

# Machine Learning for Massive MIMO Communications

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joint work with

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July 2020

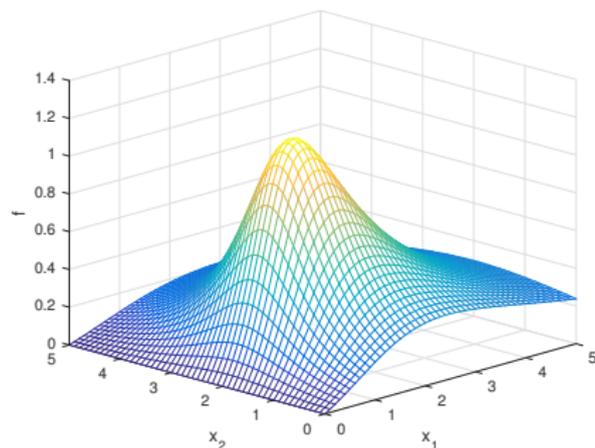


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# Why is Machine Learning so Powerful?

- **Universal** functional mapping – either by supervised or reinforcement learning
- Incorporating **vast** amount of data over **poorly defined** problems
- **Highly parallel** implementation architecture

- Mathematical optimization requires **highly structured** models over **well defined** problems.
- Finding solution efficiently relies on specific and often **convex** optimization landscape.



- Traditional approach for communication engineering is to **model-then-optimize**.
- Machine learning approach allows us to be **data driven** thereby skipping models altogether!

- Traditionally, communication engineers have invested heavily on channel models.
  - However, models are inherently only an **approximation** of the reality;
  - Moreover, model parameters need to be estimated – with inherent **estimation error**.
- Machine learning approach allows us to skip channel modeling altogether!
  - **End-to-end communication system design**
  - **Implicitly accounting for channel uncertainty**
- This talk will provide two examples in massive MIMO design for mmWave communications
  - Multiuser channel estimation and feedback for FDD massive MIMO
  - Constellation design for symbol-level precoding in TDD massive MIMO

- **Motivation:** mmWave massive MIMO for enhanced mobile broadband in the downlink.
- **Key problem:** How to obtain channel state information (CSI)?
- **Time-Division Duplex (TDD) Massive MIMO:**
  - Channel reciprocity can be assumed.
  - Uplink pilot transmission followed by CSI estimation at BS and downlink transmission.
- **Frequency-Division Duplex (FDD) Massive MIMO:**
  - Channel reciprocity does not necessarily hold in different frequencies
  - Downlink pilot transmission followed by CSI estimation and feedback at the users.

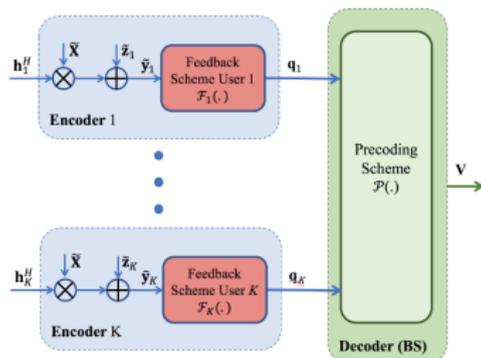
## Part I

# Channel Estimation and Feedback for FDD Massive MIMO

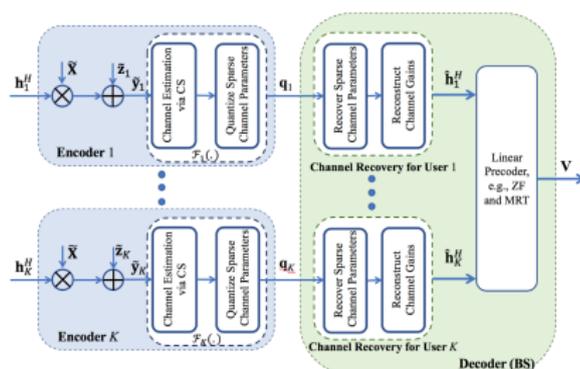
Conventional downlink FDD wireless system design involves:

- Independent **channel estimation** at each UE based on downlink pilot.
- Independent **quantization** and **feedback** of each user's channel to the BS.
- Multiuser **precoding** at the BS based on channel feedback from ALL the users.

**Key Observation:** Single-user channel feedback for multiuser precoding is NOT optimal.



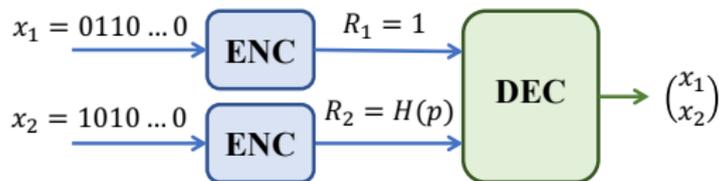
FDD downlink precoding as a DSC problem.



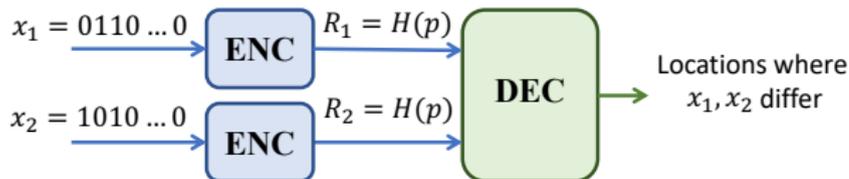
The conventional scheme amounts to a separate source coding strategy.

- The FDD feedback/precoding problem is a distributed source coding (DSC) problem.
- Much more efficient distributed feedback scheme can be designed.

- The information theoretic study of distributed source coding originated in the 1970's.
- Recovering correlated sources with separate encoders and joint decoder:
  - [Slepian and Wolf, 1973] shows that optimal lossless DSC of correlated sources can be much more efficient than independent encoding/decoding.
  - Example:  $x_1, x_2 \in \text{Ber}(0.5)$  but differing with probability  $p$  in each position.



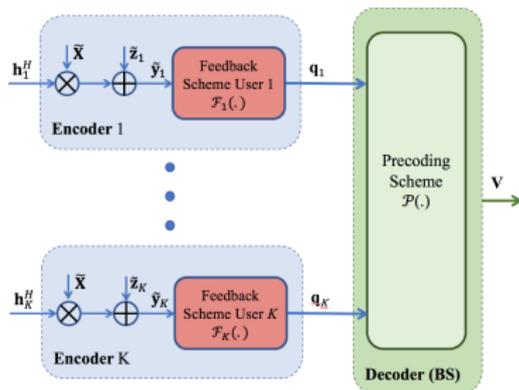
- [Wyner and Ziv, 1976] extends the results to lossy compression.
- Computing a function of multiple sources:
  - [Korner and Marton, 1979] shows how to compute mod-2 sum of two correlated sequences.



- [Nazer and Gastpar, 2007] shows DSC has benefit even when the sources are independent.

# Channel Estimation and Feedback as Distributed Source Coding

- We recognize that the end-to-end design of a downlink FDD precoding system can be regarded as a DSC problem of computing a function (the downlink precoding matrix) of independent sources (channels) under finite feedback rate constraints.



- The design of the optimal DSC strategy is, however, a difficult problem in general.
  - Statistics of the source needs to be known.
  - Optimal distributed source coding method needs to be designed.
- **This motivates us to propose a deep-learning methodology to jointly design:** (i) the pilot; (ii) a deep neural network (DNN) at each UE for channel feedback, and (iii) a DNN at the BS for precoding to achieve much better performance without explicitly channel estimation.

## Why is deep learning well suited to tackle the DSC design problem?

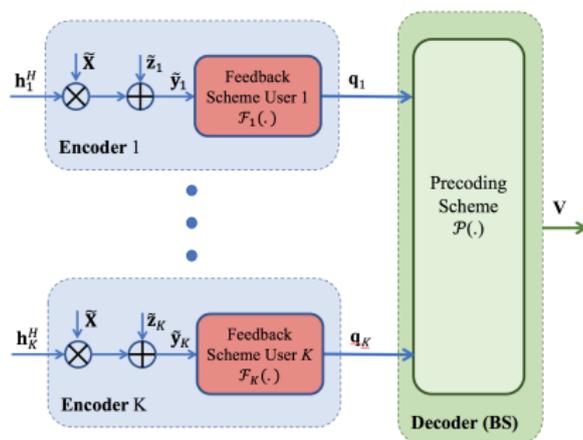
- Different from the conventional design methodology, deep learning can jointly design all the components for end-to-end performance optimization.
- Deep learning implicitly learns the channel distributions in a data-driven fashion without requiring tractable mathematical channel models.
- Computation using trained DNN can be highly parallelized, so that the computational burden of DNN is manageable.

