Sparse Activity Detection in Multi-Cell Massive MIMO Exploiting Channel Large-Scale Fading

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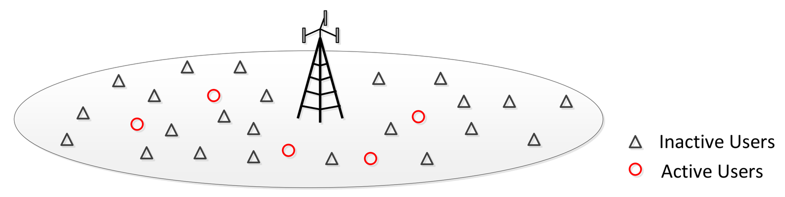
1. Massive Random Access



- A large number of devices with sporadic activity
- Non-orthogonal signature sequences for all the users
- User activity detection performed at base station (BS)

2. System Model

- BS equipped with M antennas
- N single-antenna devices, K of which are active at a time
- Each device is assigned a length-L unique signature $\mathbf{s}_n$
- Channel $\mathbf{h}_n$ includes both Rayleigh and large-scale fading
- For single-cell system, received signal $\mathbf{Y} \in \mathbb{C}^{L\times M}$ at the BS is

$$\mathbf{Y} = \sum_{n=1}^{N} \alpha_n \mathbf{s}_n \mathbf{h}_n^T + \mathbf{Z} = \mathbf{S}\mathbf{X} + \mathbf{Z}, \tag{1}$$

- $\alpha_n \in \{1,0\}$ activity indicator;
- $\mathbf{S} \triangleq [\mathbf{s}_1, \ldots, \mathbf{s}_N] \in \mathbb{C}^{L\times N}$; $\mathbf{X} \triangleq [\alpha_1 \mathbf{h}_1, \cdots, \alpha_N \mathbf{h}_N]^T \in \mathbb{C}^{N\times M}$

3. Joint Activity and Large-Scale Fading Estimation

Assumption: We only need activity α_n and do not need $\mathbf{h}_n$.

Recast as a large-scale-fading estimation problem:

$$\mathbf{Y} = \sum_{n=1}^{N} \alpha_n \mathbf{s}_n \mathbf{h}_n^T + \mathbf{Z} \triangleq \mathbf{S}\mathbf{\Gamma}^{\frac{1}{2}}\tilde{\mathbf{H}} + \mathbf{Z} \tag{2}$$

where $\mathbf{\Gamma} \triangleq \text{diag}\{\alpha_1\beta_1, \alpha_2\beta_2, \cdots, \alpha_N\beta_N\} \in \mathbb{R}^{N\times N}$.

The maximum likelihood estimation of $\mathbf{\Gamma}$ can be formulated as

$$\begin{aligned}\min_{\mathbf{\Gamma}\geq \mathbf{0}} -\log p(\mathbf{Y}|\mathbf{\Gamma}) &\propto \min_{\mathbf{\Gamma}\geq \mathbf{0}} -\frac{1}{M}\sum_{m=1}^{M}\left(\log\frac{1}{|\pi\mathbf{\Sigma}|}\exp\left(-\mathbf{y}_m^H\mathbf{\Sigma}^{-1}\mathbf{y}_m\right)\right)\\ &= \min_{\mathbf{\Gamma}\geq \mathbf{0}} \log|\mathbf{\Sigma}| + \text{tr}\left(\mathbf{\Sigma}^{-1}\hat{\mathbf{\Sigma}}\right) + const. \qquad (3)\\ &= \min_{\mathbf{\Gamma}\geq \mathbf{0}} \log|\mathbf{S}\mathbf{\Gamma}\mathbf{S}^H + \sigma^2\mathbf{I}| + \text{tr}\left((\mathbf{S}\mathbf{\Gamma}\mathbf{S}^H + \sigma^2\mathbf{I})^{-1}\hat{\mathbf{\Sigma}}\right)\end{aligned}$$

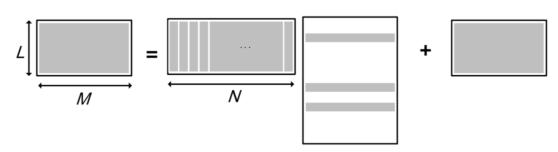
for which the sample covariance matrix is a sufficient statistics

$$\hat{\mathbf{\Sigma}} \triangleq \frac{1}{M}\sum_{m=1}^{M}\mathbf{y}_m\mathbf{y}_m^H = \frac{1}{M}\mathbf{Y}\mathbf{Y}^H. \tag{4}$$

