Coding for the Blackwell Channel: A Survey Propagation Approach

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***Abstract*— Practical implementation of random binning is one of the key challenges in achieving the largest available rate regions for many multiuser channels. This paper explores the use of low-density parity-check (LDPC) like codes for a particular kind of deterministic broadcast channel called the Blackwell channel and illustrates that random linear codes can be used to construct practical binning schemes at rates close to the capacity region of the Blackwell channel. The key ingredient is an encoding algorithm known as "survey propagation" which is a generalization of the well-known belief propagation algorithm for LDPC codes. Survey propagation has been previously devised for a class of constraint satisfaction problems called K-SAT. This paper shows that the encoding problem for the Blackwell channel contains the same features as the constraint satisfaction problem and that the survey propagation algorithm, when concatenated with an outer error correcting code, works well at rates close to the Blackwell channel capacity region.**

I. Introduction

In a broadcast channel, a single transmitter sends independent information to multiple receivers at the same time. A key idea in coding for the broadcast channel is the binning strategy, which allows the transmitter to embed information by selecting a codeword in an appropriate bin. The receivers recover information by identifying the correct bin index rather than the codeword itself. The largest achievable rate region for the broadcast channel using the binning strategy is known as the Marton's region. For a discrete memoryless broadcast channel $p(y_1, y_2|x)$, the Marton's region is the union of:

$$\begin{aligned} R_1 &\leq I(U_1; Y_1) \\ R_2 &\leq I(U_2; Y_2) \\ R_1 + R_2 &\leq I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2) \end{aligned} \quad (1)$$

over joint distributions $p(u_1, u_2)p(x|u_1, u_2)$, where U_1 and U_2 are auxiliary random variables. Although a general converse is not yet known, the Marton's region has been proved to be optimal for degraded, less noisy, more capable, deterministic, semi-deterministic, and recently the Gaussian vector broadcast channel [1] [2]. A comprehensive review of the broadcast channel problem can be found in [1] and references therein.

A sketch of the achievability proof for the Marton's region goes as follows. The transmitter generates $2^{nI(U_1;Y_1)}$ u_1^n's and $2^{nI(U_2;Y_2)}$ u_2^n's at random. These u_1^n's and u_2^n's are binned into 2^{nR_1} and 2^{nR_2} bins, respectively. To transmit a pair of indices $m_1 \in \{1, \cdots, 2^{nR_1}\}$ and $m_2 \in \{1, \cdots, 2^{nR_2}\}$, the encoder searches in the joint bin indexed by m_1 and m_2 and finds a jointly typical pair of (u_1^n, u_2^n) in the joint bin. The transmitter then finds an x^n jointly typical with (u_1^n, u_2^n), and sends x^n. The decoders receive y_1^n and y_2^n respectively, look for $\hat{u}_1^n$ jointly typical with y_1^n, and $\hat{u}_2^n$ jointly typical with y_2^n, and recover the respective bin indices of $\hat{u}_1^n$ and $\hat{u}_2^n$. The probability of encoding and decoding error goes to zero as n goes to infinity if R_1 and R_2 are within the capacity region given in (1).

This paper deals with practical implementation of random binning for the broadcast channel. In addition to the broadcast channel problem, random binning is also at the heart of several other important results in multiuser information theory. For example, binning is an essential part of the coding theorem for the distributed source coding problem (the lossless version of which is often referred to as the Slepian-Wolf problem.) Likewise, binning also plays a key role in source and channel coding problems with side information. Thus, practical implementation of binning is of great interests.

Recently a great deal of progress has been made in the practical implementation of binning for lossless source coding. In this case, the existing practical single-user channel coding methods based on low-density parity-check (LDPC) codes are directly applicable. In the single-user channel coding problem, random LDPC codes with iterative sum-product decoding algorithm have been shown to achieve very close to the single-user capacity. This remarkable development in LDPC codes is directly applicable to binning for the Slepian-Wolf problem and the source coding with side information problem (e.g. [3] [4], [5]), because the encoding process in source coding is exactly that of finding a closest codeword based on a correlated observation sequence.