## Some recent work on the use of DNNs for FDD system design:

- Single-user scenario with no interference:
  - [Wen, Shih, and Jin, 2018] and [Jang, Lee, Hwang, Ren, and Lee, 2020].
- Channel reconstruction at the BS under perfect CSI assumption:
  - [Lu, Xu, Shen, Zhu, and Wang, 2019] and [Guo, Yang, Wen, Jin, and Li, 2020].

## This work:

- Considers the **multiuser case** and take the **channel estimation process** into account.
- Provides end-to-end training, including pilot design, channel estimation process and precoder design, to **directly maximize the system throughput**.



- $K$ -user FDD downlink precoding system involves two phases:

- 1 Downlink training and Uplink feedback phase:

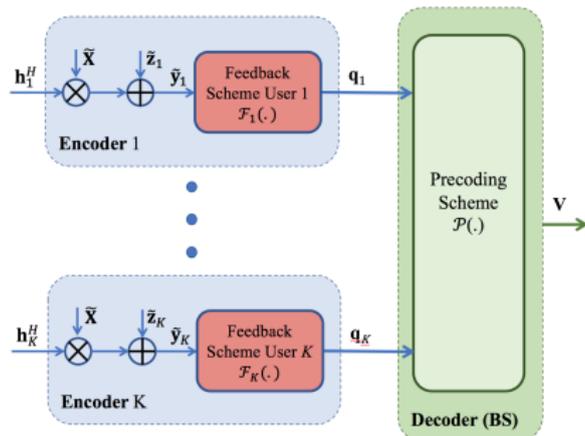
$$\tilde{y}_k = h_k^H \tilde{X} + \tilde{z}_k, \quad \blacktriangleright \text{BS broadcasts } L \text{ downlink pilots.}$$

$$q_k = \mathcal{F}_k(\tilde{y}_k), \quad \blacktriangleright \text{Each user feedbacks } B \text{ bits.}$$

- 2 Downlink precoding for data transmission:

$$V = \mathcal{P}(q_1, \dots, q_K), \quad \blacktriangleright \text{BS maps } KB \text{ bits to precoder on } M \text{ antennas.}$$

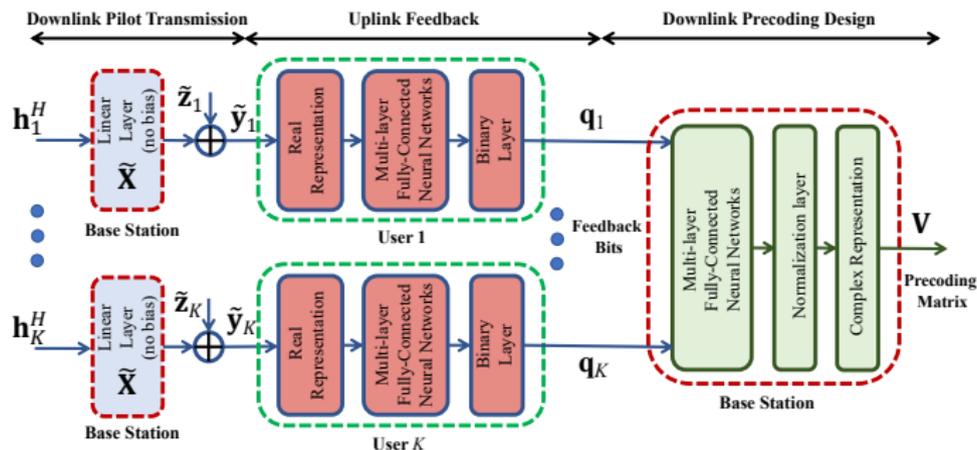
- **Goal:** Designing **training pilots**, **feedback scheme at the users**, and **precoding scheme at the BS** to maximize throughput.



- Problem of Interest: Sum rate maximization problem under power constraint  $P$ :

$$\begin{aligned}
 & \underset{\tilde{\mathbf{X}}, \{\mathcal{F}_k(\cdot)\}_{k=1}^K, \mathcal{P}(\cdot)}{\text{maximize}} && \sum_{k=1}^K \log_2 \left( 1 + \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma^2} \right) \\
 & \text{subject to} && \mathbf{V} = \mathcal{P} \left( \mathcal{F}_1(\mathbf{h}_1^H \tilde{\mathbf{X}} + \tilde{\mathbf{z}}_1), \dots, \mathcal{F}_K(\mathbf{h}_K^H \tilde{\mathbf{X}} + \tilde{\mathbf{z}}_K) \right), \\
 & && \text{Tr}(\mathbf{V}\mathbf{V}^H) \leq P, \\
 & && \|\tilde{\mathbf{x}}_\ell\|^2 \leq P,
 \end{aligned}$$

# Proposed DNN Architecture



- **Downlink Pilot Transmission:** Modelled by a linear neural layer followed by additive noise.
- **Uplink Feedback:** Modelled by an  $R$ -layer DNN with  $B$  binary activation neurons at the last layer:  $q_k = \text{sgn} \left( W_R^{(k)} \sigma_{R-1} \left( \cdots \sigma_1 \left( W_1^{(k)} \tilde{y}_k + b_1^{(k)} \right) \cdots \right) + b_R^{(k)} \right)$ .
- **Downlink Precoding Design:** Modelled by a  $T$ -layer DNN with normalization activation function at the last layer:  $v = \tilde{\sigma}_T \left( \tilde{W}_T \tilde{\sigma}_{T-1} \left( \cdots \tilde{\sigma}_1 \left( \tilde{W}_1 q + \tilde{b}_1 \right) + \cdots \right) + \tilde{b}_T \right)$ .
- Sum rate maximization can be cast as the following learning problem:

$$\tilde{\mathcal{X}}, \left\{ \Theta_R^{(k)} \right\}, \Theta_T \max \mathbb{E}_{\mathbf{h}, \tilde{\mathbf{z}}} \left[ \sum_k \log_2 \left( 1 + \frac{|\mathbf{h}_k^H \mathbf{v}_k|^2}{\sum_{j \neq k} |\mathbf{h}_k^H \mathbf{v}_j|^2 + \sigma^2} \right) \right], \quad (2)$$

# Training for Discrete Feedback

- DNN training is performed using stochastic gradient descent (SGD) via back-propagation.
- **Challenge:** The gradient of the binary hidden layer is always zeros.
- **Solution:** Approximate  $\text{sgn}(u)$  in back-propagation phase with a differentiable function,  $f(u)$ .
- **Straight-through (ST)** [Hinton's Lectures]:

$$f(u) = u.$$

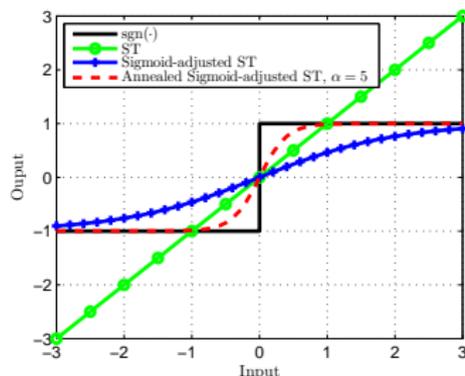
- **Sigmoid-adjusted ST** [Bengio, Léonard, and Courville, 2013]:

$$f(u) = 2 \text{sigm}(u) - 1.$$

- **Annealed Sigmoid-adjusted ST** [Chung, Ahn, and Bengio, 2016]:

$$f(u) = 2 \text{sigm}(\alpha^{(i)} u) - 1, \text{ where } \alpha^{(i)} \geq \alpha^{(i-1)}.$$

- In this work, we adopt sigmoid-adjusted ST with the annealing trick.