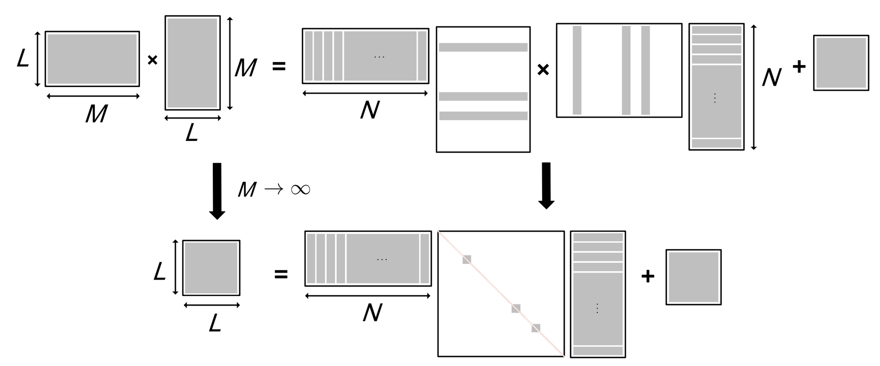
This problem can be solved efficiently using coordinate descent.

4. Covariance Based Sparse Activity Detection

For the channel model:



We now estimate large-scale fading $\mathbf{\Gamma}$ based on $\hat{\mathbf{\Sigma}} = \frac{1}{M}\mathbf{Y}\mathbf{Y}^H$:



Advantage: Instead of detecting KM variables based on LM observations, we now detect K variables based on L^2 observations!

5. Analysis of Maximum Likelihood Estimation

Theorem 1. *Consider a single-cell sparse user activity detection problem. Let $\mathcal{I}$ be the index set corresponding to inactive users, i.e., $\mathcal{I} \triangleq \{i \mid \gamma_i^0 = 0\}$. Define two sets $\mathcal{N}, \mathcal{C}$ in $\mathbb{R}^N$ as follows:*

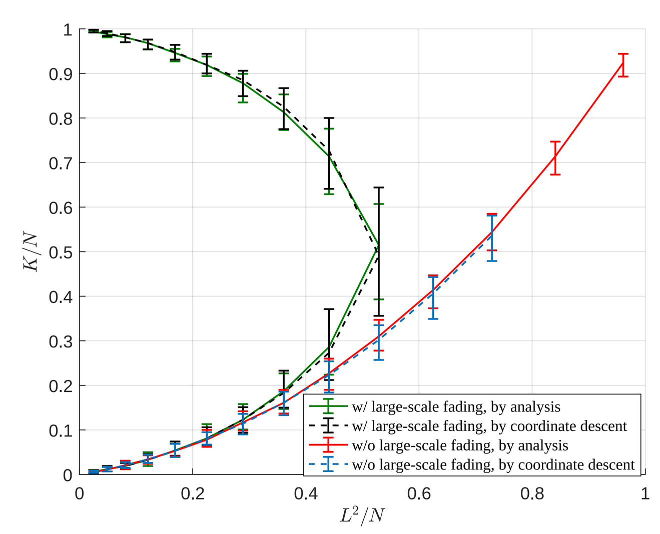
$$\mathcal{N} \triangleq \{\mathbf{x} \mid \mathbf{x}^T\mathbf{J}(\boldsymbol{\gamma}^0)\mathbf{x} = 0, \mathbf{x} \in \mathbb{R}^N\}, \tag{5}$$
$$\mathcal{C} \triangleq \{\mathbf{x} \mid x_i \geq 0, i \in \mathcal{I}, \mathbf{x} \in \mathbb{R}^N\}, \tag{6}$$

where x_i is the i-th entry of $\mathbf{x}$, and $\mathbf{J}(\boldsymbol{\gamma}^0)$ is the Fisher information matrix. Then a necessary and sufficient condition for the consistency of $\hat{\gamma}^{ML}$, i.e., $\hat{\gamma}^{ML} \to \gamma^0$ as $M \to \infty$, is $\mathcal{N} \cap \mathcal{C} = \{\mathbf{0}\}$.