However, results from LDPC codes do not translate directly to practical implementation of binning for the broadcast channel problem or the channel coding with side information problem. This is because the iterative sum-product decoder is not a true maximum likelihood decoder. The iterative decoding algorithm works whenever the received signal is a noisy version of the transmitted codeword. This is the case in single-user channel coding problems, where the decoding process consists of recovering the codeword based on a perturbed observation sequence. Decoding is possible when the noise sphere due to the perturbation is sufficiently small. In contrast,

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for the broadcast channel, the message is sent via the bin index. The encoding process consists of finding a sequence in a particular bin that is jointly typical with some existing sequence. This is akin to the quantization process in which the sequence to be quantized is not necessarily in a noise sphere. While a true maximum likelihood decoder would have been equally applicable to both problems, for an iterative sub-optimal algorithm, the decoding of the single-user channel code is considerably easier.

Existing practical binning schemes for channel coding are often based on structured codes and maximum likelihood decoding. In [6], Zamir, Shamai and Erez explored the use of lattice codes for multiterminal binning. It is shown that lattice codes are capacity achieving. However, the complexity of lattice decoding grows rapidly with the dimension of the code. In [7], Erez and ten Brink used nested convolutional and turbo codes to approach the Gaussian channel capacity with side information where the binning process is implemented using maximum likelihood Viterbi algorithm at the encoder. Likewise, a related problem is solved in [8] for the binary symmetric channel with side information. All the above work use structured codes for binning.

Random practical codes are first used for quantization in the recent work of Martinian and Yedidia [9]. In [9], random codes and the iterative decoding algorithm are used for the quantization of a binary erasure source. However, the quantization process of [9] works for erasure sources only and is not applicable to the general broadcast channel.

This paper takes a different direction and directly explores practical random binning methods for the broadcast channel. Toward this end, we illustrate that low-density parity-check codes can be used as a practical binning scheme for a particular kind of deterministic broadcast channel called the Blackwell channel. Our use of the LDPC codes for binning in a broadcast channel is different from the use of LDPC codes for binning in the Slepian-Wolf problem. As mentioned earlier, the former is similar to a quantization process which is more difficult than the decoding of channel codes in the Slepian-Wolf setting. Consequently, decoding algorithms which are more powerful than the iterative sum-product algorithm (which works well in the Slepian-Wolf problem) is needed. The sum-product algorithm is also known as the belief propagation algorithm. The main contribution of this paper is the use of a new algorithm called "survey propagation" for binning. Survey propagation was first proposed for solving constraint satisfaction problems [10] [11]. This paper shows that binning for the broadcast channel is similar to constraint satisfaction, and that survey propagation, when combined with random parity-check codes and with a concatenated outer code, works well at rates close to the Blackwell channel capacity region.

Our work is most related to the previous algebraic construction of zero-error codes for the Blackwell channel by Vanroose and van der Meulen [12] where optimal codes with small block length (of less than 10) are constructed. Our approach is quite different in that random codes with large block length are constructed and a probabilistic decoding algorithm is used.

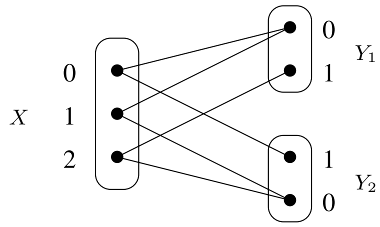

Fig. 1. The Blackwell Channel

II. Blackwell Channel

A. Capacity Region

The Blackwell channel belongs to a class of deterministic (also known as noiseless) broadcast channels, where the channel transition is deterministic (or $p(y_1, y_2|x)$ is a 0-1 function.) For the deterministic broadcast channel, the Marton's region is optimal [13]. This can be seen as follows. Choose the auxiliary random variables U_1 and U_2 in (1) as $U_1 = Y_1$ and $U_2 = Y_2$. (Such a choice is possible because Y_1 and Y_2 are known at the transmitter.) In this case, the Marton's region reduces to:

$$\begin{aligned} R_1 &\leq H(Y_1) \\ R_2 &\leq H(Y_2) \\ R_1 + R_2 &\leq H(Y_1, Y_2). \end{aligned} \quad (2)$$

Clearly, this is the best possible region under any fixed input distribution $p(x)$. The entire capacity region is then the union of the above region over all $p(x)$.

