Interference Mitigation with Joint Beamforming and Common Message Decoding in Multicell Systems

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Abstract—**Conventional multicell wireless systems operate with out-of-cell interference treated as noise—interference detection is infeasible as intercell interference is typically weak. This paper considers the benefit of designing decodable interference signals by allowing common-private message splitting at the transmitter and common message decoding by users in adjacent cells. In particular, we consider a downlink scenario, where base-stations are equipped with multiple transmit antennas, the remote users are equipped with a single antenna, and multiple remote users are active simultaneously via spatial division multiplexing. We solve a network optimization problem of jointly determining the appropriate users in adjacent cells for rate splitting, the optimal beamforming vectors for both common and private messages, and the optimal common-private rates to minimize the total transmit power across the base-stations subject to service rate requirements for remote users. We observe that for fixed user selection and fixed common-private rate splitting, the optimization of beamforming vectors can be performed using a semidefinite programming approach. Further, this paper proposes a heuristic user-selection and rate splitting strategy to maximize the benefit of common message decoding. Simulation results show that common message decoding can significantly improve both the total transmit power and the feasibility region for cell-edge users.**

I. INTRODUCTION

In a conventional wireless cellular system, each base-station communicates with the remote terminals independently. Out-of-cell interference is treated as noise. Multiuser detection, while feasible for intracell users (e.g. in a CDMA system), is difficult to implement for out-of-cell users, because out-of-cell interference is typically quite weak. Conventional cellular networks, however, are also typically designed to be interference-limited. This is especially so as networks are increasingly designed with aggressive frequency reuse factors. Thus, although out-of-cell interference is weak, it can still be significantly above the background noise level. This opens up the possibility of designing transmit signals for the purpose of multiuser detection at adjacent cells for interference mitigation.

The multicell system can be modelled as an interference network. From an information theoretical perspective, the largest known achievable rate region for the two-user interference channel is due to Han and Kobayashi [1], who devised a common-private message splitting strategy where each user's transmit signal is splitted into two parts: a private message to be decoded by the intended receiver only, and a common message to be decoded by both receivers for the sole purpose of interference mitigation. Recently, Etkin, Tse and Wang [2] offered a key insight into the optimization of Han-Kobayashi strategy by showing that a simple scheme of setting the private message power at the opposite receiver to be at the background noise level achieves within one bit of the capacity region of the interference channel. Thus, the part of the out-of-cell interference above the background noise level should essentially be regarded as common message and be decoded.

This paper takes advantage of the above insight by showing that a Han-Kobayashi common-private message splitting strategy can indeed bring a significant benefit to conventional wireless cellular networks. This paper goes beyond the simple two-user single-input single-output model of [1], [2], and considers a multicell downlink system where the base-stations are equipped with multiple antennas, and the remote receivers are equipped with a single antenna each and are separated via spatial multiplexing using downlink beamforming. In this case, the problem of designing the optimal common-private splitting scheme becomes intertwined with user selection and with the design of respective downlink beamformers across the cells. Toward this problem, this paper adopts a design criterion of minimizing the total transmit power across all the base-stations subject to rate constraints for each user. We propose a numerical algorithm for determining the most suitable out-of-cell users for common message decoding, the appropriate rate splitting levels, and the optimal beamforming vectors for both common and private messages at the base-stations. Although the proposed algorithm involves heuristic greedy discrete optimization and convex relaxation as its main components, the results of this paper nevertheless show that common message decoding by the out-of-cell users can be quite effective in mitigating intercell interference, thereby improving the overall network performance.

Common message decoding has been considered previously in a digital subscriber line setting [3], where the problem of jointly optimizing transmit spectra and common-private rate splitting is considered. For wireless systems, the downlink beamforming problem has been invesitgated extensively in the literature both for single-cell (e.g. [4], [5], [6], [7]) and multicell systems (e.g. [8], [9]), but only for systems with private messages only. In particular, this paper uses a technique known as semidefinite programming (SDP) relaxation, first proposed for single-cell downlink beamforming in [5], as a key component of the proposed joint beamforming and common-private rate splitting algorithm.

