm, background noise at -169dBm/Hz, and SNR gap of 6dB. We assume that 5% outage probability is tolerable for all D2D links. The robust sum-rate is then the summation of the 5-percentile rate that each link achieves over different channel realizations. We incorporate the following channel uncertainty models for each direct and interferring links.

- Shadowing with log-normal distribution with 8dB standard deviation.
- Rayleigh fading with i.i.d complex circular symmetric Gaussian distribution with unit variance.

We sample 500 channel realizations during testing to obtain an empirical approximation of the per-link robust rate.

Table 1. Wireless Environment

Setting	Number of Links	Region Area (m^2)	Direct-Link Distance Distribution
A	20	1500×1500	10m$\sim$40m
B	20	2000×2000	5m$\sim$70m

Table 2. Robust Sum-Rate Performance

	A	B
FP	104.2Mbps	149.6Mbps
Deep Learning **without** Uncertainties Injection	112.9Mbps	155.2Mbps
Deep Learning **with** Uncertainties Injection	**127.7Mbps**	**188.1Mbps**
Percentage Improvement	13%	21%

4.2. Neural Network Training and Testing

Following the input layer of path-loss values, the proposed neural network has three fully connected hidden layers, each with $4N^2$ neurons with *ReLu* non-linearity (where N is the number of links). The output layer has N units, each with *sigmoid* non-linearity, corresponding to $\{x_i\}$ in (3).

Because the path-losses of the direct and interferring links have a large range, it is often difficult to obtain meaningful updates at the beginning of training due to numerical issues. To make training effective, we adopt *input normalization* on the input path-loss values (for training and testing), where each of the N^2 inputs of path-loss values is normalized independently with statistics computed from the entire training set.

We present test results on the robust sum-rate obtained with only path-loss values as the inputs to the neural network. We include two highly competitive benchmarks:

- **Deep Learning without Uncertainties Injection:** We train a neural network with the identical architecture and dataset, without injecting uncertainty realizations.
- **Fractional Programming (FP):** We run the state-of-art power control algorithm FPLinQ[5] for 100 iterations based on the path-loss values alone.

To validate the generalization ability of our model under various link density level (correlated to the interference level) and transceiver distance distribution, we test with two different settings, over 1000 wireless networks under each setting, as in Table 1. Numerical results are shown in Table 2. Fig. 2 presents CDF curves of the robust sum-rates over all 1000 testing wireless networks, generated under test setting B.

4.3. Result and Analysis

For various interference levels and transceiver distance distributions, the proposed neural network consistently produces more robust solutions. The performance gain due to the proposed uncertainty injection training ranges from **13%-21%**. By examining the 5%-outage rate, it is clear that the power control strategy has learned to more heavily utilize the stronger links and to de-emphasize the weaker links. The effect can be visualized in the rate profiles in Fig. 3, generated under test setting B. This strategy makes sense, because by giving up the weaker links, it reduces the number of non-zero interference terms in (1). These terms are subject to channel uncertainty fluctuations and can be detrimental to the sum-rate under unfavorable channel conditions.

As an additional benefit for giving up weaker links, our model achieves the robust performance while allocating much less power (only **39.55%** in Setting A and **48.79%** in Setting B of FPLinQ's average allocated power at testing), thus being power-efficient in the meantime.

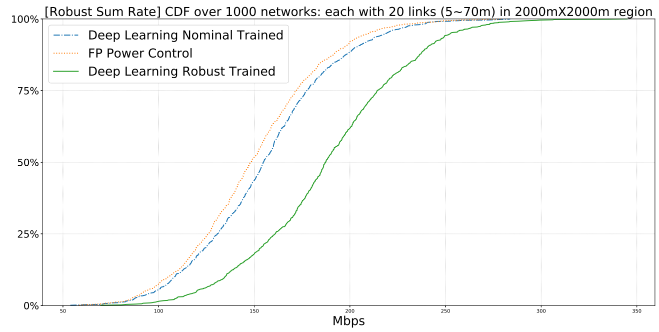

Fig. 2. Cumulative distribution of robust sum-rates.

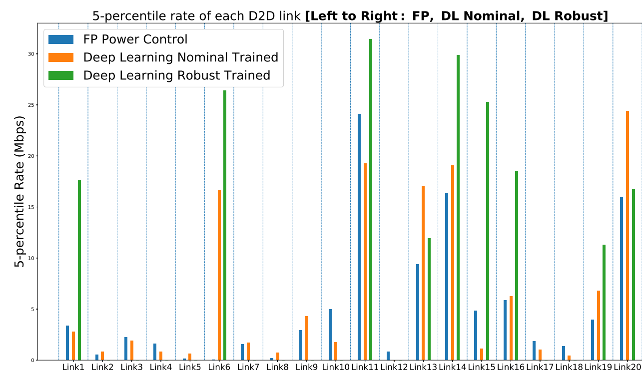

Fig. 3. Robust rate for each link in a wireless network.

5. CONCLUSIONS

Traditional robust optimization requires detailed mathematical characterizations of parameter uncertainties, which are often difficult to obtain for realistic settings. This paper proposes a novel robust optimization approach via deep learning that requires only samples of uncertainty realizations. We show that the proposed neural network can implicitly learn the channel uncertainty and achieve a better performance.

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