Active Beamforming for Integrated Sensing and Communication

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***Abstract*—This work considers multiple-input multiple-output beamforming design for integrated sensing and multi-user communication. The existing beamforming strategies focus on the non-adaptive setting wherein the beamforming matrix is held fixed over a sensing interval that spans several communication transmission blocks. In contrast, this work examines the active sensing setup in which the base station sequentially updates the beamforming matrix over consecutive time slots in order to adaptively perform the sensing task. Particularly, we formulate the beamforming design problem in a Bayesian sequential framework that selects the current beamformer according to a posterior distribution of the unknown parameter to be estimated, while simultaneously guaranteeing quality-of-service constraints for the communication users. The posterior distribution is then updated based on the current observation, thereby allowing the next beamformer to be designed. Numerical simulations indicate that the proposed active beamforming strategy outperforms the nonactive counterparts.**

I. Introduction

Integrated Sensing and Communications (ISAC) is a promising technology envisioned to make its way into future generations of wireless networks and their emerging applications such as autonomous vehicles and smart homes [1]–[3]. Compared with traditional systems, the integration of communication and radar offers better utilization of the wireless spectrum, increased hardware and cost efficiency, and reduced power consumption. This is especially true for the so-called Dual-Functional-Radar-Communication (DFRC) framework in which the communication and radar systems operate simultaneously over a single hardware unit and utilize the same resources to accomplish their respective tasks.

This work is concerned with the design of the transmit waveform, used for simultaneous communication and sensing, for a base station (BS) employing a DFRC unit and utilizing a multiple-input multiple-output (MIMO) antenna array. More specifically, we consider a model in which the BS aims to construct a transmit beamforming matrix with the goal of serving multiple communication users while simultaneously estimating the channel parameters (i.e., path gain and angle) of an unknown point target of interest. In the literature, this model has been previously considered in several existing works [4]–[6]. For instance, the work in [4] develops a beamforming design strategy in which the information-precoded waveform (i.e., the communication waveform) is also used to synthesize a predefined beampattern corresponding to the target's location. The authors of [5] later extend this beampattern strategy by expressing the transmit waveform as the sum of an information-precoded waveform and a precoded radar waveform. The advantage of using this signaling scheme is that the radar waveform is permitted to have maximum degrees of freedom, thereby allowing for a more flexible beampattern design. Finally, the work in [6] applies the signaling scheme of [5] to develop a beamforming strategy based on the optimization of the classical Cramér-Rao bound (CRB). In all previous works, the beamformers are assumed to remain fixed across the transmission blocks of a given coherence interval.

The work considered herein differs from existing works [4]–[6] in that we allow the BS to adaptively adjust its beamformers across transmission blocks so that the sensing task can now be performed in an active fashion. More specifically, the BS will now seek to actively query the environment using a sequence of carefully chosen beamformers, rather than passively building up its knowledge of the unknown parameters using a fixed beamformer. Our motivation behind adopting such a strategy comes from the success of active sensing schemes in tackling problems in which the measurements arrive in a sequential fashion. For instance, existing works [7]–[9] have already demonstrated that active beamforming strategies have the ability to exceed the performance of the nonactive counterparts in several sensing-only scenarios. With this in mind, it is not clear whether active strategies will remain beneficial in ISAC applications since the BS is required to maintain a "static" link with the communication users, which in turn, requires the beamformers to remain fixed in some sense. The main goal of this work is to show that active beamforming strategies can still prove successful even while imposing additional communication constraints.

The proposed active strategy not only improves on previous works but is also applicable in scenarios where one cannot immediately apply the existing strategies. In particular, all previous works assume knowledge of a coarse or initial estimate prior to the beamforming design. For instance, the classical CRB approach [6] uses an optimization criterion that depends on the unknown parameter to be estimated. Similarly, [5] adopts a beampattern-based design which requires some prior information of where the target might be so that beam can be steered toward the desired direction. In many cases of interest, such prior knowledge may not be readily available. In those cases, the framework we present herein is still applicable.

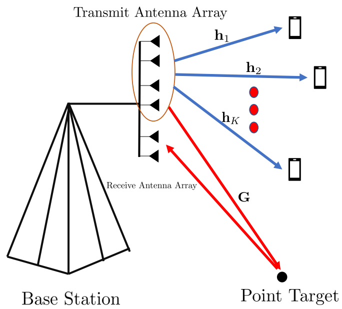

Fig. 1. The ISAC system considered in this work. The transmit and receive arrays are assumed to have N_{T} and N_{R} arrays, respectively.

We shall study the problem of adaptively designing the transmit beamformers under a Bayesian framework. More specifically, the sequential active beamforming strategy we present herein draws upon a Bayesian sequential framework that has been previously employed in several radar tracking applications, e.g., [10], [11]. Under such a framework, the BS encodes its knowledge of previous measurements in the form of a posterior distribution of the unknown parameters. This posterior distribution is constantly updated upon the arrival of new measurements and is subsequently used to design future beamforming matrices. In the proposed strategy, the beamforming design problem follows an optimization criterion that aims at maximizing the received power at target locations, subject to quality of service (QoS) constraints for the communication users. We show that such an optimization strategy possesses a convex formulation as a semi-definite program (SDP). Thus, the exact solution to the beamforming design problem can be efficiently obtained in polynomial time. Further, we also present an efficient scheme that implements the posterior update step. Finally, we provide numerical simulations demonstrating how the performance of the proposed active strategy exceeds that of the nonactive counterparts in several regimes of interest.