A well-known example of the deterministic broadcast channel is the Blackwell channel as illustrated in Fig. 1. In a Blackwell channel, the input has three symbols and the outputs have two symbols each. Out of four possible output combinations, one of them is not allowed. The other three combinations can be reached by selecting one of the three input symbols. As shown in Fig. 1, the disallowed combination is $(1, 1)$. The Blackwell channel is one of the first non-trivial broadcast channels studied in depth. The Blackwell channel is interesting because it clearly illustrates the conflict between transmitting information to Y_1 and transmitting to Y_2.

In the Blackwell channel, X can be directly determined from (Y_1, Y_2). Thus, $H(Y_1, Y_2) = H(X)$. It is then easy to see that the input distribution $p(x) = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$ maximizes the sum rate of the Blackwell channel. For the rest of this paper, we fix this particular choice of input distribution and aim to approach the resulting rate region with practical coding methods. With this input distribution, the Marton's region becomes:

$$\begin{aligned} R_1 &\leq H\left(\frac{1}{3}\right) \\ R_2 &\leq H\left(\frac{1}{3}\right) \\ R_1 + R_2 &\leq \log_2(3). \end{aligned} \quad (3)$$

This region is a pentagon as shown in Fig. 2. One of its corner points is $R_1 = H\left(\frac{1}{3}\right)$ and $R_2 = \log_2(3) - H\left(\frac{1}{3}\right) = \frac{2}{3}$. Note that the sum rate is as high as if the receivers also cooperate.

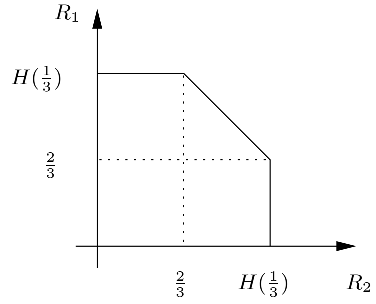

Fig. 2. Rate region for the Blackwell channel with $p(x) = \{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$

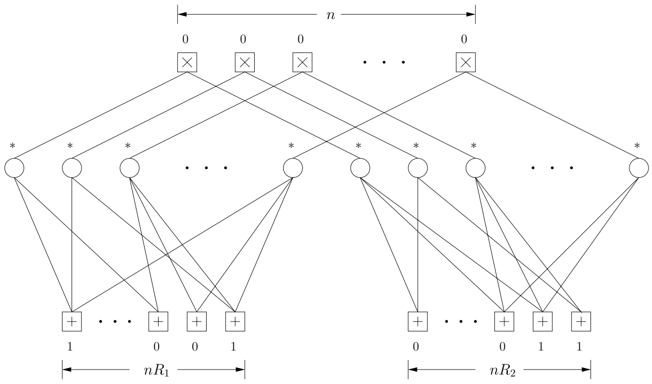

Fig. 3. Coding for the Blackwell channel

B. Coding Scheme

The main coding idea for the Blackwell channel is binning. As mentioned earlier, the optimal U_1 and U_2 for the Blackwell channel is $U_1 = Y_1$ and $U_2 = Y_2$. The code construction involves the generation of $2^{nH(Y_1)}$ y_1^n sequences and $2^{nH(Y_2)}$ y_2^n sequences, and a random assignment of these sequences into 2^{nR_1} and 2^{nR_2} bins, respectively. To transmit a particular pair of bin indices (i, j), the transmitter looks for a pair of y_1^n in bin i and y_2^n in bin j such that they are jointly typical. The transmitter then selects the input symbols to produce this pair of (y_1^n, y_2^n) at the output.

In this paper, we consider the use of linear coset codes for binning. In this case, binning can be accomplished via parity-check codes. The set of y_1^n's in a bin is defined as the set of n-sequences that satisfy a particular set of parity-check values. Thus, the values of parity checks (or the syndrome) is exactly the bin index. In Marton's coding scheme, the bin indices are message bits. Thus, with linear coset codes the messages are the values of parity checks.