## Robustness:

- The DNNs are trained under varying different channel models to ensure robustness.

## Enhancing generalizability for arbitrary $K$ :

- All different users adopt a common set of DNN parameters.
- The DNN parameters and the pilot sequences are designed by end-to-end training of a single-user system.
- The BS-side DNN are obtained by training a  $K$ -user system with the user-side DNNs fixed.

## Enhancing generalizability for arbitrary $B$ :

- **Goal:** Design a common user-side DNN to operate over a wide range of feedback rates.
- Modify user-side DNN to output soft information (which can be quantized later at different values of  $B$ ) by using a  $\tanh()$  function at the output layer.
- Train the modified user-side DNN to obtain its parameter and the pilot sequences.
- Apply different quantization resolutions to the user-side DNN, then conduct another round of training to design the BS-side DNN

## Channel Model:

- We consider a limited-scattering propagating environment, e.g., mmWave channels:

$$h_k = \frac{1}{\sqrt{L_p}} \sum_{\ell=1}^{L_p} \alpha_{\ell,k} \mathbf{a}_t(\theta_{\ell,k}),$$

- $L_p$  is the number of propagation paths,
- $\alpha_{\ell,k} \sim \mathcal{CN}(0, 1)$  is the complex gain of the  $\ell^{\text{th}}$  path,
- $\theta_{\ell,k} \sim \mathcal{U}(-30^\circ, +30^\circ)$  is the AoD of the  $\ell^{\text{th}}$  path,
- $\mathbf{a}_t(\cdot)$  is the array response vector, e.g.,  $\mathbf{a}_t(\theta) = [1, e^{j\pi \sin(\theta)}, \dots, e^{j\pi(M-1)\sin(\theta)}]$ .

## DNN Implementation:

- **Implementation platform:** TensorFlow and Keras.
- **Optimization method:** Adam optimizer with an adaptive learning rate initialized to 0.001.
- **# hidden layers:**  $T = 4$  and  $R = 4$ .
- **# hidden neurons/layer:**  $[1024, 512, 256, B]$  for the user-side DNNs,  
 $[1024, 512, 512, 2KM]$  for the BS-side DNN.
- **Activation function of the hidden layers:** Rectified linear units (ReLUs).

# Numerical Results: Performance Comparison

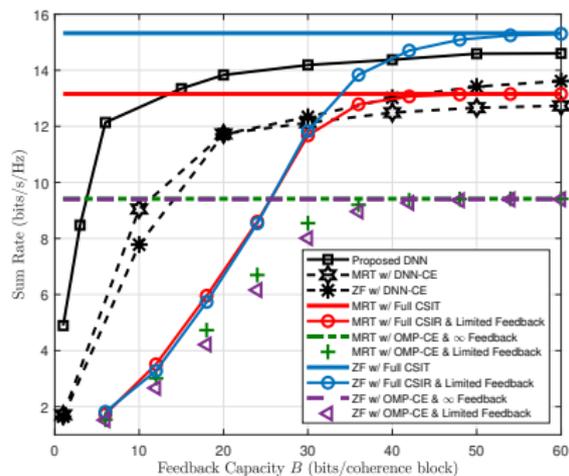


Figure: Sum rate achieved by different methods in a 2-user FDD system with  $M = 64$ ,  $L = 8$ ,  $L_p = 2$ , and  $\text{SNR} \triangleq 10 \log_{10}(\frac{P}{\sigma^2}) = 10\text{dB}$ .

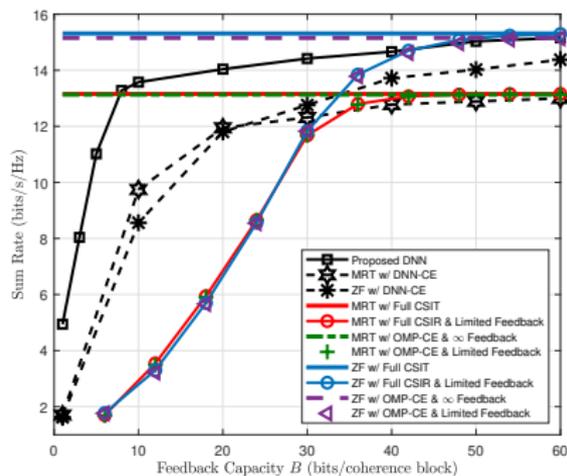


Figure: Sum rate achieved by different methods in a 2-user FDD system with  $M = 64$ ,  $L = 64$ ,  $L_p = 2$ , and  $\text{SNR} \triangleq 10 \log_{10}(\frac{P}{\sigma^2}) = 10\text{dB}$ .

# Numerical Results: Generalizability in $L_p$

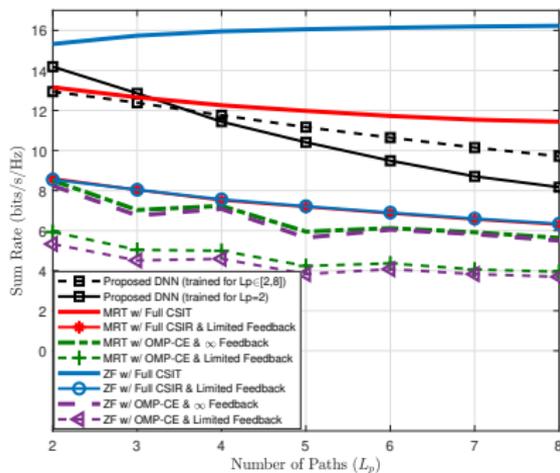


Figure: Sum rate achieved by different methods in a 2-user FDD system with  $M = 64$ ,  $L = 8$ ,  $B = 30$ , and  $\text{SNR} = 10\text{dB}$ .

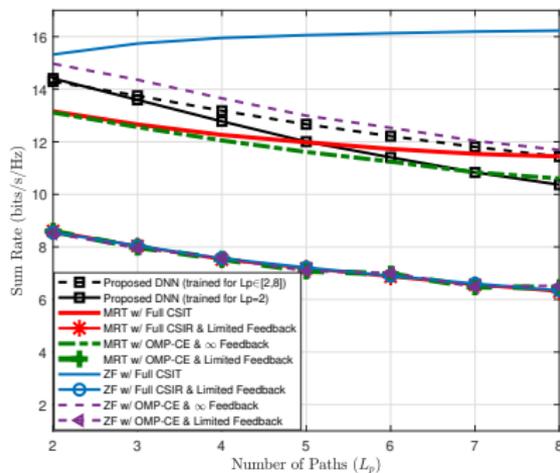
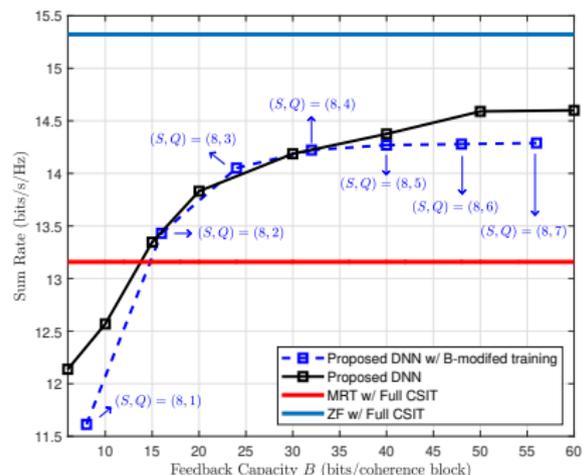
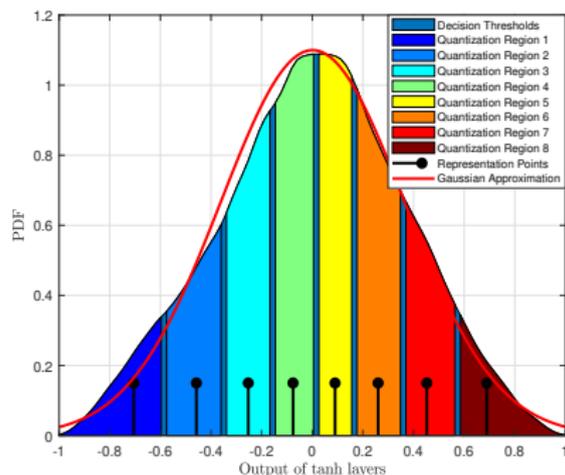


Figure: Sum rate achieved by different methods in a 2-user FDD system with  $M = 64$ ,  $L = 64$ ,  $B = 30$ , and  $\text{SNR} = 10\text{dB}$ .