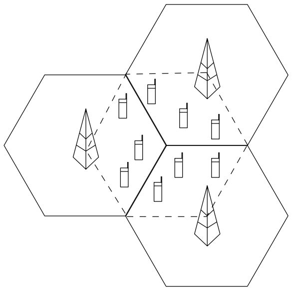

Fig. 1. A wireless network with three base-stations and three users per cell sectors.

II. System Model and Problem Formulation

A. Channel Model

Consider a multicell multiuser spatial multiplex system with N cells and K users per cell with N_t antennas at each base-station and a single antenna at each remote user. Transmit beamforming is employed at the base-station to separate users within each cell.

This paper proposes a joint beamforming and common message decoding scheme to alleviate intercell interference. In particular, each of the jth user in ith cell splits its data stream into two parts: $x^p_{i,j}$, which is a complex scalar denoting the private information with $\mathbf{w}^p_{i,j} \in \mathcal{C}^{N_t \times 1}$ as the associated beamforming vector, and $x^c_{i,j}$, which denotes the common information signal with $\mathbf{w}^c_{i,j} \in \mathcal{C}^{N_t \times 1}$ as the associated beamforming vector. The user (i,j)'s common message $x^c_{i,j}$ is intended to be decoded by both the user (i,j)'s own receiver and by some unique out-of-cell lth user in the mth cell with $m \neq i$. The user (i,j)'s receiver, on the other hand, is designed to decode an unique common message from an out-of-cell user $(\hat{i},\hat{j})$ with $\hat{i} \neq i$. In general, (m,l) does not need to be the same as $(\hat{i},\hat{j})$.

The channel model can be written down as follows:

$$
\begin{aligned}
y_{i,j} = & \sum_l \mathbf{h}^H_{i,i,j} \left(\mathbf{w}^p_{i,l} x^p_{i,l} + \mathbf{w}^c_{i,l} x^c_{i,l} \right) \\
& + \sum_{m \neq i, n} \mathbf{h}^H_{m,i,j} \left(\mathbf{w}^p_{m,n} x^p_{m,n} + \mathbf{w}^c_{m,n} x^c_{m,n} \right) + z_{i,j} \quad (1)
\end{aligned}
$$

where $y_{i,j} \in \mathcal{C}$ is the received signal at the jth user in the ith cell, $\mathbf{h}_{l,i,j} \in \mathcal{C}^{N_t \times 1}$ is the vector channel from the base-station of the lth cell to the jth user in the ith cell, and $z_{i,j}$ is the additive white Gaussian noise with power σ^2. Fig. 1 illustrates the system model for a network with three cells and three users per cell sectors.

B. Problem Formulation

This paper formulates an overall optimization problem of minimizing the total transmit power across all the base-stations subject to fixed target rates at the remote users. For each particular jth user in the ith cell, we need to choose an appropriate $\hat{j}$th user in the $\hat{i}$th cell whose common information user (i,j) will decode. We also need to choose the common and private beamforming vectors and the common-private rate splitting for each user to optimize the overall objective.

This paper uses a successive decoding strategy at each receiver with a fixed order of decoding the common message from its own transmitter first, then the common message from the out-of-cell transmitter, and finally the private message from its own transmitter. Although successive decoding with a fixed decoding order is not necessarily optimal from an information theoretic perspective, the above decoding order is reasonable for the following reason. The underlying interference channel typically has weaker interfering links as compared to direct links, so the common information rate is typically constrained by the interfering link. Hence, it is sensible to decode the common information from one's own transmitter first to help the decoding of common information from the other transmitter. Further, private message should be decoded last to take advantage of the reduced interference due to common message decoding.