II. System Model

We consider an ISAC system, depicted in Fig. 1, over a coherence interval of L transmission blocks. In such a system, a BS with co-located N_{T} transmit antenna array and N_{R} receive antenna array wishes to send downlink information to $K \leq N_{\mathrm{T}}$ single-antenna users while simultaneously learning the channel parameters for a target of interest. Of particular interest is the beamforming model proposed in [5] where the ℓ-th transmitted waveform is given by

$$\tilde{\mathbf{x}}[\ell] = \mathbf{V}_{\mathrm{C}}\mathbf{c}[\ell] + \mathbf{V}_{\mathrm{S}}\mathbf{s}[\ell], \quad \ell \in \{0, \ldots, L-1\}, \tag{1}$$

where $\mathbf{c}[\ell] \triangleq [c_{1,\ell}, \ldots, c_{K,\ell}]^{\mathsf{T}} \in \mathbb{C}^K$ is a zero-mean vector of communication symbols intended to the communication users during the ℓ-th transmission block with $\mathbb{E}\{\mathbf{c}[\ell]\mathbf{c}[\ell]^{\mathsf{H}}\} = \mathbf{I}_K$, $\mathbf{s}[\ell] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_{\mathrm{T}}}) \in \mathbb{C}^N$ is probing sequence assumed to be statistically independent of $\mathbf{c}[\ell]$. Finally, $\mathbf{V}_{\mathrm{C}} \in \mathbb{C}^{N_{\mathrm{T}} \times K}$ and $\mathbf{V}_{\mathrm{S}} \in \mathbb{C}^{N_{\mathrm{T}} \times N_{\mathrm{T}}}$ are linear beamformers applied to $\mathbf{c}[\ell]$ and $\mathbf{s}[\ell]$. This beamforming model can be seen as a generalization to the conventional beamforming strategy obtained by setting $\mathbf{V}_{\mathrm{S}}$ to zero. In ISAC applications, it is generally preferable to consider the model in (1) over the conventional strategy due to the added degrees of freedom.

In this work, we further generalize (1) by allowing the beamformers to be different across different transmission blocks of the coherence interval. In other words, the transmitted signal in the ℓ-th transmission block is now given by

$$\mathbf{x}[\ell] = \mathbf{V}_{\mathrm{C}}[\ell]\mathbf{c}[\ell] + \mathbf{V}_{\mathrm{S}}[\ell]\mathbf{s}[\ell], \quad \ell \in \{0, \ldots, L-1\}, \tag{2}$$

where the matrices of $\mathbf{V}_{\mathrm{C}}[0], \ldots, \mathbf{V}_{\mathrm{C}}[L-1]$ and $\mathbf{V}_{\mathrm{S}}[0], \ldots, \mathbf{V}_{\mathrm{S}}[L-1]$ are linear beamformers of dimensions $N_{\mathrm{T}} \times K$ and $N_{\mathrm{T}} \times N_{\mathrm{T}}$, respectively, and whose design is the main focus of this paper. Clearly, setting $\mathbf{V}_{\mathrm{C}}[\ell] = \mathbf{V}_{\mathrm{C}}$, and $\mathbf{V}_{\mathrm{S}}[\ell] = \mathbf{V}_{\mathrm{S}}, \forall \ell$ reduces the beamforming model in (2) to the one in (1). Hence, the work proposed herein encompasses the work of [5] and [6] as special cases.

In each transmission block, we impose a constraint on the transmitted power from all antennas elements, i.e., $\mathbb{E}\{\|\mathbf{x}[\ell]\|^2\} \leq P_{\mathrm{D}}$, where P_{D} is the total downlink power. This implies the following constraint on the ℓ-th beamformers.

$$\mathrm{Tr}\left(\mathbf{V}_{\mathrm{C}}[\ell]^{\mathsf{H}}\mathbf{V}_{\mathrm{C}}[\ell]\right) + \mathrm{Tr}\left(\mathbf{V}_{\mathrm{S}}[\ell]^{\mathsf{H}}\mathbf{V}_{\mathrm{S}}[\ell]\right) \leq P_{\mathrm{D}}, \quad \forall \ell. \tag{3}$$

Our main goal is to design the beamformers adaptively. Specifically, at any given transmission block, the radar receiver is assumed to have already observed the measurements due to transmissions in previous blocks, which is generally true since the round-trip delay is typically negligible compared to the length of one transmission block. Hence, we can utilize these historical measurements to construct the next beamformers in an intelligent/active manner, thus enabling the estimation performance to exceed that of the nonactive schemes. We now outline both radar and communication models below.