# Numerical Results: Generalizability in $B$

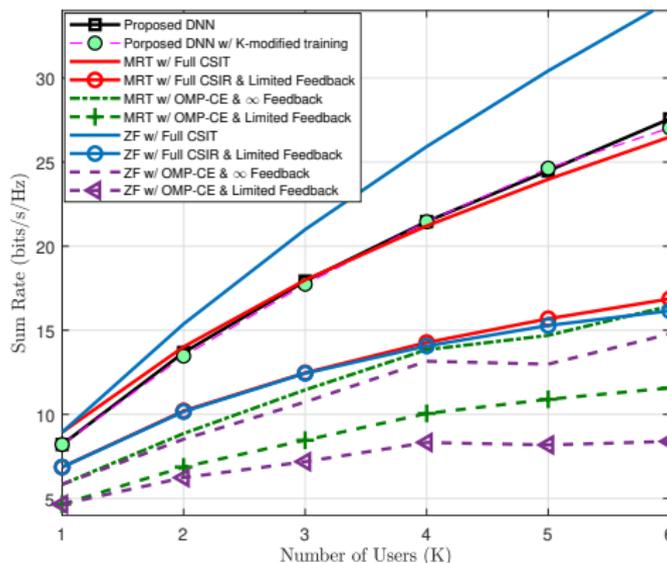


**Figure:** Sum rate achieved by different methods in a 2-user FDD system with  $M = 64$ ,  $L = 8$ ,  $L_p = 2$ , and  $\text{SNR} = 10\text{dB}$ .



**Figure:** The empirical PDF of the soft output layer in the modified user-side DNN, trained for  $M = 64$ ,  $K = 2$ , and  $L = 8$ . This figure also indicates the quantization regions and the corresponding representation points for the optimal 3-bit quantizer.

# Numerical Results: Generalizability in $K$



**Figure:** Sum rate achieved by different methods in a  $K$ -user FDD system with  $M = 64$ ,  $L = 8$ ,  $B = 30$ ,  $L_p = 2$ , and SNR = 10dB.

- As the input dimension of the decoding DNN is  $KB$ , for larger values of  $K$  we need to increase the capacity of the BS's DNN in order to fully process the input signals.
- In this simulations, we employ a 4-layer DNN at the BS with  $[2048, 1024, 512, 2MK]$  number of neurons per layer.

- This work shows that the design of a downlink FDD massive MIMO system with limited feedback can be formulated as a DSC problem.
- To solve such a challenging DSC problem, we propose a novel deep learning framework.
- In particular, we represent an end-to-end FDD downlink precoding system, including the downlink training phase, the uplink feedback phase, and the downlink precoding phase, using a user-side DNN and a BS-side DNN.
- We propose a machine learning framework to jointly design:
  - The pilots in the downlink training phase,
  - The channel estimation and feedback strategy adopted at the users,
  - The precoding scheme at the BS.
- We also investigate how to make the proposed DNN architecture more generalizable to different system parameters.
- Numerical results show that the proposed DSC strategy for FDD precoding, which bypasses explicit channel estimation, can achieve an outstanding performance.

## Part II

# Symbol-Level Precoding for TDD Massive MIMO

In TDD systems, CSI can be estimated in the uplink for downlink beamforming due to reciprocity.

- **Fully Digital Beamforming**

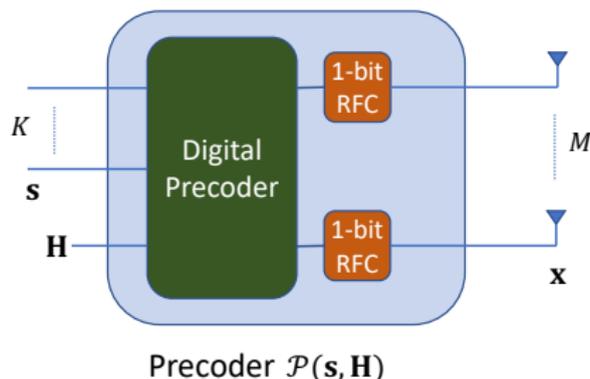
- Requires one high-resolution RF chain per antenna element.
- Has high power consumption and hardware complexity.

- **Lower-Complexity Architectures:**

- Analog Beamforming
- Antenna Switching
- Hybrid Beamforming
- One-Bit Precoding ✓

Beamforming design is a challenging problem. [Further, how to take CSI uncertainty into account?](#)

# One-Bit Precoding Architecture

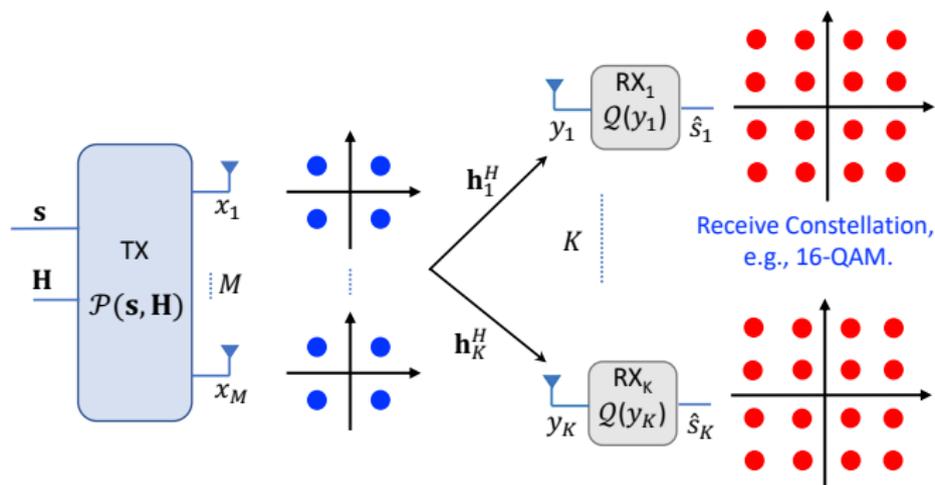


- One RF chain is dedicated to each antenna but with only 1-bit resolution per dimension.
- The transmitted signal of each antenna is chosen from:  $\mathcal{X} = \left\{ \frac{1}{\sqrt{2}} (\pm 1 \pm i) \right\}$ .
- Power saving due to low-resolution digital-to-analog converter.

# How to Perform One-Bit Precoding?

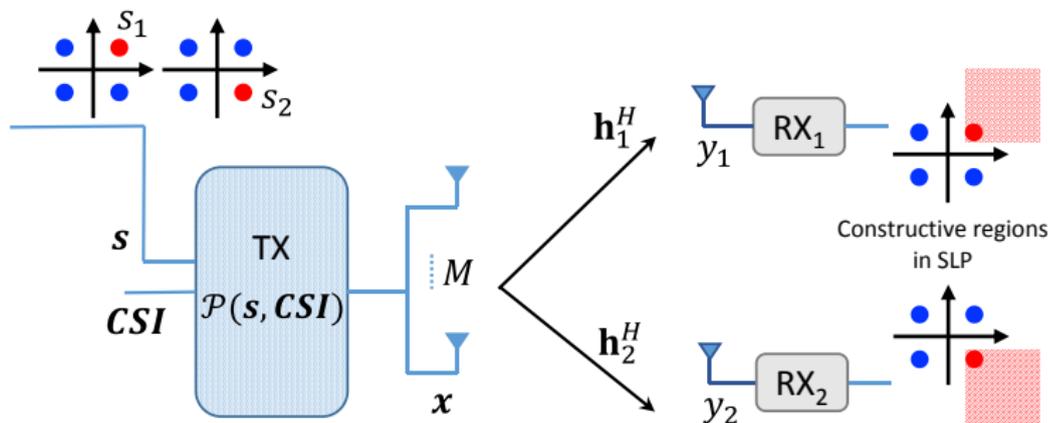
- Quantized-ZF one-bit precoding: [Saxena, Fijalkow, and Swindlehurst, 2016].
  - Performance at moderate-to-high SNRs is limited by quantization noise.
- One-bit beamforming at both transmitter and receivers: [Usman, Jedda, Mezghani, and Nossek, 2016].
  - Restricted to the QPSK constellation.
- One-bit precoding for higher order modulations:
  - Examples: POKEMON [Castañeda, Goldstein, and Studer, 2017], SQUID [Jacobsson, Durisi, Coldrey, Goldstein, and Studer, 2016], and Greedy-exhaustive one-bit precoding [Sohrabi, Liu, and Yu, 2018].
  - Restricted to the conventional QAM and PSK constellations.
- We can actually *jointly* design the **receive constellation** and **one-bit precoder**.
  - Machine learning, specifically the concept of **autoencoder**, allows us to do this efficiently.