For the Blackwell channel, the joint typicality of y_1^n and y_2^n (or equivalently that of u_1^n and u_2^n) is equivalent to being consistent with respect to the channel constraint. Since an output $(1, 1)$ cannot occur in a Blackwell channel, the problem of finding a jointly typical (y_1^n, y_2^n) is precisely the problem of finding y_1^n and y_2^n such that $(1, 1)$ never occurs in any position.

Fig. 3 illustrates the graphical coding structure for achieving an arbitrary rate pair (R_1, R_2). In the figure, circles are used to denote the variable nodes and squares are used to denote constraint nodes. The variable nodes store the values of y_1^n and y_2^n. There are two types of constraint nodes. The constraint nodes at the top are essentially NAND constraints (or equivalently product constraints), ensuring that the $(1, 1)$ combination does not occur. The bottom constraints are the usual parity check constraints. They are connected to the variable nodes at random. The values of these bottom parities are the bin indices, which are the message bits sent through the channel. The encoding process consists of the following. The information bits nR_1 and nR_2 are placed at the parity checks at the bottom. The encoding task is to find the set of variable assignments that satisfy the parity checks and at the same time satisfy the product constraints at the top, i.e., $y_{1,n} y_{2,n} = 0\ \forall n$. These two sets of constraints ensure that a pair of (y_1^n, y_2^n) that belongs to the appropriate bins can be found so that there exists an input x^n that can produce this pair of (y_1^n, y_2^n) at the output.

The error events occur exclusively in the encoding process. An error occurs when the encoder fails to find a set of variable assignments that are consistent with both constraints. The decoding process is simple and error free. The decoder obtains (y_1^n, y_2^n) from the channel and recovers the message by forming the parity checks.

Note that the two parity-check matrices at the bottom can be randomly chosen, while the product constraints at the top are deterministic. It is interesting to note that in spite of sharing many of the same structures, this type of codes considered for the Blackwell channel is quite different from the usual LDPC codes. For example, in the usual LDPC codes the values of the parity checks are always zero. Also, because of the constraint that $(1, 1)$ can never occur, the variable nodes in Fig. 3 have a probability of $\frac{1}{3}$ to be 1 and a probability of $\frac{2}{3}$ to be 0.

III. Encoding Algorithm

A. Belief Propagation

As illustrated in the previous section, the linear-code encoding problem for the Blackwell channel is equivalent to a constraint satisfaction problem with two different kinds of constraints. Therefore, the performance of a Blackwell channel encoding scheme hinges strongly on the ability for the encoding algorithm to solve the constraint satisfaction problem efficiently. Clearly exhaustive search is not feasible when the block length is large.

Our proposed random linear coding scheme is inspired by the phenomenal success of iterative message-passing decoding algorithms for low-density parity-check codes for channel coding. In the message-passing algorithm, the variable nodes and the constraint nodes pass messages back and forth, where the messages represent the beliefs whether a variable node should be assigned 0 or 1. The message-passing algorithm is also known as the belief propagation algorithm.

Despite the similarity between the encoding problem outlined in the previous section and the decoding problem for LDPC codes in a single-user channel, strong differences also exist. In belief propagation decoding of LDPC codes, a prior bias is available to the decoder based on the channel output. The decoding task is equivalent to finding a codeword that is closest to the observation sequence. Such a strong bias is crucial and is absent in the Blackwell encoding problem. In the Blackwell channel, the encoder needs to find a satisfying sequence without knowing approximately where such a sequence is. Without a strong prior bias, the belief propagation algorithm usually fails. This is akin to the quantization problem where iterative decoding is also not known to perform well. Thus, belief propagation is not suitable for the broadcast channel.

