Sparse Activity Detection in Multi-Cell Massive MIMO
Exploiting Channel Large-Scale Fading
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1. Massive Random Access

- A large number of devices with sporadic activity
- Non-orthogonal signature sequences for all the users
- User activity detection performed at base station (BS)

2. System Model

- BS equipped with \( M \) antennas
- \( N \) single-antenna devices, \( K \) of which are active at a time
- Each device is assigned a length-\( L \) unique signature \( s_n \)
- Channel \( h_n \) includes both Rayleigh and large-scale fading
- For single-cell system, received signal \( Y \in C^{N \times M} \) at the BS is

\[
Y = \sum_{n=1}^{N} a_n s_n h_n^T + Z = XN + Z
\]

3. Joint Activity and Large-Scale Fading Estimation

Assumption: We only need activity \( a_n \), and do not need \( h_n \).
Recast as a large-scale fading estimation problem:

\[
Y = \sum_{n=1}^{N} a_n b_n^T + Z \propto XN + Z
\]

where \( \Gamma = \text{diag}(a_1, a_2, \ldots, a_N) \in \mathbb{R}^{N	imes N} \). The maximum likelihood estimation of \( \Gamma \) can be formulated as

\[
\hat{\Gamma} = \arg \min_{\Gamma \in \mathbb{R}^{N\times N}} \frac{1}{M} \sum_{m=1}^{M} \log \left| \sum_{x=1}^{X} \left( XN + Z \right) \right|
\]

4. Covariance Based Sparse Activity Detection

For the channel model:

\[
Y = \sum_{n=1}^{N} a_n s_n h_n^T + Z = XN + Z
\]

We now estimate large-scale fading \( \Gamma \) based on \( S = \frac{1}{M} Y Y^H \):

\[
\hat{\Gamma} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M) \in \mathbb{R}^{M\times M}
\]

5. Analysis of Maximum Likelihood Estimation

Theorem 1. Consider a single-cell sparse user activity detection problem. Let \( I \) be the index set corresponding to inactive users, i.e., \( Z \in \{1 | \gamma = 0 \} \). Define two sets \( \mathcal{N}, \mathcal{C} \in \mathbb{R}^{N\times N} \) as follows:

\[
\mathcal{N} = \{ x \ | x^T \gamma x = 0, x \in \mathbb{R}^N \},
\]

\[
\mathcal{C} = \{ x \ | x \geq 0, x^T \gamma x \in \mathbb{R}^N \}
\]

where \( \gamma \) is the \( \text{th} \) entry of \( \gamma \), and \( \mathcal{N} \bigcap \mathcal{C} = \emptyset \). The activity indicator \( \hat{\chi} \) can be estimated as

\[
\hat{\chi} = \arg \min_{\chi \in [0,1]^N} \frac{1}{M} \sum_{m=1}^{M} \log | \mathcal{N} \bigcap \mathcal{C} |
\]

6. Cooperative Activity Detection in Multicell Systems

- Multi-cell system with \( B \) BSs each equipped with \( M \) antennas;
- Received signal \( Y_k \in C^{N \times M} \) at BS \( k \) is

\[
Y_k = \sum_{n=1}^{N} a_n b_n^T + \sum_{j=1}^{B} \sum_{l \neq k} a_j b_j^T h_{j,l} + Z_k
\]

where \( a_{\alpha} \in \{1,0\} \) is the activity indicator.

Cooperative detection: To alleviate the impact of inter-cell interference, we consider a BS cooperation architecture by assuming that all BSs are connected to a central unit, where all received signals are jointly processed.

- When \( G_{\alpha,j} \) is unknown, we need to estimate \( \Gamma_{\alpha,j} = A_{\alpha,j} G_{\alpha,j} \):  
  \( N \times N \) unknown parameters
- When \( G_{\alpha,j} \) are known, we only need to estimate \( A_{\alpha,j} \):
  \( N \times N \) unknown parameters

Theorem 2. Suppose that large-scale fading \( b_n \) is known, for estimating the activity indicator \( a_n \in \{0,1\} \), we define

\[
A_{\alpha} = \{ x \ | x^T \gamma x = 0, x \in \mathbb{R}^R \},
\]

\[
\mathcal{C} = \{ x \ | x \geq 0, x^T \gamma x \in \mathbb{R}^R \}
\]

then a necessary and sufficient condition for the consistency of \( A_{\alpha} \), i.e., \( A_{\alpha} \rightarrow A_{\alpha}^* \) as \( M \rightarrow \infty \), is \( A_{\alpha} \bigcap \mathcal{C} \neq \emptyset \).

Theorem 3. Consider the problem of maximum likelihood estimation of user activities in a multicell cooperative system assuming large-scale fading. The probability of error (PF = PM) is

\[
\text{PF} = \mathbb{P}(\hat{A}_{\alpha,j} \neq A_{\alpha,j})
\]

8. Conclusions

- Device activity detection for massive random access is a sparse recovery problem.
- This paper extends the analysis and algorithm for covariance based MLE of device activity detection to multicell systems.
- Multicell cooperative detection has a similar phase transition as single-cell system, but only if large-scale fading is known.
- In practice, there is significant performance degradation due to intercell interference.

References