# One-Bit Symbol-Level Precoding



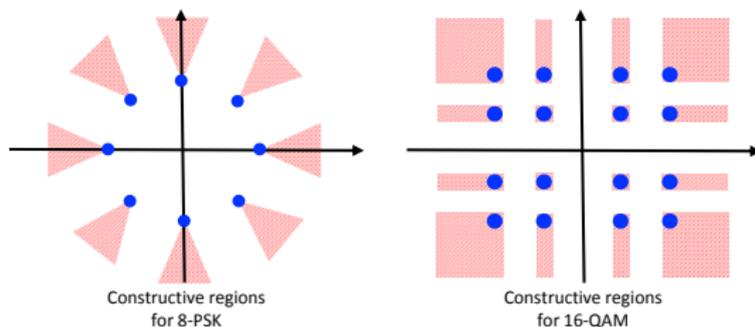
- Target constellation point  $s$  is taken from a constellation conventionally QAM or PSK.
- **Symbol-by-symbol** precoding:  $x = \mathcal{P}(s, \mathbf{H})$ , where  $x \in \mathcal{X}^M = \left\{ \frac{1}{\sqrt{2}} (\pm 1 \pm \iota) \right\}^M$ .
- Received signal at the  $k^{\text{th}}$  user:  $y_k = \sqrt{\frac{P}{M}} \mathbf{h}_k^H x + z_k$ .
- Signal recovery at the receiver:  $\hat{s}_k = Q(y_k)$ .
- **Goal:** Design **the receive constellation** and **precoder**  $\mathcal{P}(s, \mathbf{H})$  to minimize average SER.

- The one-bit precoding architecture is an example of **symbol-level precoding**.
- **Traditional Multiuser Precoding:**
  - Focuses on **eliminating interference** between different users.
  - Designs precoders only based on **channel state information (CSI)**.
- **Symbol-Level Precoding (SLP):**
  - Exploits **constructive interference** for enhancing received signal power.
  - Designs precoders by exploiting **the knowledge of users' data symbol**, in addition to **CSI**.



- **Symbol-level Precoding Main Idea:**

- Design **precoders** such that received symbols for all users lie in the **constructive regions**.
- Such a precoding design involves formulating/solving non-trivial optimization problems.
- The idea of SLP is pioneered in [Alodeh, Chatzinotas, Ottersten, 2015] and [Masouros, G. Zheng, 2015].

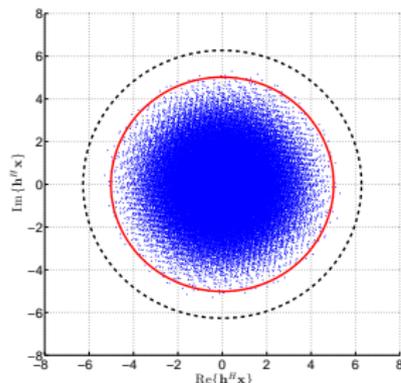


- Most previous works on SLP focus on PSK modulations.
  - This is because the decision boundaries in PSK are easier to characterize.
  - Examples: [Li and Masouros, 2018] and [Law and Masouros, 2018].
- Some recent works consider SLP design for QAM modulations.
  - Examples: [Kalantari et al., 2018] and [Li, Masouros, Li, Vucetic, and Swindlehurst, 2018].

- Precoder design problem given the constellation point  $s^j$ :

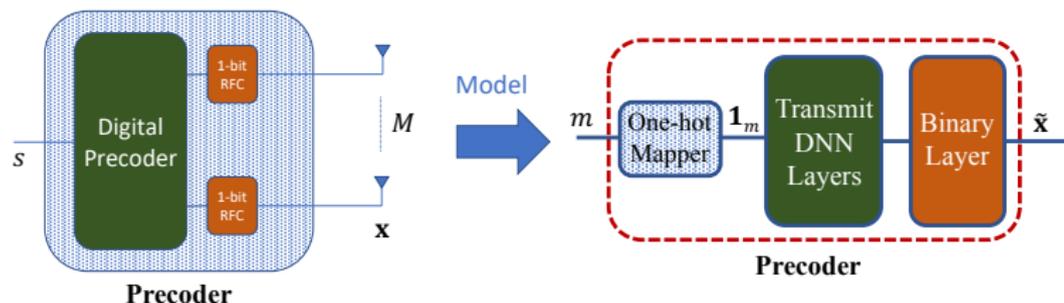
$$x_i^* = \underset{x_i \in \mathcal{X}^M}{\operatorname{argmin}} \left| \sqrt{\frac{P}{M}} \mathbf{h}^H x_i - s^j \right|. \quad (3)$$

- **Observation:** For a fixed channel, the possible realizations of  $\mathbf{h}^H \mathbf{x}$  when  $\mathbf{x} \in \mathcal{X}^M$  are distributed densely close to the origin, e.g.,



- [Sohrabi, Liu, Yu '08]: Set the range to be  $\sqrt{\frac{2}{\pi}}$ , or 80% of the infinite resolution case
- Can we use a neural network to “discover” the optimal constellation and precoder?

# Neural Network Representation: Transmitter Side



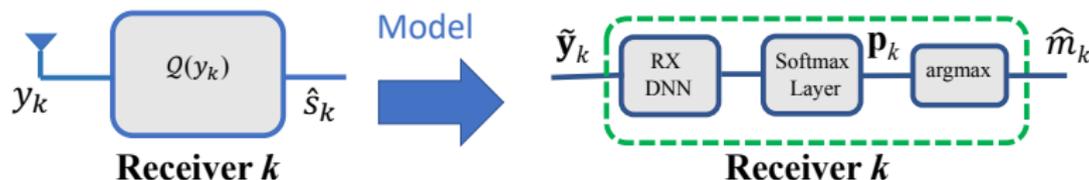
- The real-valued received signal model:

$$\underbrace{\begin{bmatrix} \Re\{y_k\} \\ \Im\{y_k\} \end{bmatrix}}_{\triangleq \tilde{y}_k} = \rho \underbrace{\begin{bmatrix} \Re\{h_k^H\} & -\Im\{h_k^H\} \\ \Im\{h_k^H\} & \Re\{h_k^H\} \end{bmatrix}}_{\triangleq \tilde{H}_k} \underbrace{\begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix}}_{\triangleq \tilde{x}} + \underbrace{\begin{bmatrix} \Re\{z_k\} \\ \Im\{z_k\} \end{bmatrix}}_{\triangleq \tilde{z}_k}.$$

- The precoder is modeled by a DNN with  $T$  dense layers followed by a binary layer:

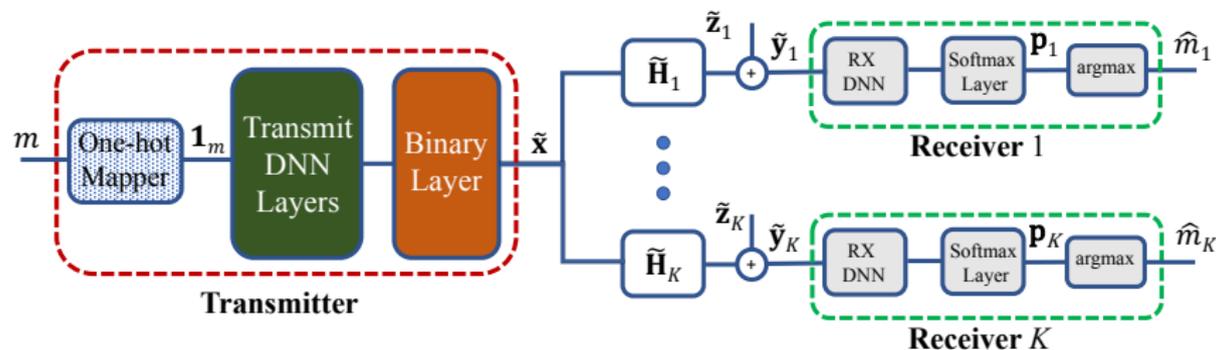
$$\tilde{x} = \text{sgn}(W_T \sigma_{T-1}(\cdots W_2 \sigma_1(W_1 \mathbf{1}_m + \mathbf{b}_1) + \cdots \mathbf{b}_{T-1}) + \mathbf{b}_T),$$

- $m \in \{1, \dots, |\mathcal{C}|\}$  denotes the index of the intended symbol.
- $\mathbf{1}_m \in \mathbb{R}^{|\mathcal{C}|}$  denotes the one-hot representation of  $m$ .
- $\sigma_t$  is the activation function for the  $t^{\text{th}}$  layer.
- Binary layer ensures that the one-bit constraints on the elements of  $\tilde{x}$  are met.



- The receivers' operations are modeled by another DNN with  $R$  dense layers.
- Softmax activation function in the last layer:
  - To generate  $\mathbf{p}_k \in (0, 1)^{|C|}$ , where its  $i^{\text{th}}$  element indicates the probability that the index of the intended symbol is  $i$ .
- Receiver  $k$  declares  $\hat{m}_k$ , which corresponds to the index of largest  $\mathbf{p}_k$ .
- We consider one common DNN to represent the decoding procedure of different users.
  - Reduces dimensions of the receivers' trainable parameters.  
⇒ Faster training procedure.
  - The BS needs to broadcast the common constellation parameters to all the users.  
⇒ Reduction in amount of required feedback.

# End-to-End Autoencoder Representation



- As proof of concept, consider the case that a common symbol is sent to multiple users.
- **Input:** Index of the intended symbol.
- **Outputs:** Index of the intended symbol decoded at the receivers.
- After the network being trained for a fixed  $\{\tilde{\mathbf{H}}_k\}_{k=1}^K$ , we obtain:
  - The precoding procedure at the transmitter.
  - The constellation design and decision boundaries at the receivers.
- How to train this network?
  - SGD-based training via **back-propagation**.
  - The binary layer is approximated by **annealed sigmoid-adjusted straight-through**.

- **Implementation platform:** TensorFlow.
- **Optimization method:** Adam optimizer with an adaptive learning rate initialized to 0.001.
- **# hidden layers:**  $T_x = 12$  and  $R_x = 5$ .
- **# hidden neurons/layer:**  $6M$  for the transmitter and  $2M$  for the receiver.
- **Activation function of the hidden layers:** Exponential linear units (ELUs).
- **Loss Function:** Cross entropy between  $1_m$  and the probability vectors,  $p_k$ :

$$\mathcal{L}_{CE} = -\mathbb{E}_{\text{training samples}} \left[ \frac{1}{K|C|} \sum_{k=1}^K \sum_{m=1}^{|C|} \log p_{k,m} \right]. \quad (4)$$

- **Annealing parameter update rule:**

$$\alpha^{(i)} = 1.002\alpha^{(i-1)} \quad (5)$$

with  $\alpha^{(0)} = 1$  such that  $1.002^{2000} \approx 55$ .

- In the training stage, the noise variance is randomly generated so that:

$$\text{SNR} \triangleq 10 \log_{10} \left( \frac{P}{2\sigma^2} \right) \in [4\text{dB}, 16\text{dB}]. \quad (6)$$

# Numerical Results: Autoencoder-Based Constellation Design

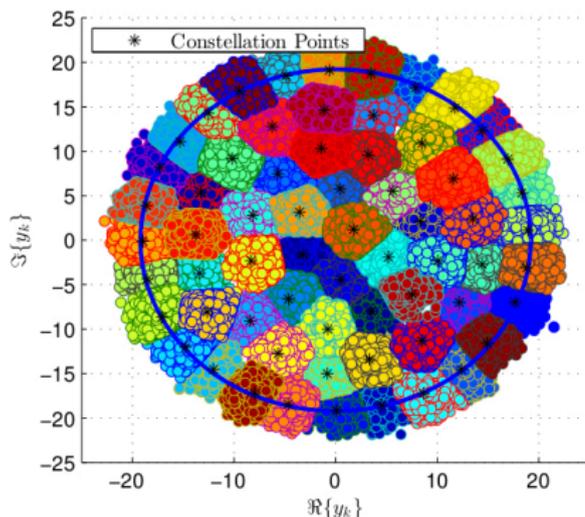


Figure: The receive constellation points and their corresponding decision boundaries obtained from a trained autoencoder in a system with  $M = 128$ ,  $K = 4$ , and  $|C| = 64$ .

- The furthest constellation points are located at the following distance from the origin:

$$d^* = \sqrt{\frac{\frac{2}{\pi} P}{\mathbf{1}^T (\mathbf{H}\mathbf{H}^H)^{-1} \mathbf{1}}}, \quad (7)$$

matching the heuristic 0.8 constellation range result in [Sohrabi, Liu, Yu '08].

# Numerical Results with Varying Channels

- **Constellation range needs to adapt to the channel:**
  - Consider the constellation designed for one particular  $H$ .
  - Rescale that constellation for other  $H$  so that the constellation range becomes  $d^*$ .

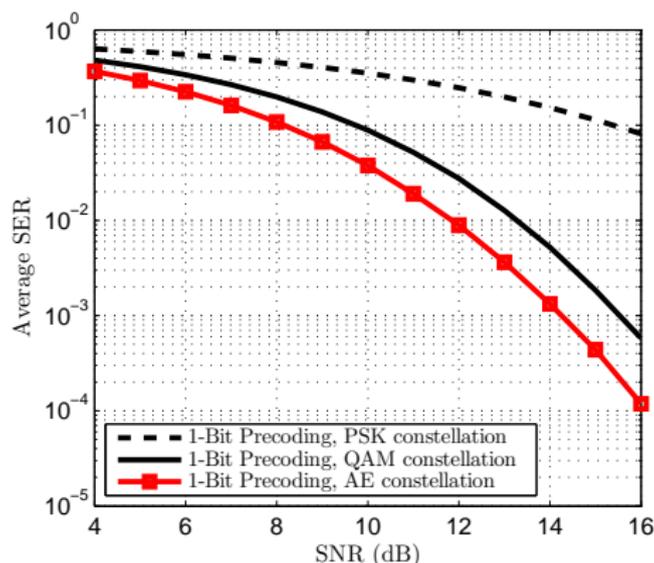
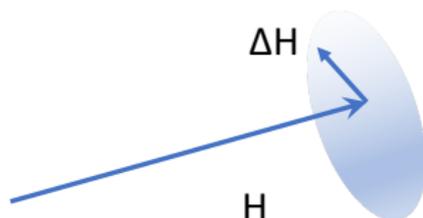


Figure: Average SER versus SNR in a system with  $M = 128$ ,  $K = 4$ , and  $|C| = 64$  using the greedy plus exhaustive search based one-bit precoding algorithm of [Sohrabi, Liu, and Yu, 2018].

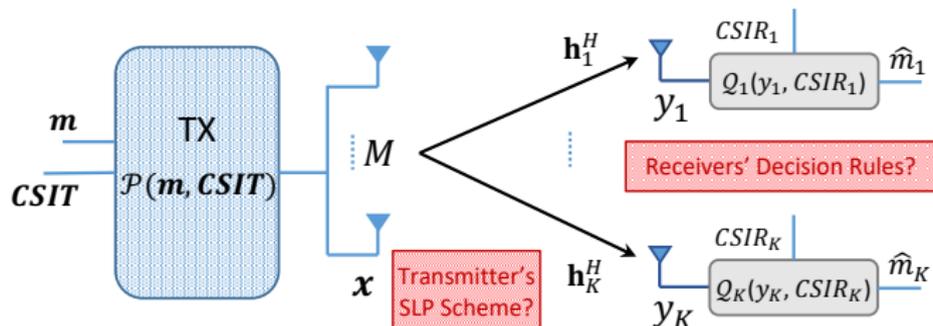
- CSI is never perfect in practice due to several reasons such as:
  - Imperfect channel estimation,
  - Limited/delayed feedback in FDD systems,
  - Mismatch in channel reciprocity in TDD systems.