A. Radar Model

We consider a point-target radar model in which the $N_{\mathrm{R}} \times 1$ backscattered waveform, received at the BS, is expressed by

$$\mathbf{y}_{\mathrm{S}}[\ell] = \mathbf{G}\mathbf{x}[\ell] + \mathbf{n}[\ell], \quad \ell \in \{0, \ldots, L-1\}. \tag{4}$$

Here, $\mathbf{n}[\ell] \sim \mathcal{CN}\left(\mathbf{0}, \sigma_{\mathrm{s}}^2 \mathbf{I}_{N_{\mathrm{R}}}\right)$ is a Gaussian noise vector and $\mathbf{G} \in \mathbb{C}^{N_{\mathrm{R}} \times N_{\mathrm{T}}}$ is the "round trip" channel between the transmit and receive antenna arrays and whose functional form is

$$\mathbf{G} \triangleq \alpha \mathbf{a}_{\mathrm{R}}(\theta)\mathbf{a}_{\mathrm{T}}(\theta)^{\mathsf{H}}, \tag{5}$$

where $\mathbf{a}_{\mathrm{T}}(\cdot)$ and $\mathbf{a}_{\mathrm{R}}(\cdot)$ are the transmit and receive steering vectors, respectively, and α and θ are the path gain and angle. Here, we make the following assumptions: i) The BS employs uniform transmit and receive antenna arrays with a half-wavelength antenna separation, in which case the steering vectors are given by

$$\mathbf{a}_{\mathrm{D}}(\theta) = \frac{1}{\sqrt{N_{\mathrm{D}}}}\left[e^{\imath\pi 0 \sin(\theta)}, \ldots, e^{\imath\pi(N_D-1)\sin(\theta)}\right]^{\mathsf{T}}, \mathrm{D} \in \{\mathrm{R}, \mathrm{T}\}.$$

ii) The path parameters α and θ are both random and unknown. Further, these parameters (and hence $\mathbf{G}$) are independent of the communication channel and remain constant over the coherence interval. iii) The radar is interested in the joint estimation of α and θ but has no prior knowledge on either.

From a radar perspective, the goal is to estimate the parameters after L transmission blocks, where the performance is measured in terms of the mean-squared error (MSE):

$$\text{mse} = \mathbb{E}\left[(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}_L)^2\right], \tag{6}$$

where $\boldsymbol{\eta} \triangleq \begin{bmatrix}\Re(\alpha) & \Im(\alpha) & \theta\end{bmatrix}^{\mathsf{T}}$ and $\hat{\boldsymbol{\eta}}_L \triangleq \hat{\boldsymbol{\eta}}(\mathbf{x}[0], \mathbf{y}_{\mathrm{S}}[0], \ldots, \mathbf{x}[L-1], \mathbf{y}_{\mathrm{S}}[L-1])$ is an estimator constructed from all L observations.

B. Communication Model

Upon transmitting $\mathbf{x}[\ell]$, the received waveform at the k-th communication user is given by

$$\begin{aligned} y^{\mathrm{c}}_{k,\ell} &= \mathbf{h}_k^{\mathsf{H}}\mathbf{x}[\ell] + n_{k,\ell}, \quad \ell \in \{0,\ldots,L\}, \\ &= \mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[\ell]c_{k,\ell} + \mathbf{h}_k^{\mathsf{H}}\left(\sum_{i\neq k}\mathbf{v}_i[\ell]c_{i,\ell} + \mathbf{V}_{\mathrm{S}}[\ell]\mathbf{s}[\ell]\right) + n_{k,\ell}, \end{aligned} \tag{7}$$

where $\mathbf{h}_k$ is the channel between the BS and the k-th user and whose elements are assumed to be independent identically-distributed (i.i.d) complex Gaussian $\mathcal{CN}(0,1)$ and $\mathbf{v}_k[\ell]$ is the k-th vector of $\mathbf{V}_{\mathrm{C}}[\ell]$, i.e., $\mathbf{V}_{\mathrm{C}}[\ell] \triangleq [\mathbf{v}_1[\ell],\ldots,\mathbf{v}_K[\ell]]$. For simplicity, we assume that $\mathbf{H} \triangleq [\mathbf{h}_1,\ldots,\mathbf{h}_K] \in \mathbb{C}^{N_T\times K}$ is perfectly known at the BS. The term $n_{k,\ell} \sim \mathcal{CN}\left(0,\sigma_c^2\right)$ denotes the additive noise term. Note that the impact of $\mathbf{V}_{\mathrm{S}}[\ell]$ on the communication channel manifests itself as interference.

In this paper, we assume that the communication links are established with the purpose of satisfying certain QoS requirements for the communication users. In particular, the users have minimum data rate demands that they must meet. This corresponds to the following set of constraints:

$$\frac{\left|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[\ell]\right|^2}{\sum_{i\neq k}\left|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_i[\ell]\right|^2 + \mathbf{h}_k^{\mathsf{H}}\mathbf{V}_{\mathrm{S}}[\ell]^{\mathsf{H}}\mathbf{V}_{\mathrm{S}}[\ell]\mathbf{h}_k + \sigma_c^2} \geq \gamma_k, \ \forall \ell, k,$$

where γ_k is the signal-to-interference-and-noise-ratio (SINR) threshold for user k.

III. Active Beamforming Design

The proposed active beamforming scheme follows a general framework known as Bayesian Sequential Inference; see [10] and [11]. In this framework, the radar receiver keeps track of a posterior distribution of the unknown parameter of interest. Such posterior distribution is constantly updated upon the arrival of new observations. At any time instance, the next transmit waveform is then selected based on the current posterior distribution, usually according to some optimization criterion of a certain performance metric. For the ISAC system previously described, the general procedure can be summarized in terms of the following stages:

1) Environment Interrogation: At time ℓ, the BS is presented with the beamforming matrices $\mathbf{V}_{\mathrm{S}}[\ell]$ and $\mathbf{V}_{\mathrm{C}}[\ell]$, a set of communication symbols $\mathbf{c}[\ell]$, and a probing sequence $\mathbf{s}[\ell]$. The BS forms $\mathbf{x}[\ell]$ in (2), transmits it over the ISAC channel, and receives $\mathbf{y}_{\mathrm{S}}[\ell]$ in (4).