With this fixed decoding order, we can write down the signal-to-interference-and-noise (SINR) expressions for the common and private messages for each user. Assume that the $\hat{j}$th user in the $\hat{i}$th cell shares its common information with the jth user in the ith cell. Let Γ^p_{ij}, $\Gamma^c_{ij,ij}$ and $\Gamma^c_{\hat{i}\hat{j},ij}$ denote the SINRs for the private message of the user (i,j), the common message from user (i,j)'s own transmitter, and the common message from the out-of-cell user $(\hat{i},\hat{j})$, respectively. We can write $\Gamma^c_{ij,ij}$ as

$$
\Gamma^c_{ij,ij} = \frac{|\mathbf{h}^H_{i,i,j} \mathbf{w}^c_{i,j}|^2}{I^p_{i,j} + I^c_{i,j} + O^p_{i,j} + O^c_{i,j} + \sigma^2} \quad (2)
$$

where $I^p_{i,j}$ represents the interference due to the intracell private messages, $I^c_{i,j}$ represents the interference due to the intracell common messages, $O^p_{i,j}$ is the interference due to the out-of-cell private messages, and $O^c_{i,j}$ is the interference due to the out-of-cell common messages. More explicitly,

$$
\begin{aligned}
I^p_{i,j} &= \sum_l |\mathbf{h}^H_{i,i,j} \mathbf{w}^p_{i,l}|^2 && (3) \\
I^c_{i,j} &= \sum_{l \neq j} |\mathbf{h}^H_{i,i,j} \mathbf{w}^c_{i,l}|^2 && (4) \\
O^p_{i,j} &= \sum_{m \neq i, n} |\mathbf{h}^H_{m,i,j} \mathbf{w}^p_{m,n}|^2 && (5) \\
O^c_{i,j} &= \sum_{m \neq i, n} |\mathbf{h}^H_{m,i,j} \mathbf{w}^c_{m,n}|^2 && (6)
\end{aligned}
$$

Let $T^c_{i,j}$ denote the overall total interference for the detection of the common message $x^c_{i,j}$ at the user (i,j), i.e., $T^c_{i,j} = I^p_{i,j} + I^c_{i,j} + I^p_{i,j} + O^c_{i,j}$. We can write $\Gamma^c_{\hat{i}\hat{j},ij}$ and Γ^p_{ij} as

$$
\begin{aligned}
\Gamma^c_{\hat{i}\hat{j},ij} &= \frac{|\mathbf{h}^H_{\hat{i},i,j} \mathbf{w}^c_{\hat{i},\hat{j}}|^2}{T^c_{i,j} - |\mathbf{h}^H_{\hat{i},i,j} \mathbf{w}^c_{\hat{i},\hat{j}}|^2 + \sigma^2} && (7) \\
\Gamma^p_{ij} &= \frac{|\mathbf{h}^H_{i,i,j} \mathbf{w}^p_{i,j}|^2}{T^c_{i,j} - |\mathbf{h}^H_{\hat{i},i,j} \mathbf{w}^c_{\hat{i},\hat{j}}|^2 - |\mathbf{h}^H_{i,i,j} \mathbf{w}^p_{i,j}|^2 + \sigma^2} && (8)
\end{aligned}
$$

Let $R_{i,j}$ be the fixed target rate for the jth user in the ith cell, which is split into a private part $R^p_{i,j}$ and a common part

$R^c_{i,j}$ with $R^p_{i,j}+R^c_{i,j}=R_{i,j}$. The overall optimization problem can now be formulated as

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} \|\mathbf{w}^p_{i,j}\|^2 + \|\mathbf{w}^c_{i,j}\|^2 \\ \text{subject to} \quad & \Gamma^p_{i,j} \geq 2^{R^p_{i,j}} - 1, \\ & \Gamma^c_{ij,ij} \geq 2^{R^c_{i,j}} - 1, \\ & \Gamma^c_{\hat{i}\hat{j},ij} \geq 2^{R^c_{\hat{i},\hat{j}}} - 1, \\ & R^p_{i,j} + R^c_{i,j} = R_{i,j}, \qquad \forall i,j \end{aligned} \tag{9}$$

where the minimization is over $\mathbf{w}^p_{i,j}$, $\mathbf{w}^c_{i,j}$, all possible rate splittings, and all possible assignment of user $(\hat{i},\hat{j})$ to each of the user (i,j). Note that as mentioned earlier, each user (i,j) only has a single $(\hat{i},\hat{j})$ assigned to it. Likewise, each $(\hat{i},\hat{j})$ is assigned to only a single (i,j). In addition, throughout this paper, we assume that the optimization problem (9) is feasible.