⇒ Robust symbol-level precoding design is crucial.



- A robust SLP scheme has recently been proposed in [Haqiqatnejad, Kayhan, and Ottersten, 2019]:
  - Restricted to spherical bounded model and stochastic Gaussian model.
  - Based on the assumption that CSI uncertainty model is accurate.
- In contrast, a data-driven robust SLP design can implicitly account for channel uncertainty.

# Symbol-Level Precoding with CSI Uncertainty



- Target message  $m_k$  of  $B$ -bits for each user is uniformly taken from  $\{1, \dots, 2^B\}$ .
- **Symbol-by-symbol precoding**:  $x = \mathcal{P}(m, CSIT)$ , satisfying  $\|x\|^2 \leq P$ .
- Received signal at the  $k^{\text{th}}$  user:  $y_k = h_k^H x + z_k$ .
- Message recovery at the  $k^{\text{th}}$  user:  $\hat{m}_k = Q_k(y_k, CSIR_k)$ .
- **Goal**: Design the **precoder function**  $\mathcal{P}(\cdot)$  and the **receivers' decision rules**  $Q_k(\cdot), \forall k$ , to minimize average SER.

- We consider a propagating environment with sparse channels, e.g., mmWave channels:

$$\mathbf{h}_k = \frac{1}{\sqrt{L}} \sum_{\ell=1}^L \alpha_{\ell,k} \mathbf{a}_t(\theta_{\ell,k}),$$

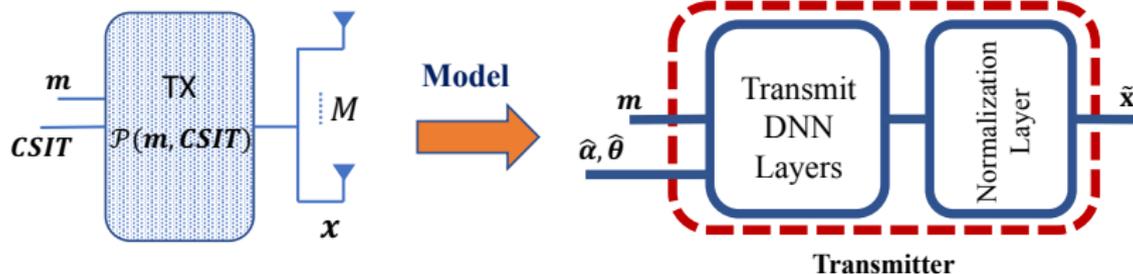
- $L$  is the number of propagation paths,
  - $\alpha_{\ell,k}$  is the complex gain of the  $\ell^{\text{th}}$  path,
  - $\theta_{\ell,k}$  is the AoD of the  $\ell^{\text{th}}$  path,
  - $\mathbf{a}_t(\cdot)$  is the array response vector, e.g.,  $\mathbf{a}_t(\theta) = [1, e^{j\pi \sin(\theta)}, \dots, e^{j\pi(M-1) \sin(\theta)}]$ .
- We assume that the available CSI is in the form of imperfect estimation of the sparse channel parameters as:

$$\begin{aligned} \hat{\alpha}_{\ell,k} &= \alpha_{\ell,k} + \Delta\alpha_{\ell,k}, \\ \hat{\theta}_{\ell,k} &= \theta_{\ell,k} + \Delta\theta_{\ell,k}, \end{aligned}$$

where  $\Delta\alpha_{\ell,k} \sim \mathcal{CN}(0, \sigma_{\Delta\alpha}^2)$  and  $\Delta\theta_{\ell,k} \sim \mathcal{U}(-\Delta\theta_{\max}, \Delta\theta_{\max})$ .

- Summary of the CSI model:  $CSIT = \{\hat{\alpha}_{\ell,k}, \hat{\theta}_{\ell,k}\}_{\forall \ell,k} = \{\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\theta}}\}$   
 $CSIR_k = \{\hat{\alpha}_{\ell,k}, \hat{\theta}_{\ell,k}\}_{\forall \ell}$

# Neural Network Representation: Transmitter Side



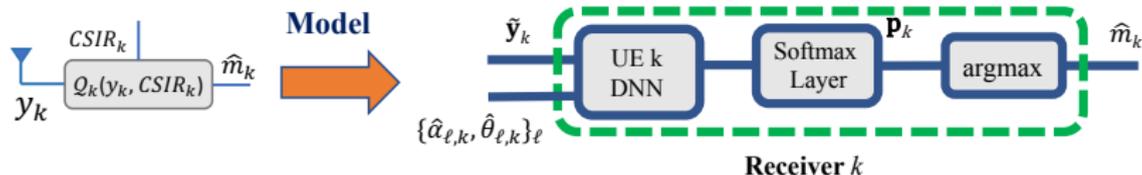
- The real-valued received signal model:

$$\underbrace{\begin{bmatrix} \Re\{y_k\} \\ \Im\{y_k\} \end{bmatrix}}_{\tilde{y}_k} = \underbrace{\begin{bmatrix} \Re\{h_k^H\} & -\Im\{h_k^H\} \\ \Im\{h_k^H\} & \Re\{h_k^H\} \end{bmatrix}}_{\tilde{H}_k} \underbrace{\begin{bmatrix} \Re\{x\} \\ \Im\{x\} \end{bmatrix}}_{\tilde{x}} + \underbrace{\begin{bmatrix} \Re\{z_k\} \\ \Im\{z_k\} \end{bmatrix}}_{\tilde{z}_k}.$$

- The precoder is modeled by a DNN with  $T$  dense layers followed by a normalization layer:

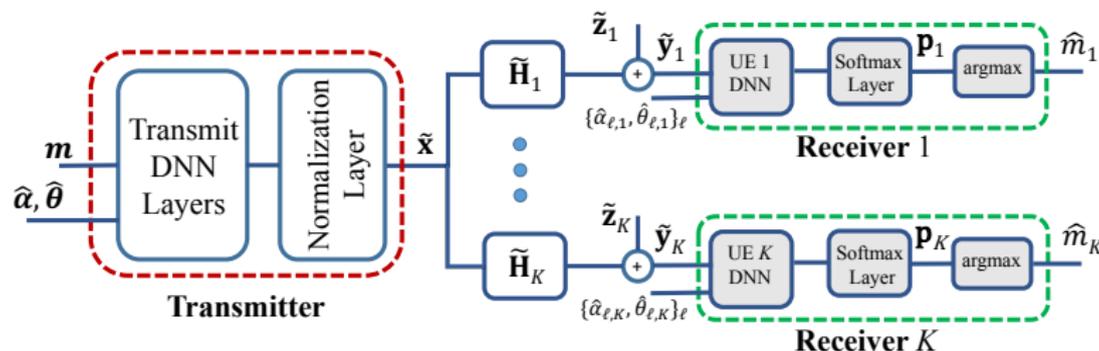
$$\tilde{x} = \sigma_T(W_T \sigma_{T-1}(\dots W_2 \sigma_1(W_1 v + b_1) + \dots) + b_T),$$

- $\sigma_t$ ,  $W_t$ , and  $b_t$  are the activation function, the weights, and the biases in the  $t^{\text{th}}$  layer.
- $v = [\hat{\alpha}, \hat{\theta}, m]$  is the input vector to the DNN.
- Normalization layer,  $\sigma_T(x) = \min(\sqrt{P}, \|x\|) \frac{x}{\|x\|}$ , ensures that the power constraint is met.



- The receivers' operations are modeled by another DNN with  $R$  dense layers.
- Softmax activation function in the last layer:
  - To generate  $\mathbf{p}_k \in (0, 1)^{|C|}$ , where its  $i^{\text{th}}$  element indicates the probability that the index of the intended symbol is  $i$ .
- Receiver  $k$  declares  $\hat{m}_k$ , which corresponds to the index of largest  $\mathbf{p}_k$ .