Theorem 2. *Suppose that large-scale fading β_n is known, for estimating the activity indicator $\alpha_n \in [0,1]$ in $\gamma_n = \alpha_n\beta_n$, we define*

$$\mathcal{N}' \triangleq \{\mathbf{x} \mid \mathbf{x}^T\mathbf{J}(\boldsymbol{\gamma}^0)\mathbf{x} = 0, \mathbf{x} \in \mathbb{R}^N\}, \tag{7}$$
$$\mathcal{C}' \triangleq \{\mathbf{x} \mid x_i \geq 0, i \in \mathcal{I}, x_i \leq 0, i \notin \mathcal{I}, \mathbf{x} \in \mathbb{R}^N\}, \tag{8}$$

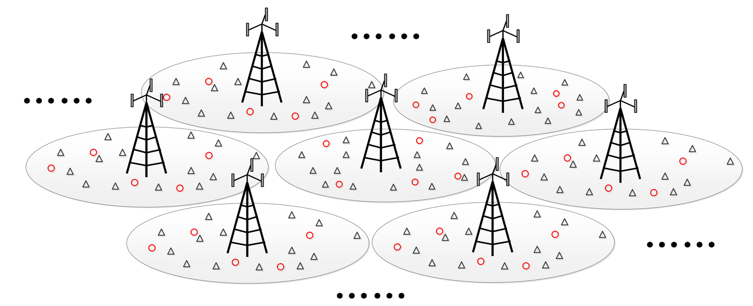
then a necessary and sufficient condition for the consistency of $\hat{\alpha}^{ML}$, i.e., $\hat{\alpha}^{ML} \to \alpha^0$ as $M \to \infty$, is $\mathcal{N}' \cap \mathcal{C}' = \{\mathbf{0}\}$.



When $\frac{K}{N} \approx 1$, then inactive users are sparse!

6. Cooperative Activity Detection in Multicell Systems

- Multi-cell system with B BSs each equipped with M antennas;



- Received signal $\mathbf{Y}_b \in \mathbb{C}^{L\times M}$ at BS b is

$$\begin{aligned}\mathbf{Y}_b &= \sum_{n=1}^{N}\alpha_{bn}\mathbf{s}_{bn}\mathbf{h}_{bbn}^T + \sum_{j=1,j\neq b}^{j=B}\sum_{n=1}^{N}\alpha_{jn}\mathbf{s}_{jn}\mathbf{h}_{bjn}^T + \mathbf{Z}_b\\ &= \mathbf{S}_b\mathbf{A}_b\mathbf{G}_{bb}^{\frac{1}{2}}\tilde{\mathbf{H}}_{bb} + \sum_{j=1,j\neq b}^{j=B}\mathbf{S}_j\mathbf{A}_j\mathbf{G}_{bj}^{\frac{1}{2}}\tilde{\mathbf{H}}_{bj} + \mathbf{Z}_b\\ &= \mathbf{S}_b\mathbf{\Gamma}_{bb}^{\frac{1}{2}}\tilde{\mathbf{H}}_{bb} + \sum_{j=1,j\neq b}^{j=B}\mathbf{S}_j\mathbf{\Gamma}_{bj}^{\frac{1}{2}}\tilde{\mathbf{H}}_{bj} + \mathbf{Z}_b \qquad (9)\end{aligned}$$

where $\alpha_{bn} \in \{1,0\}$ is the activity indicator.

Cooperative detection: To alleviate the impact of inter-cell interference, we consider a BS cooperation architecture by assuming that all BSs are connected to a central unit, where all received signals are jointly processed.

- When $\mathbf{G}_{bj}$ are unknown, we need to estimate $\mathbf{\Gamma}_{bj} = \mathbf{A}_j\mathbf{G}_{bj}$:

  B^2N unknown parameters

- When $\mathbf{G}_{bj}$ are known, we only need to estimate $\mathbf{A}_b$ with

  BN unknown parameters

Theorem 3. *Consider the problem of maximum likelihood estimation of user activities in a multicell cooperative system assuming known large-scale fading:*

$$\min_{\{\mathbf{A}_b\}} \sum_{b=1}^{B}\left(\log|\mathbf{\Sigma}_b| + \text{tr}\left(\mathbf{\Sigma}_b^{-1}\hat{\mathbf{\Sigma}}_b\right)\right) \tag{10a}$$
$$\text{s.t.} \quad \alpha_{bn} \in \{0,1\}, \forall b, n. \tag{10b}$$

