

EC431H1 Digital Signal Processing

LAB 1.

The DFT and FFT. Spectral analysis and time delay estimation.

Introduction

The purpose of this experiment is to become familiar with the utilization of the FFT (Fast Fourier Transform) in DSP applications. It is recommended that you use MATLAB to program and execute the required steps of the experiment. Your lab report (one per group of two students, 5 page max) should include: a) printouts of your programs, b) printouts/plots of the obtained results, and c) answers to questions and brief but critical discussion of the obtained results.

Experiments

PART A. Spectral analysis

We have given the analog signal

$$x_a(t) = 3 \cdot e^{-53.12t} \cdot \left[\sin(1600\pi t + \frac{\pi}{4}) \right] + e^{-45.97t} \cdot \left[\sin(2600\pi t + \frac{\pi}{2}) \right]$$

for $t = 0$ to 100 years.

and asked to estimate the magnitude spectrum $|X_a(\Omega)|$. We have decided to utilize discrete time techniques.

1. Sample $x_a(t)$ with sampling frequency $F_s = \frac{1}{T} = 12$ kHz to obtain the discrete signal $x[n]$. Provide the expression for $x[n]$.
2. Write a little routine to generate and store the samples $x[n]$ for $n = 0, 1, \dots, 127$. Plot $x[n]$.
3. Using the MATLAB `fft(.)` subroutine, compute the N -point DFT $X[k]$, $k = 0, 1, \dots, N$. Plot the quantity $20 \log_{10} |X[k]|$ (i.e., in dB scale), for the following cases:
 - i) $N = 16$, using the samples $x[0], x[1], \dots, x[15]$.
 - ii) $N = 128$, using the samples $x[0], x[1], \dots, x[15]$ plus 112 appended zeros.
 - iii) $N = 128$, using the samples $x[0], x[1], \dots, x[63]$ plus 64 appended zeros.
 - iv) $N = 128$, using the samples $x[0], 0, x[1], 0, x[2], \dots, x[62], 0, x[63], 0$.
 - ii) $N = 128$, using the samples $x[0], x[1], \dots, x[127]$.

Use the same size axis to represent the underlying range $0 \leq \omega \leq 2\pi$ in all cases. Recall that $\omega = \frac{2\pi k}{N}$. Compare and discuss the results obtained. Comment on the concepts of spectral resolution and spectral leakage. Why do the obtained spectra seem different? Do you observe aliasing effects?

PART B. Time delay estimation

A transmitting station sends the signal $y(t)$. However, due to channel attenuation and delay, the signal witnessed at a distant receiving station is $r(t) = Gy(t - D)$ where G is an attenuation factor and D a time delay. To estimate the distance between the two stations we decide to discretize the signals, estimate D from the discrete crosscorrelation function between the transmitted and received signals. Let assume that by applying a sampling rate of $F_s = 10\text{KHz}$, we have obtained

$$y[n] = \sin(2\pi \frac{n}{8} + \frac{\pi}{8}); \quad n = 0, 1, \dots, 99$$

and

$$r[n] = 0.8 \cdot \sin(2\pi \frac{n}{8} - 2\frac{\pi}{3}); \quad n = 0, 1, \dots, 99$$

The crosscorrelation between the two signals is defined as

$$C_{y,r}[m] = \frac{1}{N} \sum_{n=\max(0,-m)}^{\min(N,N-m-1)} y[n]r[n+m] = \frac{G}{N} \sum_{n=\max(0,-m)}^{\min(N,N-m-1)} y[n]y[n+m-\hat{D}] = C_{y,y}[m-\hat{D}]$$

where \hat{D} is a discrete estimate (in samples) of D . Since the maximum value of $C_{y,y}[m-\hat{D}]$ occurs for $m-\hat{D} = 0$, the maximum value of $C_{y,r}[m]$ will occur for $m = \hat{D}$. Furthermore, by inspection it is easy to realize that

$$C_{y,r}[m] = \frac{1}{N} y[m] * r[-m]$$

where $*$ stands for linear convolution. Thus, in DTFT terms

$$C_{y,r}(\omega) = \frac{1}{N} Y(\omega) R^*(\omega)$$

where now $*$ stands for complex conjugate operation. Therefore, we can use FFT operations to calculate $C_{y,r}[m]$.

1. Generate and plot $y[n]$ and $r[n]$, $n = 0, 1, \dots, 99$.
2. Append 100 zeros to each sequence and calculate the 200 point FFTs $Y[k]$ and $R[k]$, $k = 0, 1, \dots, 199$.
3. Calculate $Y[k] \cdot R^*[k]$, $k = 0, 1, \dots, 199$.
4. Calculate the 200 point inverse FFT of the product in (3) and take the real part.
5. Find the maximum absolute value of the estimated crosscorrelation and estimate the delay D in seconds (not samples).

What is the reason for zero-padding the sequences in step 2?

Now change this slightly:

1. Zero padd the product in (3) by inserting 800 zeros between samples 100 and 101.
2. Calculate the 1000 point inverse FFT of the product in the previous step and take the real part.
3. Find the maximum absolute value of the estimated crosscorrelation and estimate the delay D is seconds.

Compare to the previous estimate of D . Which one of the two estimates do you think is better? Comment appropriately.