# End-to-End Autoencoder Representation



- The BS aims to send independent messages to multiple users.
- **Inputs:** Intended messages and estimated channel parameters.
- **Outputs:** Intended messages recovered at the users.
- After the network is trained for a fixed  $\{\tilde{\mathbf{H}}_k\}_{k=1}^K$ , we obtain:
  - The precoding procedure at the transmitter.
  - The decision boundaries at the receivers.
- End-to-End SGD-based training with cross-entropy loss.

- **Implementation platform:** TensorFlow.
- **Optimization method:** Adam optimizer with an adaptive learning rate initialized to 0.001.
- **# hidden layers:**  $T = 4$  and  $R = 4$ .
- **# hidden neurons/layer:**  $[1024, 512, 512, 2M]$  for the transmitter,  
 $[256, 128, 64, 2^B]$  for the receivers.
- **Activation function of the hidden layers:** Rectified linear units (ReLU).
- In the training stage, the noise variance is generated so that:

$$\text{SNR} \triangleq 10 \log_{10}\left(\frac{P}{\sigma^2}\right) \in \mathcal{U}(5, 30)\text{dB}.$$

- We use  $10^5$  channel realizations for training and set the CSI parameters as:
  - Linear array with  $M = 128$ .
  - Single-path, i.e.,  $L_k = 1, \forall k$ .
  - $\alpha_k \sim \mathcal{CN}(0.5 + 0.5i, 1)$ ,
  - $\theta_k \sim \mathcal{U}(\phi_k - 5^\circ, \phi_k + 5^\circ), \forall k$ , with  $\{\phi_1, \phi_2, \phi_3\} = \{-30^\circ, 0^\circ, +30^\circ\}$ ,
  - $\sigma_{\Delta\alpha} = 0.001$  and  $\Delta\theta_{\max} = 1^\circ$ .

# Numerical Results: SER Performance vs SNR

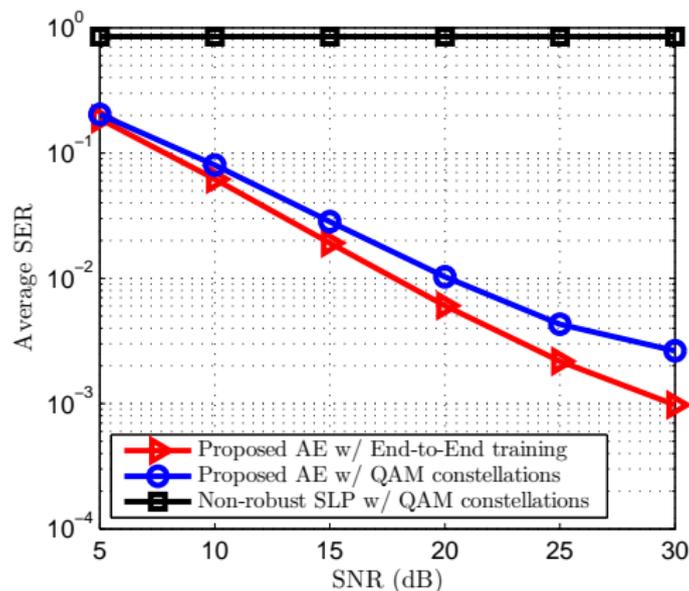
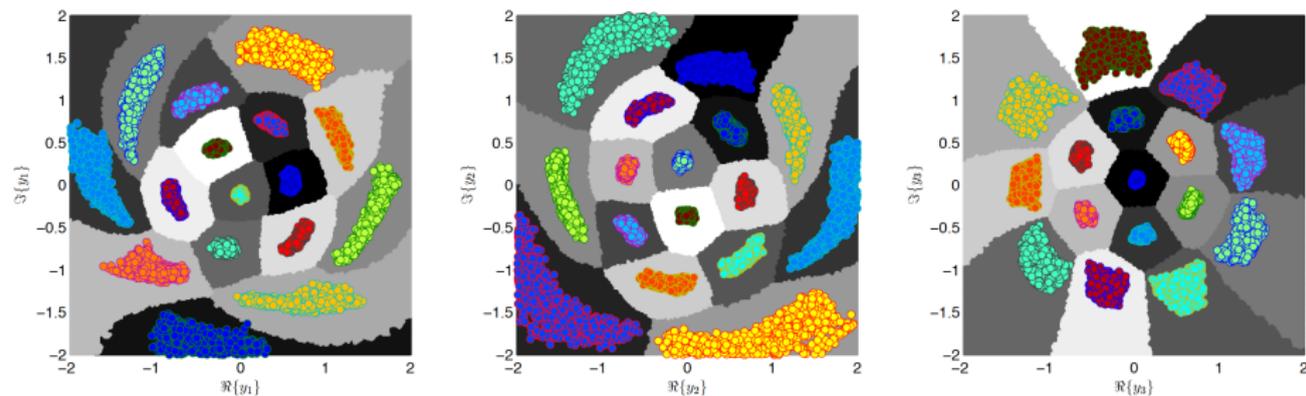


Figure: Avg. SER versus SNR in a system with  $M = 128$ ,  $K = 3$ ,  $B = 4$ bits,  $\Delta\theta_{\max} = 1^\circ$  and  $\sigma_{\Delta\alpha} = 0.001$ . “Non-robust SLP” corresponds to the SLP algorithm in [Li, Masouros, Li, Vucetic, and Swindlehurst, 2018].

# Constellation Design by Autoencoder with End-to-End Training



**Figure:** The decision boundaries (in grey scale) designed by the autoencoder together with the noiseless received signal (as circles) for a robust SLP with  $K = 3$  users.

*CSI Uncertainty is Explicitly Accounted for in Constellation Design!*

# Numerical Results: SER Performance vs CSI Uncertainty

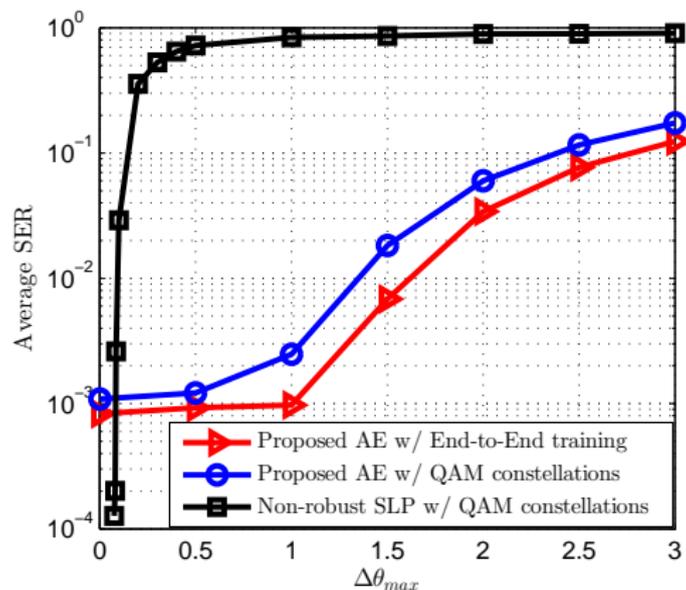


Figure: Avg. SER versus  $\Delta\theta_{max}$  in a system with  $M = 128$ ,  $K = 3$ ,  $B = 4$ bits, SNR = 30dB and  $\sigma_{\Delta\alpha} = 0.001$ . “Non-robust SLP” corresponds to the SLP algorithm in [Li, Masouros, Li, Vucetic, and Swindlehurst, 2018].

## Summary of Part II

- We propose an **end-to-end** design for one-bit precoding and for symbol-level precoding.
- We use an **DNN autoencoder** to jointly design the transceiver and the constellation.
- The design account for channel estimation and leads to a more **robust** receive constellation in a limited scattering environment.

## Concluding Remarks:

- Traditional paradigm for communication system design is to **model-then-optimize**.
- Machine learning allows a data-driven approach that
  - Perform channel estimation, feedback and precoding without explicit channel model;
  - Perform robust precoding and detection without explicit channel uncertainty model.
- Key future issues are: **generalizability**, training and computational **complexity**



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