2) Belief Update: Upon observing $\mathbf{y}_{\mathrm{S}}[\ell]$, the BS utilizes the new observation $\mathbf{z}_\ell \triangleq (\mathbf{y}_{\mathrm{S}}[\ell], \mathbf{x}[\ell])$ to update its posterior distribution of the unknown parameters, denoted by $r_\ell(\boldsymbol{\eta})$, according to Bayes' rule:

$$\begin{aligned} r_\ell(\boldsymbol{\eta}) &\propto f\left(\boldsymbol{\eta}|\mathbf{z}_{0:(\ell-1)}\right) f\left(\mathbf{z}_\ell|\boldsymbol{\eta},\mathbf{z}_{0:(\ell-1)}\right) \\ &= r_{\ell-1}(\boldsymbol{\eta}) f\left(\mathbf{z}_\ell|\boldsymbol{\eta},\mathbf{z}_{0:(\ell-1)}\right), \end{aligned} \tag{8}$$

where the likelihood is given by

$$f\left(\mathbf{z}_\ell|\boldsymbol{\eta},\mathbf{z}_{0:(\ell-1)}\right) = f\left(\mathbf{x}[\ell]\right) f\left(\mathbf{y}_S[\ell]|\mathbf{x}[\ell],\boldsymbol{\eta},\mathbf{z}_{0:(\ell-1)}\right).$$

Note that the density function $f(\mathbf{x}[\ell])$ is generally known at the BS since the densities $f(\mathbf{c}[\ell])$ and $f(\mathbf{s}[\ell])$ are both known. In particular, in the case of Gaussian signaling, we have $\mathbf{x}[\ell] \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{V}_{\mathrm{S}}[\ell]\mathbf{V}_{\mathrm{S}}[\ell]^{\mathsf{H}} + \mathbf{V}_{\mathrm{C}}[\ell]\mathbf{V}_{\mathrm{C}}[\ell]^{\mathsf{H}}\right)$.

3) Next Beamforming Selection: The BS selects its next beamforming matrices $\mathbf{V}_{\mathrm{C}}[\ell+1]$ and $\mathbf{V}_{\mathrm{S}}[\ell+1]$ so as to optimize a performance metric which depends on the current posterior. This paper proposes to maximize the signal power at target locations corresponding to the current posterior, i.e.,

$$\mathbb{E}\left[|\alpha|^2|\mathbf{a}_{\mathrm{T}}(\theta)^{\mathsf{H}}\mathbf{x}[\ell+1]|^2\middle|\mathbf{z}_{0:\ell}\right] = \mathrm{Tr}\left(\mathbf{A}[\ell+1]\mathbf{R}[\ell+1]\right), \tag{9}$$

where $\mathbf{R}[\ell+1] \triangleq \mathbf{V}_{\mathrm{C}}[\ell+1]\mathbf{V}_{\mathrm{C}}[\ell+1]^{\mathsf{H}} + \mathbf{V}_{\mathrm{S}}[\ell+1]\mathbf{V}_{\mathrm{S}}[\ell+1]^{\mathsf{H}}$ is the beamforming covariance matrix, and $\mathbf{A}[\ell+1]$ is a positive semi-definite matrix, given by:

$$\mathbf{A}[\ell+1] \triangleq \mathbb{E}\left[|\alpha|^2\mathbf{a}_{\mathrm{T}}(\theta)\mathbf{a}_{\mathrm{T}}(\theta)^{\mathsf{H}}\middle|\mathbf{z}_{0:\ell}\right]. \tag{10}$$

The interpretation of (9) is as follows: At time ℓ, the BS has access to some posterior distribution $r_\ell(\eta)$. The support of such posterior distribution corresponds to a set of candidate locations in which the target is believed to be. It is thus natural to query these target locations in the next transmission slot in an attempt to search for the target. Since we wish to query all candidate directions at once, a question that arises is how to split the total power among these different candidate locations. In the absence of communication users, maximization the above metric allocates the power in proportion to the posterior weights, which essentially capture the degree to which a target exists at a given location.

A. Beamforming Optimization

We are now ready to discuss how to design the precoding matrices at every time step. We divide the overall design into two stages: i) Beamforming for $\ell = 0$, and ii) Beamforming for $\ell \geq 1$.

1) Beamforming Design for $\ell = 0$: In the initial stage, the BS does not possess any information about the target location. Hence, from a radar point of view, it is natural to transmit a beamformer whose beampattern is omnidirectional, i.e., transmit the same amount of power in every direction:

$$\mathbf{a}^{\mathsf{T}}(\theta)\mathbf{R}[0]\mathbf{a}(\theta) = c, \quad \forall\theta, \tag{11}$$

which, in the absence of communication constraints, can be achieved by setting the covariance matrix to $\mathbf{R}[0] = \mathbf{R}_{\text{omni}} \triangleq \sqrt{\frac{P_{\mathrm{D}}}{N_{\mathrm{T}}}}\mathbf{I}_{N_{\mathrm{T}}}$. Owing to the existence of communication users, it may not always be possible to synthesize this desired beampattern. Instead, we focus on finding a pair of beamformers $\mathbf{V}_{\text{C}}[0]$ and $\mathbf{V}_{\text{S}}[0]$ whose covariance is as close as possible to $\mathbf{R}_{\text{omni}}$ without violating the SINR constraints. Mathematically, this is cast as the following optimization problem