III. JOINT BEAMFORMING AND COMMON MESSAGE DECODING

The joint beamforming, common-private rate splitting, and user selection problem is a mixed discrete and continuous optimization problem. Finding the global optimal solution for such a problem would likely require a combinatorial search with exponential complexity. Thus, instead of looking for global optimal solutions, this paper focuses on practical algorithms using heuristic greedy search and convex relaxation techniques. Although the proposed algorithm does not guarantee global optimality, it nevertheless offers significant improvement over conventional systems where only private messages are used, thereby providing one step toward making the information theoretical results of [1], [2] practical.

A. Beamforming Optimization with Fixed User Rates

The overall optimization problem (9) is a joint optimization of beamforming vectors, user selection, and common-private splitting of rates. In this section, we make a key observation that if one fixes the user selection and the common-private rate splitting, (i.e., the assignment of user $(\hat{i},\hat{j})$ whose common message user (i,j) will decode, and the associated $R^p_{i,j}$ and $R^c_{i,j}$), the optimization of beamforming vectors $\mathbf{w}^p_{i,j}$ and $\mathbf{w}^c_{i,j}$ can be handled by a SDP relaxation method, and can therefore be efficiently solved.

More specifically, let $\mathbf{V}^p_{i,j} = \mathbf{w}^p_{i,j}(\mathbf{w}^p_{i,j})^H$ and $\mathbf{V}^c_{i,j} = \mathbf{w}^c_{i,j}(\mathbf{w}^c_{i,j})^H$. The objective function of (9) can be reformulated as $\sum_{i,j} \text{tr}(\mathbf{V}^p_{i,j}) + \text{tr}(\mathbf{V}^c_{i,j})$. Also, let $\mathbf{H}_{m,i,j} = \mathbf{h}_{m,i,j}(\mathbf{h}_{m,i,j})^H$, one can rewrite $T^c_{i,j}$ defined earlier as

$$\begin{aligned} T^c_{i,j} = & \sum_{l} \text{tr}(\mathbf{H}_{i,i,j}\mathbf{V}^p_{i,l}) + \sum_{l \neq j} \text{tr}(\mathbf{H}_{i,i,j}\mathbf{V}^c_{i,l}) \\ & + \sum_{m \neq i,n} \text{tr}(\mathbf{H}_{m,i,j}\mathbf{V}^p_{m,n}) + \sum_{m \neq i,n} \text{tr}(\mathbf{H}_{m,i,j}\mathbf{V}^c_{m,n}) \end{aligned} \tag{10}$$

Then, for fixed $R^p_{i,j}$ and $R^c_{i,j}$ and for fixed $(\hat{i},\hat{j})$ for each (i,j), (9) can be written as an SDP

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} \text{tr}(\mathbf{V}^p_{i,j}) + \text{tr}(\mathbf{V}^c_{i,j}) \\ \text{subject to} \quad & \left(\frac{1}{2^{R^p_{i,j}}-1} + 1\right) \text{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^p_{i,j}) \\ & \qquad + \text{tr}(\mathbf{H}^H_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}) - T^c_{i,j} \geq \sigma^2 \\ & \left(\frac{1}{2^{R^c_{i,j}}-1}\right) \text{tr}(\mathbf{H}^H_{i,i,j}\mathbf{V}^c_{i,j}) - T^c_{i,j} \geq \sigma^2 \\ & \left(\frac{1}{2^{R^c_{\hat{i},\hat{j}}}-1} + 1\right) \text{tr}(\mathbf{H}^H_{\hat{i},i,j}\mathbf{V}^c_{\hat{i},\hat{j}}) - T^c_{i,j} \geq \sigma^2 \\ & \mathbf{V}^p_{i,j} \succeq 0, \mathbf{V}^c_{i,j} \succeq 0, \qquad \forall i,j \end{aligned} \tag{11}$$