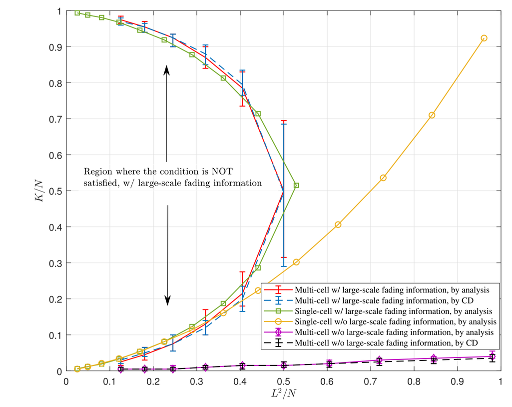
Let $\underline{\alpha} \triangleq [\alpha_1, \ldots, \alpha_{BN}]^T$ be the activity indicators, and let true value of $\underline{\alpha}$ be $\underline{\alpha}^0$. Let $\underline{\mathcal{I}}$ be index set corresponding to zero entries of $\underline{\alpha}^0$, i.e., $\underline{\mathcal{I}} \triangleq \{i \mid \underline{\alpha}_i^0 = 0\}$. We define two sets $\mathcal{N}'', \mathcal{C}''$ in the space $\mathbb{R}^{BN}$, respectively, as follows

$$\mathcal{N}'' \triangleq \{\mathbf{x} \mid \mathbf{x}^T\mathbf{J}(\boldsymbol{\gamma}^0)\mathbf{x} = 0, \mathbf{x} \in \mathbb{R}^{BN}\}, \tag{11}$$
$$\mathcal{C}'' \triangleq \{\mathbf{x} \mid x_i \geq 0, i \in \underline{\mathcal{I}}, x_i \leq 0, i \notin \underline{\mathcal{I}}, \mathbf{x} \in \mathbb{R}^{BN}\}, \tag{12}$$

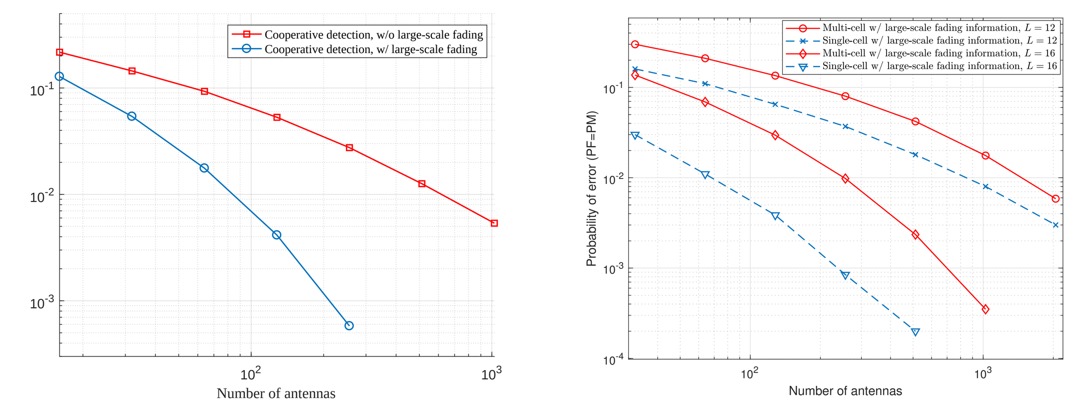
where x_i is the i-th entry of $\mathbf{x}$. Then a necessary and sufficient condition for the consistency of $\hat{\underline{\alpha}}^{ML}$, i.e., $\hat{\underline{\alpha}}^{ML} \to \underline{\alpha}^0$ as $M \to \infty$, is $\mathcal{N}'' \cap \mathcal{C}'' = \{\mathbf{0}\}$.

7. Sparse Activity Detection: Multicell vs. Single-Cell

With cooperation and assuming known large-scale fading, all dimensions of the problem scale by B, so we expect phase transition to be the same! For example, for $B = 7$, $N = 200$:



Practical detection performance: $B = 7$, $N = 200$, $K = 20$, $L = 20$



8. Conclusions

- Device activity detection for massive random access is a sparse recovery problem.
- This paper extends the analysis and algorithm for covariance based MLE of device activity detection to multicell systems.
- Multicell cooperative detection has a similar phase transition as single-cell system, but only if large-scale fading is known.
- In practice, there is significant performance degradation due to intercell interference.

References

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