$$\min_{\mathbf{V}_{\text{C}},\mathbf{V}_{\text{S}}} \quad \left\|\mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^{\mathsf{H}} + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}} - \mathbf{R}_{\text{omni}}\right\|^2 \tag{12a}$$

$$\text{subject to} \quad \frac{\left|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k\right|^2}{\sum_{i\neq k}\left|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_i\right|^2 + \mathbf{h}_k^{\mathsf{H}}\mathbf{V}_{\text{S}}^{\mathsf{H}}\mathbf{V}_{\text{S}}\mathbf{h}_k + \sigma_{\text{c}}^2} \geq \gamma_k. \tag{12b}$$

$$\operatorname{Tr}\left(\mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^{\mathsf{H}} + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}}\right) \leq P_{\text{D}}. \tag{12c}$$

We remark that (12) is a nonconvex problem due to the SINR constraints (12b). Nonetheless, it turns out that a global solution can still be obtained. To show this, we relax (12) into a convex semi-definite form by introducing the new variables

$$\mathbf{R} \triangleq \sum_k \mathbf{R}_k + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}}, \quad \mathbf{R}_k \triangleq \mathbf{v}_k\mathbf{v}_k^{\mathsf{H}}, \quad \forall k. \tag{13}$$

Next, we obtain a semi-definite relaxation (SDR) of (12) by dropping the rank-1 constraints on $\mathbf{R}_k$. Thus, we have

$$\begin{aligned} \min_{\mathbf{R},\mathbf{R}_1,\ldots,\mathbf{R}_K} \quad & \|\mathbf{R} - \mathbf{R}_{\text{omni}}\|^2 \\ \text{subject to} \quad & \operatorname{Tr}(\mathbf{R}) \leq P_{\text{D}}, \\ & \operatorname{Tr}\left(\mathbf{Q}_k\left((1+\gamma_k)\mathbf{R}_k - \gamma_k\mathbf{R}\right)\right) \geq \gamma_k\sigma_c^2 \\ & \mathbf{R} \succcurlyeq \sum_k \mathbf{R}_k, \qquad \mathbf{R}_k \succcurlyeq 0 \qquad \forall k, \end{aligned} \tag{14}$$

where $\mathbf{Q}_k \triangleq \mathbf{h}_k\mathbf{h}_k^{\mathsf{H}}$. Generally speaking, convex relaxations obtained in this manner do not necessarily provide an optimal solution to the original problem. However, in this particular case, the solution of the relaxed problem allows us to immediately construct an optimal solution for (12). For completeness, we provide the details of how to construct the optimal solution and refer the reader to [5] for proof. Let $\mathbf{R}^*, \mathbf{R}_1^*, \ldots, \mathbf{R}_K^*$, be an optimal solution of (14). The following matrices constitute an optimal solution of (12).

$$\tilde{\mathbf{V}}_{\text{C}}\tilde{\mathbf{V}}_{\text{C}}^{\mathsf{H}} = \sum_k \frac{\mathbf{R}_k^*\mathbf{Q}_k{\mathbf{R}_k^*}^{\mathsf{H}}}{\operatorname{Tr}\left(\mathbf{Q}_k\mathbf{R}_k^*\right)}, \tag{15}$$

$$\tilde{\mathbf{V}}_{\text{S}}\tilde{\mathbf{V}}_{\text{S}}^{\mathsf{H}} = \mathbf{R}_{\text{S}}^* + \sum_k \mathbf{R}_k^* - \tilde{\mathbf{V}}_{\text{C}}\tilde{\mathbf{V}}_{\text{C}}^{\mathsf{H}}. \tag{16}$$

2) Beamforming Design for $\ell \geq 1$: After the initial stage, the BS has established a link with the communication users which involves choosing certain modulation and coding (MC) parameters based on the target SINR for the communication users. In subsequent stages, the task now becomes that of maintaining such communication link (i.e., by keeping those parameters fixed and by fixing the channels seen by the users) while actively searching for the target. In particular, for $\ell \geq 1$, we have the following constraints for the k-th user

$$\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[\ell] = \mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[0], \tag{17}$$

and

$$\sum_{i\neq k}|\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_i|^2 + \mathbf{h}_k^{\mathsf{H}}\mathbf{V}_{\text{S}}^{\mathsf{H}}\mathbf{V}_{\text{S}}\mathbf{h}_k + \sigma_{\text{c}}^2 \leq d_k. \tag{18}$$

To see why (17) is necessary, note that the users typically employ a coherent receiver to decode their intended symbol. This requires knowledge of the scalar channel seen by each user, i.e., $\mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[\ell]$, for the ℓ-th transmission block. In the conventional system where the beamformers remain fixed across transmission blocks, the users learn these coefficients at the beginning of the coherence interval with the aid of a short downlink pilot sequence. Since we design a new beamformer for every transmission symbol, such scalar channels must remain fixed throughout the coherence interval. Furthermore, (18) is an interference constraint, made to ensure that the MC schemes selected in the initial stage remain operational in subsequent stages with an acceptable error probability and d_k is the maximum allowable interference level. Note that d_k is generally a deterministic function of the target SINR (i.e., γ_k) and whose exact expression depends on the selected MC scheme. To keep the discussion general, we do not specify the exact relationship herein.