B. Survey Propagation

In this paper, we advocate an alternative approach based on a recent advance in algorithms for the constraint satisfaction problem called "survey propagation" [10] [11]. Survey propagation is initially devised to solve the well-known NP-complete K-SAT problem. In a K-SAT problem, a large number of OR-clauses need to be satisfied at the same time. Each OR-clause consists of K variables. The key problem parameter is the ratio of the number of clauses and the number of variables. The K-SAT problem becomes exponentially complex as the ratio parameter approaches a critical value. The region just below the critical value is considered the hard region. Recently, based on statistical physics considerations, Braunstein, Mezard, Weigt and Zecchina [10] [11] proposed an algorithm called survey propagation and found that it outperforms traditional algorithms in the hard region. In the following, we give a briefly synopsis of survey propagation for the K-SAT problem, followed by our proposed algorithm for the Blackwell channel.

For K-SAT, standard belief propagation (BP) works well in finding solutions in the so-called easy region, where the solutions are all concentrated within one big cluster (i.e. the Hamming distance between solutions are small). As the ratio of the number of constraints and the number of variables rises, the solution space breaks up into an exponential number of distinct clusters. Here belief propagation fares poorly because for example, two adjacent parts of the factor graph can end up finding solutions which are in different clusters, and are thus incompatible. With neither side having enough momentum at the boundary, the BP algorithm fails to converge.

Survey propagation (SP) gets around this failing by introducing the concept of a joker state in addition to the 0 and 1 states. Early in the iterative process, where BP would choose an available state for a particular variable (say either 0 or 1), and thus localize itself within a particular cluster, SP introduces the joker (or * state) which represents the fact that the variable is free to be either 0 or 1. This allows SP to create "histograms of solutions across clusters" [10] [11]. After the probability histograms have converged, SP makes hard decision only on variables that are most biased toward 0 or 1. The algorithm then freezes the values of these variables. The graph is now reduced to a smaller one. SP then proceeds by repeating the same process for the rest of the factor graph until all variables are frozen.

The key to the success of survey propagation on the K-SAT problem is the inclusion of the joker state. Without the joker state, survey propagation reduces to belief propagation. The inclusion of joker state makes sense for the K-SAT problem because constraints in the K-SAT problem is an OR-constraint. The OR-constraint is satisfied whenever one variable connected to the constraint is 1. When this happens, all other variables connecting to the constraint are in the "don't-care" state where they are free to be assigned to be either 0 or 1. This corresponds to the joker state.

The inclusion of the joker state is not directly applicable to LDPC decoding where constraints are parity-check (or XOR) constraints in which no "don't-care" state exists. In this case, belief propagation remains to be the best decoding algorithm known to date.

One of the main points of this paper is the recognition that the "don't-care" states are natural for the Blackwell channel. Consider the factor graph as illustrated in Fig. 3, although the parity-check constraints at the bottom of the graph are XOR constraints, the top product constraints have the property that if one of the variables connecting to it is 0, then the other variable can be either 0 or 1. So, it can be assigned to the "don't-care" state. On the other hand, if the variable is 1, then the other variable connecting to the constraint must be assigned a 0. The existence of these "don't-care" configurations strongly suggest that survey propagation may outperform belief propagation for the Blackwell channel encoding problem.

In the following, we present the survey propagation equations. Let constraint nodes be denoted as $\{a, b, \cdots\}$ and variable nodes be denoted as $\{i, j, \cdots\}$. Let $A(i)$ be the set of constraint nodes connecting to variable i. Let $B(a)$ be the set of variable nodes connecting to the constraint node a. The message from the variable node j to the constraint node a consists of three probabilities: $\alpha_{j\to a}$, $\beta_{j\to a}$ and $\delta_{j\to a}$, denoting the probabilities of 0, 1 or joker states, respectively. The message from the constraint node a to variable node i consists of $u^{\alpha}_{a\to i}$, $u^{\beta}_{a\to i}$ and $u^{\delta}_{a\to i}$, representing beliefs for 0, 1, and joker states, respectively.

