Homework Set #5

1. The cooperative capacity of a multiple access channel. (Figure 1)

(a) Suppose $X_1$ and $X_2$ have access to both indices $W_1 \in \{1, 2^{nR_1}\}, W_2 \in \{1, 2^{nR_2}\}$. Thus the codewords $X_1(W_1, W_2), X_2(W_1, W_2)$ depend on both indices. Find the capacity region.

(b) Evaluate this region for the binary erasure multiple access channel $Y = X_1 + X_2, X_i \in \{0, 1\}$. Compare to the non-cooperative region.

2. Gaussian multiple access.

A group of $m$ users, each with power $P$, is using a Gaussian multiple-access channel at capacity, so that

$$\sum_{i=1}^{m} R_i = C\left(\frac{mP}{N}\right),$$

where $C(x) = \frac{1}{2} \log(1 + x)$ and $N$ is the receiver noise power. A new user of power $P_0$ wishes to join in.

(a) At what rate can he send without disturbing the other users?

(b) What should his power $P_0$ be so that the new users’ rate is equal to the combined communication rate $C(mP/N)$ of all the other users?

3. Rate distortion function with infinite distortion.

Find the rate distortion function $R(D) = \min I(X; \hat{X})$ for $X \sim$ Bernoulli \(\left(\frac{1}{2}\right)\) and distortion

$$d(x, \hat{x}) = \begin{cases} 0, & x = \hat{x}, \\ 1, & x = 1, \hat{x} = 0, \\ \infty, & x = 0, \hat{x} = 1. \end{cases}$$
For the case of a continuous random variable $X$ with mean zero and variance $\sigma^2$ and squared error distortion, show that

$$h(X) - \frac{1}{2} \log(2\pi e) D \leq R(D) \leq \frac{1}{2} \log \frac{\sigma^2}{D}. \quad (2)$$

For the upper bound, consider the joint distribution shown in Figure 2. Are Gaussian random variables harder or easier to describe than other random variables with the same variance?

Figure 2: Joint distribution for upper bound on rate distortion function.