From a radar perspective, recall that for any $\ell \geq 1$, the BS has already observed the measurements $\mathbf{z}_0, \ldots, \mathbf{z}_{\ell-1}$. Hence, it can use these measurements to determine the posterior distribution $r_{\ell-1}(\eta)$ and thus compute $\mathbf{A}[\ell]$ in (10). Given $\mathbf{A}[\ell]$, the main goal is to choose the beamformers that maximize the signal strength at the posterior target locations, while simultaneously satisfying the constraints imposed by the communication system. With this in mind, we can now express the beamforming design for $\ell \geq 1$ as follows

$$\min_{\mathbf{V}_{\text{C}},\mathbf{V}_{\text{S}}} \quad -\operatorname{Tr}\left(\mathbf{A}[\ell]\left(\mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^{\mathsf{H}} + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}}\right)\right) \tag{19a}$$

$$\text{subject to} \quad \mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k = c_k, \qquad \forall k, \tag{19b}$$

$$\operatorname{Tr}\left(\mathbf{Q}_k\mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^H + \mathbf{Q}_k\mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^H\right) \leq \tilde{d}_k, \tag{19c}$$

$$\operatorname{Tr}\left(\mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^H + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^H\right) \leq P_{\text{D}}, \tag{19d}$$

where we define $c_k \triangleq \mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k[0]$, and $\tilde{d}_k \triangleq d_k - \sigma_c$. Problem (19) is a quadratically-constrained quadratic problem (QCQP) in which constraints (19b)-(19d) are all convex, but the objective function is concave. The overall problem is thus nonconvex. We remark, however, that this problem is always feasible. To see this, it suffices to find a pair of matrices $(\mathbf{V}_{\text{C}}, \mathbf{V}_{\text{S}})$ that satisfy the constraints. It can be readily verified that $\mathbf{V}_{\text{C}} = \mathbf{V}_{\text{C}}[0]$ and $\mathbf{V}_{\text{S}} = \mathbf{0}$ satisfy (19b)-(19d).

To tackle (19), we reformulate it into a convex SDP. This is done by introducing the auxiliary variable $\mathbf{R} \triangleq \mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^{\mathsf{H}} + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}}$. Based on this, we can now express (19) as the equivalent problem

$$\min_{\mathbf{V}_{\text{C}},\mathbf{V}_{\text{S}},\mathbf{R}} \quad -\operatorname{Tr}\left(\mathbf{A}[\ell]\mathbf{R}\right) \tag{20a}$$

$$\text{subject to} \quad \mathbf{h}_k^{\mathsf{H}}\mathbf{v}_k = c_k, \quad \forall k, \tag{20b}$$

$$\operatorname{Tr}\left(\mathbf{Q}_k\mathbf{R}\right) \leq d_k, \quad \forall k, \tag{20c}$$

$$\operatorname{Tr}\left(\mathbf{R}\right) \leq P_{\text{D}}, \tag{20d}$$

$$\mathbf{R} = \mathbf{V}_{\text{C}}\mathbf{V}_{\text{C}}^{\mathsf{H}} + \mathbf{V}_{\text{S}}\mathbf{V}_{\text{S}}^{\mathsf{H}}. \tag{20e}$$

Aside from (20e), the objective and all other constraints are convex. Furthermore, it is easy to see that $\mathbf{V}_{\mathrm{S}}$ acts as a slack variable in (20e). As a consequence, we may equivalently express this constraint as $\mathbf{R} \succcurlyeq \mathbf{V}_{\mathrm{C}}\mathbf{V}_{\mathrm{C}}^{\mathsf{H}}$. Using the Schur complement, this new constraint can be expressed as the following convex semi-definite constraint

$$\begin{bmatrix} \mathbf{R} & \mathbf{V}_{\mathrm{C}} \\ \mathbf{V}_{\mathrm{C}}^{\mathsf{H}} & \mathbf{I}_K \end{bmatrix} \succcurlyeq 0.$$

With this constraint, problem (20) is now equivalent to

$$\min_{\mathbf{V}_{\mathrm{C}},\mathbf{R}} \quad -\operatorname{Tr}\left(\mathbf{A}[\ell]\mathbf{R}\right) \tag{21a}$$

$$\text{subject to} \quad (20\text{b}) - (20\text{d})$$

$$\begin{bmatrix} \mathbf{R} & \mathbf{V}_{\mathrm{C}} \\ \mathbf{V}_{\mathrm{C}}^{\mathsf{H}} & \mathbf{I}_{\mathbf{K}} \end{bmatrix} \succcurlyeq 0. \tag{21b}$$

Problem (21) is a convex SDP, for which efficient solvers exist, e.g., CVX [12]. After obtaining the optimal solution of (21), denoted by $\mathbf{V}_{\mathrm{C}}^{*}$ and $\mathbf{R}^{*}$, the optimal solution $\left(\tilde{\mathbf{V}}_{\mathrm{C}}, \tilde{\mathbf{V}}_{\mathrm{S}}\right)$ of (21) is given by $\tilde{\mathbf{V}}_{\mathrm{C}} = \mathbf{V}_{\mathrm{C}}^{*}$, and $\tilde{\mathbf{V}}_{\mathrm{S}}\tilde{\mathbf{V}}_{\mathrm{S}}^{\mathsf{H}} = \mathbf{R}_{\mathrm{S}}^{*} - \tilde{\mathbf{V}}_{\mathrm{C}}\tilde{\mathbf{V}}_{\mathrm{C}}^{\mathsf{H}}$.

