Writing on Colored Paper

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Abstract — **A Gaussian channel when corrupted by an additive Gaussian interfering signal that is not necessarily stationary or ergodic, but whose complete sample sequence is known to the transmitter, has the same capacity as if the interfering signal were not present.**

I. Introduction

Costa [1] showed that the capacity for an AWGN channel corrupted by an i.i.d. Gaussian interfering signal known non-causally only at the transmitter is, surprisingly, the same as when the interfering signal does not exist. This result has since been generalized to the case where the interfering process is ergodic, but with an arbitrary distribution [2]; and to the case where the interference is an arbitrary sequence if common randomness is available at both the transmitter and the receiver [3]. This paper extends Costa's result in a different direction. While retaining the Gaussian assumption, we consider the case where the interference and the noise are not necessarily stationary or ergodic. We will construct a code on a single block of n transmissions with a probability of error independent of the covariance structure of interference and noise.

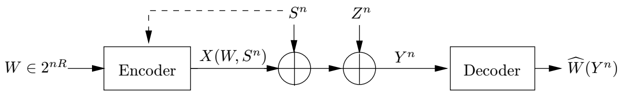

Figure 1: Gaussian channel corrupted by known interference.

II. Writing on colored paper

Gaussian processes possess an asymptotic equipartition (AEP) property that does not require the usual ergodicity condition [4]. This allows a coding theorem to be stated for the Gaussian channel $Y^n = X^n + Z^n$ such that the probability of error is independent of the covariance of Z^n. Fix $\epsilon > 0$. Consider one block of n transmissions over the Gaussian channel. Let

$$C_n = \max_{K_{X^n}} \frac{1}{2n} \log \frac{|K_{X^n} + K_{Z^n}|}{|K_{Z^n}|}, \tag{1}$$

where the maximization is over covariance matrices K_{X^n} such that $\mathrm{tr}(K_{X^n}) \leq nP/(1+\epsilon)$. If the maximizing K_{X^n} has minimum eigenvalue greater than ϵ, then we can construct a $(2^{n(C_n-\epsilon)}, n)$ code over a one-shot use of n transmissions, that satisfies power constraint P and has probability of error $P_e^{(n)} < e^{-n\alpha(\epsilon)}$, where $\alpha(\epsilon)$ does not depend on the covariance structure of Z^n.

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A similar distribution-free result may be stated for the Gaussian channel with non-causal side information. Consider the Gaussian channel $Y^n = X^n + S^n + Z^n$ shown in Figure 1, where S^n and Z^n are independent Gaussian sequences, S^n is completely known at the transmitter only, and Z^n is not known at the transmitter or at the receiver. A $(2^{nR}, n)$ code consists of 2^{nR} encoding functions $X^n(W, S^n)$ and a decoding function $\widehat{W}(Y^n)$. The power constraint P requires $\mathbf{E}||X^n(W, S^n)||^2 \leq nP$. The probability of error is defined as $P_e^{(n)} = \mathrm{Prob}\{\widehat{W}(Y^n) \neq W\}$ where $W \sim \mathrm{uniform}\{1, \cdots, 2^{nR}\}$.

Theorem 1 *Fix $\epsilon > 0$. Consider one block of n transmissions in a Gaussian channel $Y^n = X^n + S^n + Z^n$ with interference S^n known at the transmitter. Let C_n be defined as before, where $\mathrm{tr}(K_{X^n}) \leq nP/(1+\epsilon)$ and the maximizing K_{X^n} has minimum eigenvalue greater than ϵ. Then there exists a $(2^{n(C_n-\epsilon)}, n)$ code with power constraint P and probability of error $P_e^{(n)} \leq e^{-n\beta(\epsilon)}$, where $\beta(\epsilon)$ does not depend on the covariance structure of S^n and Z^n.*

The coding theorem does not involve repeated uses of the channel. Although S^n and Z^n may vary arbitrarily and C_n may fluctuate, a $(2^{n(C_n-\epsilon)}, n)$ code can be constructed over the first n transmissions with a probability of error that does not depend on the covariance structure of S^n and Z^n. The rate of convergence for the probability of error, $\beta(\epsilon)$, is explicitly given in [5].

The theorem is proved in two steps. First, we show that

$$R_n = \frac{1}{n}(I(U^n; Y^n) - I(U^n; S^n)) - \epsilon, \tag{2}$$

is achievable for all joint Gaussian distributions $p(u^n|x^n, s^n)p(x^n)p(s^n)$. Secondly, we show that an appropriate choice of U^n is $U^n = X^n + FS^n$. Further, the matrix F that maximizes $I(U^n; Y^n) - I(U^n; S^n)$ is $F = K_{X^n}(K_{X^n} + K_{Z^n})^{-1}$, and the maximum value of $I(U^n; Y^n) - I(U^n; S^n)$ is precisely C_n in equation (1). Thus the capacity is the same as if S^n is not present. Curiously, the optimal F takes the form of an optimal non-causal Wiener filter to estimate X^n from $X^n + Z^n$.

References

[1] M. Costa, "Writing on dirty paper," *IEEE Trans. on Info. Th.*, vol. 29, no. 3, pp. 439–441, May 1983.

[2] A. Cohen and A. Lapidoth, "The Gaussian watermarking game: Part I," *submitted to IEEE Trans. on Info. Th.*, 2000.

[3] U. Erez, S. Shamai, and R. Zamir, "Capacity and lattice strategies for canceling known interference," in *Proc. of Inter. Symp. on Info. Th. and its App.*, Nov. 2000.

[4] T. M. Cover and S. Pombra, "Gaussian feedback capacity," *IEEE Trans. on Info. Th.*, vol. 35, no. 1, pp. 37–43, Jan. 1989.

[5] W. Yu, A. Sutivong, D. Julian, T. M. Cover and M. Chiang, "Writing on colored paper," *preprint.*