where $T^c_{i,j}$ is as defined in (10) and the minimization is over the Hermitian positive semidefinite matrices $\mathbf{V}^p_{i,j}$ and $\mathbf{V}^c_{i,j}$. The above reformulation is a relaxation of (9) because the original problem requires the matrices $\mathbf{V}^p_{i,j}$ and $\mathbf{V}^c_{i,j}$ to be rank-1, while the relaxation does not necessarily produce a rank-1 solution. Nevertheless, because the SDP is a convex optimization problem for which efficient numerical algorithms are available, it offers an efficient way of finding good solutions to the original problem (9).

The use of SDP relaxation for solving downlink beamforming problem is originally due to [5], where it is proved that for the single-cell system the relaxation actually admits a rank-1 optimal solution to the original problem. The same is true for the multicell problem [9] if no common-private information splitting is employed. With common-private rate splitting, the SDP in general does not always admit a rank-1 optimal solution. In this case, randomization techniques can be used to produce an approximate rank-1 solution, which is often a good solution to the original optimization problem [10], [11]; see also [12].

Interestingly, there is a special case in which even with private-common splitting the SDP relaxation (11) does admit an optimal rank-1 solution. This happens when there are at most two pairs of information splittings in the network. The proof of this fact can be obtained from the recent result of [13], where it is shown that a high-rank solution can be reduced to a lower-rank one if the size of the optimization problem satisfies certain condition. This condition is satisfied when the number of information splittings in (11) is no more than two.

B. Joint Beamforming, Rate Splitting and User Selection

We now consider the problem of joint beamforming, common-private rate splitting and user-selection strategy in a multicell system, such as the one illustrated in Fig. 1. The optimization variables are the user common-message decoding pair $(\hat{i},\hat{j}) \rightarrow (i,j)$ combinations (so that user (i,j) decodes common message from user $(\hat{i},\hat{j})$), and the corresponding common and private rates for each pair. Note that as mentioned earlier, each user is allowed to share common information with only one out-of-cell user. In addition, each user may only decode common message from one out-of-cell user.

A brute force approach to solving the joint optimization problem would involve searching over all possible user decoding-pair combinations, and all possible common-private rate splittings. For each combination, the SDP relaxation approach in the previous section can be used to find the beamforming vectors and the resulting total power. The optimal decoding-pair and rate splitting is the combination that gives the minimal overall power. This exhaustive search strategy is clearly infeasible for any reasonably sized network, as the search space is exponentially large.

This paper proposes a heuristic approach of pairing users according to the respective interference-to-noise ratios (INR). The intuition is that to achieve within one bit of the capacity region, any interference above the noise level should be regarded as common information [2]. Thus, the INR of user $(\hat{i}, \hat{j})$ at user (i, j) gives an indication as to whether common-message splitting at user $(\hat{i}, \hat{j})$ is worthwhile for user (i, j).

In fact, the user pair $(\hat{i}, \hat{j}) \to (i, j)$ (assuming $\hat{i} \neq i$) with the highest INR can be seen as the best candidate for common-private message splitting. Thus, we can simply search through the INRs of all $(i, j) \to (\hat{i}, \hat{j})$ user pairs and select the M pairs with the largest INRs, while satisfying the condition that each user splits rate for only one other user and decodes common message from only one other user. In the following, we denote the INR of user $(\hat{i}, \hat{j})$ at user (i, j) as $\text{INR}_{(i,j)\to(\hat{i},\hat{j})}$. Note that there are $(N-1)K$ such INR entries for each receiver, and the network has $(N-1)NK^2$ entries in total.