B. Posterior Tracking

The proposed algorithm requires storing and tracking of $r_\ell(\boldsymbol{\eta})$ in order to compute $\mathbf{A}[\ell]$ in each step. Since $r_\ell(\boldsymbol{\eta})$ is a continuous distribution, a natural question that arises is how to efficiently parameterize $r_\ell(\boldsymbol{\eta})$ so that the posterior update step can be carried out numerically? This subsection addresses this question. To this end, we consider a grid assumption for θ. Let N_{g} denote the grid size and $\theta_1, \ldots, \theta_{N_{\mathrm{g}}}$ be the grid values. At any time ℓ, the continuous angle posterior distribution is approximated by the discrete distribution

$$p\left(\theta|\mathbf{z}_{0:\ell}\right) = \sum_{n=1}^{N_{\mathrm{g}}} w_{n,\ell}\delta(\theta - \theta_n), \tag{22}$$

where $w_{n,\ell}$ is the probability that $\theta = \theta_n$, with $\sum_n w_{n,\ell} = 1$, and $\delta(\cdot)$ is the dirac-delta function. As a result, we have

$$r_\ell(\boldsymbol{\eta}) \approx \sum_{n=1}^{N_{\mathrm{g}}} w_{n,\ell} f\left(\alpha|\theta_n, \mathbf{z}_{0:\ell}\right)\delta(\theta - \theta_n).$$

Thus, to track $r_\ell(\boldsymbol{\eta})$, we instead track the weights $\{w_{n,\ell}\}_{n=1}^{N_g}$ and the family of posterior distributions $\{f(\alpha|\theta_n, \mathbf{z}_{0:\ell})\}_{n=1}^{N_g}$. The next result provides the means of doing this.

Proposition 1. *Suppose that α and θ are initially independent with $\alpha \sim \mathcal{CN}\left(\mu_0, \sigma_0^2\right)$ and let the angle prior be any arbitrary discrete distribution $p\left(\theta\right) \triangleq \sum_n w_{n,0}\delta(\theta - \theta_n)$. For any n and $\ell \geq 1$, define $\tilde{\mathbf{x}}_n[\ell] \triangleq \mathbf{a}_{\mathrm{R}}(\theta_n)\mathbf{a}_{\mathrm{T}}(\theta_n)^H\mathbf{x}[\ell]$. Then, we have:*

- $\alpha|\mathbf{z}_{0:\ell}, \theta_n \sim \mathcal{CN}\left(\mu_{n,\ell}, \sigma_{n,\ell}^2\right)$, *with*

$$\sigma_{n,\ell}^2 = \frac{\sigma_{n,(\ell-1)}^2\sigma_s^2}{\|\tilde{\mathbf{x}}_n[\ell]\|^2\sigma_{n,(\ell-1)}^2 + \sigma_s^2}, \tag{23}$$

$$\mu_{n,\ell} = \mu_{n,(\ell-1)} + \frac{\sigma_{n,\ell}^2}{\sigma_s^2}\left(\tilde{\mathbf{x}}_n^H[\ell]\mathbf{y}^s[\ell] + \mu_{n,(\ell-1)}\|\tilde{\mathbf{x}}_n[\ell]\|^2\right) \tag{24}$$

- $p(\theta|\mathbf{z}_{0:\ell}) = \sum_n w_{n,\ell}\delta(\theta - \theta_n)$, *with*

$$w_{n,\ell} = \frac{w_{n,(\ell-1)}e^{b_{n,\ell}}}{\sum_i w_{i,(\ell-1)}e^{b_{i,\ell}}}, \tag{25}$$

where $b_{n,\ell} \triangleq -\frac{\left|\mu_{n,(\ell-1)}\right|^2}{\sigma_{n,(\ell-1)}^2} + \frac{|\mu_{n,\ell}|^2}{\sigma_{n,\ell}^2} + \log\frac{\sigma_{n,\ell}^2}{\sigma_{n,(\ell-1)}^2}$.

Proof. The proof depends on classical results in estimation theory. The details are omitted for space limitations. □

The previous proposition states that, if the prior for α is Gaussian, then the posterior distribution of α, when conditioned on a specific $\theta = \theta_n$, is Gaussian whose mean and variance depend on θ_n as shown in (23) and (24). Hence, the family of distributions $\{f(\alpha|\theta_n, \mathbf{z}_{0:\ell})\}_{n=1}^{N_g}$ is parameterized by $\{\mu_{n,\ell}, \sigma_{n,\ell}^2\}_{n=1}^{N_{\mathrm{g}}}$, and the overall posterior $r_\ell(\boldsymbol{\eta})$ is completely specified by the parameters $\{w_{n,\ell}, \mu_{n,\ell}, \sigma_{n,\ell}^2\}_{n=1}^{N_{\mathrm{g}}}$.

IV. Simulation Results

We now examine the performance of the proposed adaptive beamforming design. In the following, we consider an ISAC system with $N_{\mathrm{T}} = 8$ transmit antennas, $N_{\mathrm{R}} = 3$ receive antennas, and $K = 3$ communication users having SINR thresholds of 10, 8, and 12 dB. Further, we assume that the BS antennas cover a single sector that spans the angular interval $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$. The communication symbols are drawn from a QPSK constellation. For the proposed beamforming design, we choose a uniform prior for the angle over $\left[-\frac{\pi}{3}, \frac{\pi}{3}\right]$ and a Gaussian prior $\mathcal{CN}(0,1)$ for the path gain. The grid size N_{g} is set to 240, which corresponds to an angle separation $\Delta\theta = 0.5^\circ$ between adjacent grid points. In the simulations, we report the average performance over many iterations where the true location of the target is chosen randomly over $[-\frac{\pi}{3}, \frac{\pi}{3}]$. Finally, we define the radar receive SNR as $\mathrm{SNR}_{\mathrm{RX}} = 10\log_{10}\frac{|\alpha|^2 P_{\mathrm{D}}}{\sigma^2}$.