The survey propagation equations for the even parity XOR-constraints are as follows (the odd parity equations are analogous):

$$\begin{aligned} u^{\alpha}_{a\to i} &= \frac{\prod_{j\in B(a)/i}(\alpha_{j\to a}+\beta_{j\to a}) + \prod_{j\in B(a)/i}(\alpha_{j\to a}-\beta_{j\to a})}{2} \\ u^{\beta}_{a\to i} &= \frac{\prod_{j\in B(a)/i}(\alpha_{j\to a}+\beta_{j\to a}) - \prod_{j\in B(a)/i}(\alpha_{j\to a}-\beta_{j\to a})}{2} \\ u^{\delta}_{a\to i} &= 1 - u^{\alpha}_{a\to i} - u^{\beta}_{a\to i}. \end{aligned} \tag{4}$$

Briefly, $u^{\alpha}_{a\to i}$ and $u^{\beta}_{a\to i}$ represents the probability that constraint node a requires variable i to be 0 or 1, respectively. If neither is the case, a joker probability is assigned.

The messages $\alpha_{j\to a}$ $\beta_{j\to a}$ and $\delta_{j\to a}$ for the XOR-constraints are computed as follows:

$$\alpha_{j\to a} = \prod_{b\in A(j)/a} (1-u^{\beta}_{b\to j}) - \prod_{b\in A(j)/a} (1-u^{\beta}_{b\to j}-u^{\alpha}_{b\to j}),$$
$$\beta_{j\to a} = \prod_{b\in A(j)/a} (1-u^{\alpha}_{b\to j}) - \prod_{b\in A(j)/a} (1-u^{\beta}_{b\to j}-u^{\alpha}_{b\to j}),$$
$$\delta_{j\to a} = \prod_{b\in A(j)/a} (1-u^{\beta}_{b\to j}-u^{\alpha}_{b\to j}) \quad (5)$$

Briefly, $\alpha_{j\to a}$ is the probability that no constraints (other than a) request variable node j to be 1 and at least one constraint requests j to be 0 (by subtracting from it the probability that all messages are jokers.) Similar interpretation exists for $\beta_{j\to a}$ and $\delta_{j\to a}$. Note that $\alpha_{j\to a}$, $\beta_{j\to a}$, and $\delta_{j\to a}$ need to be normalized to be 1 to exclude contradictory messages.

The survey propagation equations for the product (or NAND) constraints are as follows. As all such constraints are of degree 2, the equations are simply:

$$u^{\alpha}_{a\to i} = \beta_{j\to a} \quad (6)$$
$$u^{\beta}_{a\to i} = 0 \quad (7)$$
$$u^{\delta}_{a\to i} = 1-\beta_{j\to a} \quad (8)$$

As $(1,1)$ configuration can never occur, the message from the constraint to variable nodes is either 0 or "don't-care", depending on whether the incoming message is 1 or not.

Taking these equations together, survey propagation runs as follows. All variable nodes are assigned $(0.5, 0.5, 0)$ at the beginning. The survey equations are allowed to propagate until the surveys converge (or a maximum number of iterations, typically 100, is reached.) We then compute the overall (α, β, δ) using (5) but without the exclusion of a in $A(j)$ in the product operation. We then find the most biased variable nodes (i.e. the node whose overall $|\alpha-\beta|$ is the largest) and freeze these variables accordingly. The factor graph is now reduced to a smaller one, on which survey propagation may be run again. The second iteration freezes more variables. The process goes on until all variables are set.

IV. Results

We experimented with random code constructions using survey propagation for encoding on the Blackwell channel. Random codes with small degree distributions have been found to work well experimentally. We experimented with several rate pairs with $R_1 + R_2 = 1.5$, which is about 95% of the sum capacity. (The sum capacity is $\log_2(3) = 1.5850$.) One variable is frozen per iteration.

At the middle point $R_1 = R_2 = 0.75$, the two random parity check matrices are chosen so that all variable nodes have degree 2, with $\frac{1}{3}$ of the parity checks have degree 3 and $\frac{2}{3}$ of the parity checks have degree 2. The graphs are checked to remove cycles of length 4. The probability of bit error is $5\cdot 10^{-3}$ at a moderate block length 4,000.

Near the corner point with $R_1 = 0.6$ and $R_2 = 0.9$, the random parity check matrix associated with R_1 is chosen so that $\frac{2}{3}$ of the parity checks have degree 3 and $\frac{1}{3}$ of the parity checks have degree 4. The parity check matrix associated with R_2 is chosen so that $\frac{7}{9}$ of the parity checks have degree 2 and $\frac{2}{9}$ of the parity checks have degree 3. At a moderate block length of 4,000, the probability of bit error is about $2\cdot 10^{-3}$.