Once the set of user pairs are chosen, we can then choose the common-private rates for each pair using a linear search. In theory, the common-private rate splittings of the user pairs are interdependent. In practice, to reduce the complexity of exhaustive search, we simply first find the optimal splitting for the user pair with the highest INR using a linear search, then fix the first splitting and search for the optimal splitting for the user pair with the next highest INR, etc.

The proposed heuristic strategy is summarized below:

1) Find the set of beamformers corresponding to the conventional transmission strategies with private information only by solving the SDP relaxation (11) with $R^p_{i,j} = R_{i,j}\ \forall (i, j)$. Calculate the corresponding set of $(N-1)NK^2$ entries of $\text{INR}_{(i,j)\to(\hat{i},\hat{j})}$ with $i \neq \hat{i}$.
2) Sort the list of $\text{INR}_{(i,j)\to(\hat{i},\hat{j})}$ in decreasing order.
3) Starting from the top of the sorted list, select M user pairs, while ensuring no user appears more than once on either the right hand side of the "→" or the left hand size of the "→" in the subscripts of INR's.
4) Starting from the top of the selected user pairs, for each $(\hat{i}, \hat{j}) \to (i, j)$ pair, find the optimal private rates $R^p_{\hat{i},\hat{j}}$ through a linear search by calling the SDP relaxation routine for all possible value of $0 \le R^p_{\hat{i},\hat{j}} \le R_{\hat{i},\hat{j}}$, while keeping all other rates fixed. Repeat over all M pairs until convergence.

The above algorithm may be improved by updating the INR list as soon as a rate splitting is determined, or by iteratively updating the optimal rate splittings, at additional complexity.

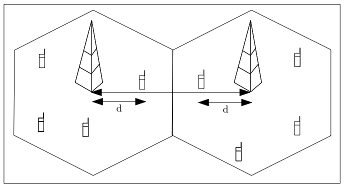

Fig. 2. A two-cell four-user per cell configuration with two users located between two base-stations at distance d.

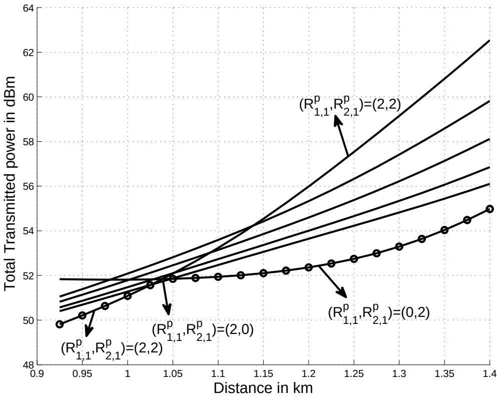

Fig. 3. Total transmitted power versus distance in km for different rate splits for two-cell network with four users per cell.

IV. SIMULATIONS

This section illustrates the benefit of joint beamforming and common information decoding for multicell networks by simulation. Consider first a 2-cell network with 4 users per cell as shown in Fig. 2 where common message decoding is performed only for the two users situated directly between the two base-stations. The base-stations are equipped with 4 antennas each. Realistic channel models are used in the simulation: the noise power spectral density is set to -162 dBm/Hz; the channel vectors are chosen according to a distance-dependent path loss $L = 128.1 + 37.6 \log_{10}(d)$, where d is the distance in kilometers, with log-normal shadowing with 8dB variance, and a Rayleigh component. The distance between neighboring base-stations is 2.8km; an antenna gain of 15dBi is assumed.