To evaluate the performance of the proposed strategy, we consider comparisons against the following schemes: (i) Nonactive CRB-based Beamforming [6], (ii) Nonactive Beampattern-based Beamforming [5], and (iii) Time Sharing with Active Sensing Strategy. Note that the beamformers designed using the nonactive benchmarks are selected at the beginning of the coherence interval and held fixed in all subsequent transmission blocks. Whereas the proposed active strategy and the time-sharing benchmark are both active. For the CRB-based benchmark, we optimize a Bayesian version of the CRB-expression in [6], since the classical CRB expression of [6] requires some knowledge of the true values of the channel parameters which we do not assume to be available herein. Similarly, we choose an omnidirectional beampattern for benchmark of [5] due to the lack of any prior information. Finally, the time-sharing benchmark corresponds to the case in which there exists no integration between the communication and radar systems. In which case, we split the BS operation into communication-only and radar-only modes, which last for βL and $(1-\beta)L$ transmission blocks, with $\beta \in [0,1]$. For communication, we use a zero-forcing beamformer.

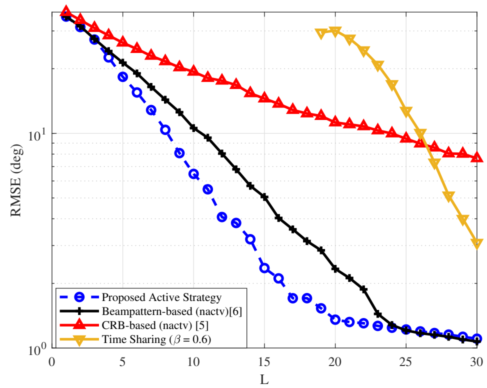

Fig. 2. The performance of the proposed design and existing benchmarks for fixed $\mathrm{SNR}_{\mathrm{RX}} = -5$ dB at different values of L.

In Fig. 2, we compare the performance of different beamforming approaches, measured in the root-MSE (RMSE) in angle estimation, against the number of transmission blocks for a fixed value of the received SNR. From this figure, it is seen that the proposed active beamforming can outperform all existing benchmarks. In particular, we observe that the proposed approach is able to correctly predict the true value of the AoA (i.e., within an error value of $\approx 1.5°$) after $L = 20$ transmission blocks, as compared to $L = 25$ for its closest competitor.

Finally, to give some insight into the operation of the proposed strategy, in Fig. 3, we investigate the evolution of the angle posterior over a duration of 6 transmission blocks. The results are shown for a true target located at $-40°$. In this figure, the first column corresponds to the angle posterior distribution, whereas the second column corresponds to the array response of the chosen beamformers. It is seen from this figure that the proposed active strategy initially selects a beamformer with a relatively uniform beam. Note, however, that the initial array response does not look exactly uniform due to the presence of a user at roughly $20°$. The algorithm then proceeds to generate the sequence of beamformers investigating several directions of where the true angle might be. Eventually, after a sufficient number of iterations, the proposed scheme is able to correctly predict the true angle value.

V. Conclusion

This work considers the beamforming design problem for an integrated sensing and multiuser communication system in which the base station seeks to learn the channel parameters of a target over a single coherence interval consisting of several transmission slots using a sequence of actively chosen beamformers. To tackle this problem, we propose a sequential framework wherein previous radar observations are incorporated into a posterior distribution the unknown parameters. Such posterior distribution is then used to determine the next beamformer according to an optimization criterion of the received signal strength. Such optimization criterion is shown to have an exact convex semi-definite formulation and therefore a global solution can be found in polynomial time.

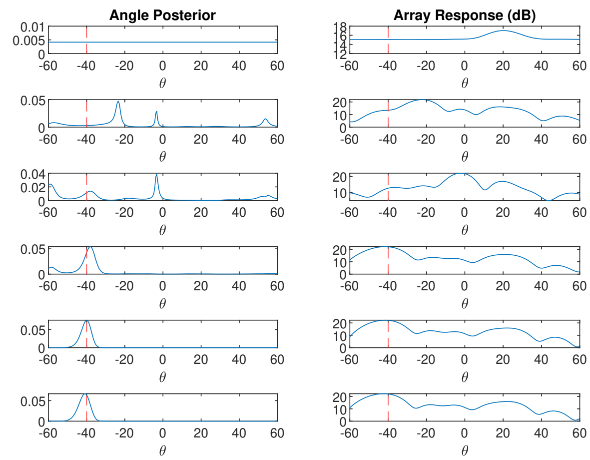

Fig. 3. The evolution of the AoA posterior distribution (first column) and the corresponding array response of the chosen beamformer (second column) at $\mathrm{SNR}_{\mathrm{RX}} = -5$ dB. In this figure, the radar target is at $-40°$ and there is a communication user at $20°$.

Numerical simulations reveal that the performance of the proposed methodology exceeds that of the nonactive counterparts.

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