First, we would like to acknowledge that the derivation (12)-(13) in [1] is not correct. Fortunately, [1, Theorem 1] remains valid, and the derivation can be corrected by following the steps (23)-(25) in [2] using [2, Lemma 1].

However, it has been pointed out to us that [2, Lemma 1] as stated is also problematic. A counterexample is that Lemma 1 does not hold if $\mathbf{b}$ is in the null space of $\mathbf{A}$. Fortunately, the following modified version of Lemma 1 can be used instead in order to prove the desired result.

Lemma 1 *Let* $\mathbf{A}$ *be an* $n \times n$ *symmetric positive semidefinite matrix and* $\mathbf{b}$ *be an* $n \times 1$ *vector* **in the row space of $\mathbf{A}$**. *Then,* $\mathbf{A} \succeq \mathbf{b}\mathbf{b}^H$ *if and only if* $\mathbf{b}^H \mathbf{A}^\dagger \mathbf{b} \leq 1$.

The problem with the orignal proof of [2, Lemma 1] is in the last chain of inequalities on p. 2659. The extra condition that $\mathbf{b}$ is in the row space of $\mathbf{A}$ allows us to fix the chain of inequalities.

First, since $\mathbf{A}$ is positive semidefinite, there exists a unitary matrix $\mathbf{U} = [\mathbf{U}_1^T, \mathbf{U}_2^T]^T$, such that $\mathbf{U}\mathbf{A}\mathbf{U}^H = \mathrm{diag}\{\mathbf{U}_1\mathbf{A}\mathbf{U}_1^H, \mathbf{0}\}$. In other words, $\mathbf{U}_1$ and $\mathbf{U}_2$ are the row space and the null space of $\mathbf{A}$, respectively. Note that since $\mathbf{A}$ is symmetric, a vector $\mathbf{b}$ is in the row space if and only if $\mathbf{U}_2\mathbf{b} = 0$. Then, we have

$$\mathbf{A} \succeq \mathbf{b}\mathbf{b}^H \Leftrightarrow \mathbf{U}\left(\mathbf{A} - \mathbf{b}\mathbf{b}^H\right)\mathbf{U}^H \succeq \mathbf{0} \Leftrightarrow \mathbf{U}_1\mathbf{A}\mathbf{U}_1^H \succeq \mathbf{U}_1\mathbf{b}\mathbf{b}^H\mathbf{U}_1^H \Leftrightarrow \mathbf{b}^H\mathbf{A}^\dagger\mathbf{b} \leq 1$$

where the second step uses $\mathbf{U}_2\mathbf{b} = \mathbf{0}$ and the third step follows from the positive definite case in the early part of the proof of Lemma 1. This proves the modified Lemma 1.

Now, we apply the modified Lemma 1 to the proof of [2, Theorem 1]. Consider $\mathbf{A} = \sum_{j=1}^K \lambda_j \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}$ and $\mathbf{b} = \sqrt{(1 + 1/\gamma_i)\lambda_i} \cdot \mathbf{h}_i$. Our proof of [2, Theorem 1] requires us to show that $\mathbf{A} \succeq \mathbf{b}\mathbf{b}^H$ if and only if $\mathbf{b}^H\mathbf{A}^\dagger\mathbf{b} \leq 1$. This is obvious when $\lambda_i = 0$. Thus we only need to consider the case of $\lambda_i > 0$. But it is easy to see that in this case we do have $\mathbf{U}_2\mathbf{b} = 0$ because of the following:

$$\mathbf{0} = \mathbf{U}_2\mathbf{A}\mathbf{U}_2^H = \mathbf{U}_2\left(\sum_{j\neq i}^K \lambda_j \mathbf{h}_j \mathbf{h}_j^H + \mathbf{Q}\right)\mathbf{U}_2^H + \lambda_i \mathbf{U}_2 \mathbf{h}_i \mathbf{h}_i^H \mathbf{U}_2^H \succeq \lambda_i \mathbf{U}_2 \mathbf{h}_i \mathbf{h}_i^H \mathbf{U}_2^H.$$

Since $\lambda_i \mathbf{U}_2 \mathbf{h}_i \mathbf{h}_i^H \mathbf{U}_2^H$ must be positive semidefinite, we conclude $\mathbf{U}_2\mathbf{h}_i = \mathbf{0}$, which also means $\mathbf{U}_2\mathbf{b} = \mathbf{0}$. Then the modified Lemma 1 can be applied to finish the proof of [2, Theorem 1].

References

[1] H. Dahrouj and W. Yu, "Coordinated Beamforming for the Multicell Multi-Antenna Wireless System," *IEEE Trans. Wireless Commun.*, vol. 9, no. 5, pp. 1748–1759, May 2010.

[2] W. Yu and T. Lan, "Transmitter Optimization for the Multi-Antenna Downlink With Per-Antenna Power Constraints," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2646–2660, June 2007.