Fig. 3 shows the minimum total transmit power when every user in the 2-cell network is assigned a target rate of 2 bits/sec/Hz. The two users with common-message decoding are situated at distance d away from their respective base-stations. (The other users are located randomly within each cell.) The minimum total transmit power is plotted as a function of d for various common-private rate splittings. For example, the line marked with $(R^p_{1,1}, R^p_{2,1}) = (2, 2)$ represents the case of private message only—this is actually optimal when d is less than about 1.03 km. As the users move closer to the cell edge, assigning $(R^p_{1,1}, R^p_{2,1}) = (2, 0)$ or $(R^p_{1,1}, R^p_{2,1}) = (0, 2)$ becomes optimal. The minimum transmit power over all possible common-private rate splittings is the lower envelope of all these curves. Fig. 3 shows that the benefit

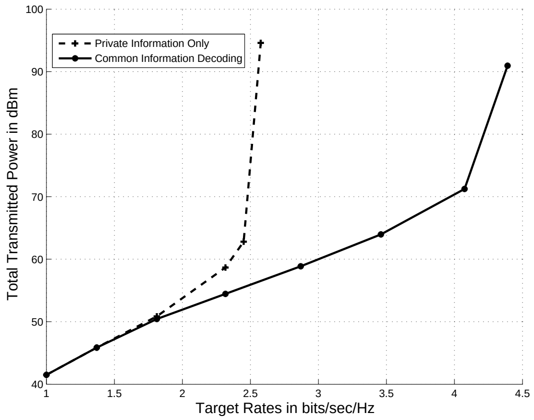

Fig. 4. Total transmitted power versus the rate targets for both the case of private-message only and the case of common-message decoding in a two-cell network with four users per cell.

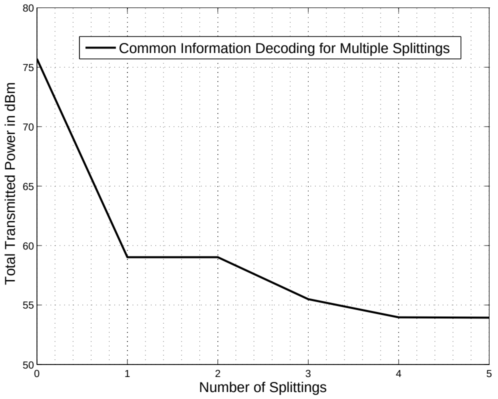

Fig. 5. Total transmitted power versus the number of splittings for three-cell network with three users per cell sector.

of common message decoding in term of total transmit power reduction at the base-stations is substantial. It can be up to 7dB when the users are at the cell edge, where out-of-cell interference is the largest. Fig. 3 also shows that determining the appropriate rate splitting is crucial, and that the optimal rate splitting is channel dependent.

Fig. 4 shows the minimal transmit power as a function of target rate for the 2-cell scenario at a fixed $d = 1.1$km. The plot illustrates that with private information only, the system becomes infeasible at about 2.5 bits/sec/Hz. With common message decoding, the feasible target rate can be expanded to about 4.5 bits/sec/Hz, which is a significant improvement.

Finally, we simulate a 3-cell network with 3 users per cell sector shown in Fig. 1. Again, realistic channel models are used. In addition, the antenna element responses here also include a directional component due to sectorization. The users are randomly located close to the cell edge. We implement the joint beamforming, user selection and rate-splitting algorithm of Section III-B, and plot the minimal total transmit power as a function of the number of user common-message decoding pairs in Fig. 5. As Fig. 5 illustrates, a splitting of a single user pair already produces substantial power reduction. Additional performance improvement is obtained when common-message splitting is implemented with multiple user pairs.

V. CONCLUDING REMARKS

Information theoretical studies have long suggested that common-private message splitting at the transmitters and common-message decoding at the receivers have the potential to significantly improve the achievable rate region of the interference channel. This paper is an effort toward making the information theoretical insight practical for a realistic multiuser multiantenna multicell network. By formulating the problem in a tractable form of minimizing the total transmit power subject to target rate constraints, and by incorporating the additional components such as multiantenna beamforming at the transmitter and user selection among the multiple cells, this paper shows that common-message decoding can indeed bring substantial benefit to a practical multicell network. The main ingredients of the proposed joint optimization algorithm are a convex relaxation approach for transmit beamforming design and a heuristic user-pairing and rate-splitting strategy for common-message decoding.

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