Similar results are obtained at $R_1 = 0.85$ and $R_2 = 0.65$. In this case, a fraction of $(0.647, 0.353)$ of the parity checks associated with R_1 are chosen to have degrees 2 and 3 respectively, and a fraction of $(0.232, 0.461, 0.307)$ of the parity checks associated with R_2 are chosen to have degrees 2, 3 and 4, respectively. The average probability of error is $1.8\cdot 10^{-3}$ at block length 4,000.

The probability of bit error goes down with a larger block length. For the middle point, the probability of bit error is reduced to 10^{-3} at a block length of 40,000 in 10 simulation runs. Similar results are obtained for other points as well. It should be noted that survey propagation is $O(n)$ slower than belief propagation, as it consists of $O(n)$ steps, each having a complexity approximately equal to that of belief propagation. Thus, it is difficult to drive the probability of bit error down with very large block lengths. Nevertheless, our result is promising as it appears to be the first attempt in the construction and the encoding/decoding of random codes for deterministic broadcast channels. Comparing to the algebraic coding approach of [12], where zero-error codes up to a block length of 10 are constructed, our approach is more flexible and achieves a higher rate. To further reduce the probability of error in the present scheme, a high-rate outer error-correcting code may be concatenated with the survey propagation based inner code at a small rate loss.

References

[1] T. Cover, "Comments on broadcast channels," *IEEE Trans. Info. Theory*, vol. 44, no. 6, pp. 2524–2530, Oct. 1998.

[2] H. Weingarten, Y. Steinberg, and S. Shamai, "The capacity region of the Gaussian MIMO broadcast channel," *submitted to IEEE Trans. Info. Theory*, 2004.

[3] S. S. Pradhan and K. Ramchandran, "Distributed source coding using syndromes (DISCUS): design and constructions," *IEEE Trans. Info. Theory*, vol. 49, no. 3, pp. 626–643, March 2003.

[4] D. Schonberg, K. Ramchandran, and S. S. Pradhan, "LDPC codes can approach the Slepian-Wolf bound for general binary sources," in *Allerton Conf. Comm., Control and Computing*, Oct 2002.

[5] A. D. Liveris, Z. Xiong, and C.N. Georghiades, "Compression of binary sources with side information at the decoder using LDPC codes," *IEEE Comm. Letters*, vol. 6, pp. 440–442, Oct 2002.

[6] R. Zamir, S. Shamai, and U. Erez, "Nested linear/lattice codes for structured multiterminal binning," *IEEE Trans. Info. Theory*, vol. 48, no. 6, pp. 1250–1276, June 2002.

[7] U. Erez and S. ten Brink, "Approaching the dirty paper limit for canceling known interference," in *Allerton Conf. Comm. Control and Computing*, Oct 2003.

[8] G. Caire, A. Bennatan, D. Burshtein, and S. Shamai, "Coding schemes for the binary symmetric channel with known interference," in *Allerton Conf. Comm. Control and Computing*, Oct 2003.

[9] E. Martinian and J. Yedidia, "Iterative quantization using codes on graphs," in *Allerton Conf. Comm. Control and Computing*, Oct 2003.

[10] A. Braunstein, M. Mezard, M. Weigt, and R. Zecchina, "Constraint satisfaction by survey propagation," Sep. 2003, arXiv:cond-mat/0212451.

[11] A. Braunstein, M. Mezard, and R. Zecchina, "Survey propagation: an algorithm for satisfiability," Nov. 2003, arXiv:cs.CC/0212002 v3.

[12] P. Vanroose and E.C. van der Meulen, "Coding for the Blackwell broadcast channel," in *Proc. 4th Joint Soviet-Swedish Workshop on Information Theory*, Aug 1989, Gotland, Sweden.

[13] S. I. Gel'fand, "Capacity of one broadcast channel," *Problemy Peredachi Informatsii*, vol. 13(3), pp. 106–108, 1977.