Gaussian Z-Interference Channel with a Relay Link: Achievable Rate Region and Asymptotic Sum Capacity

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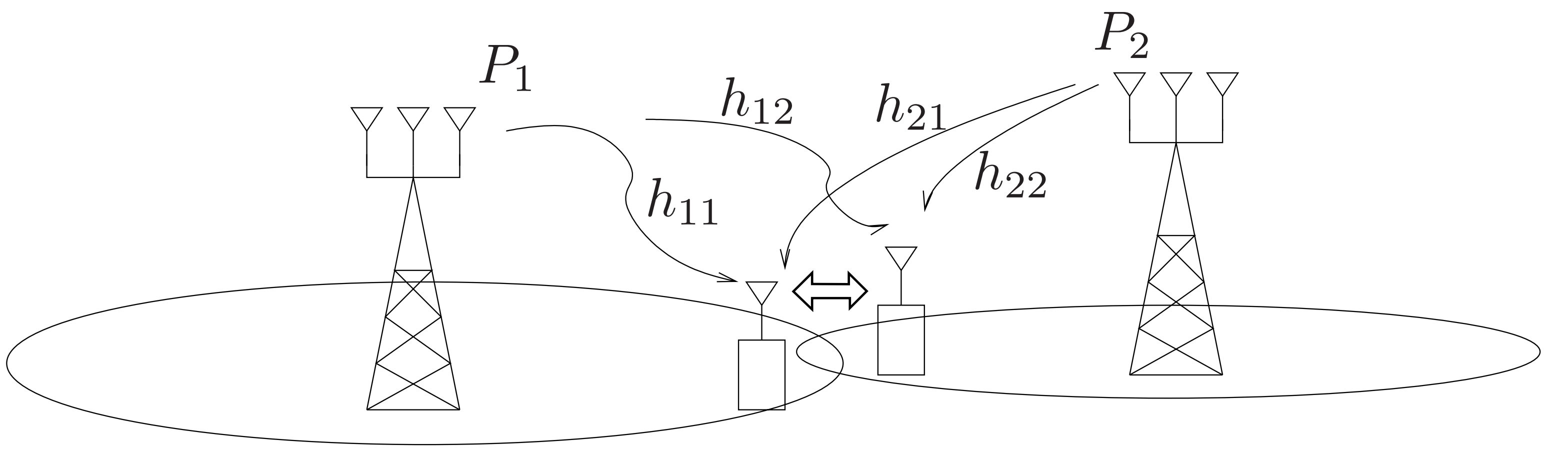
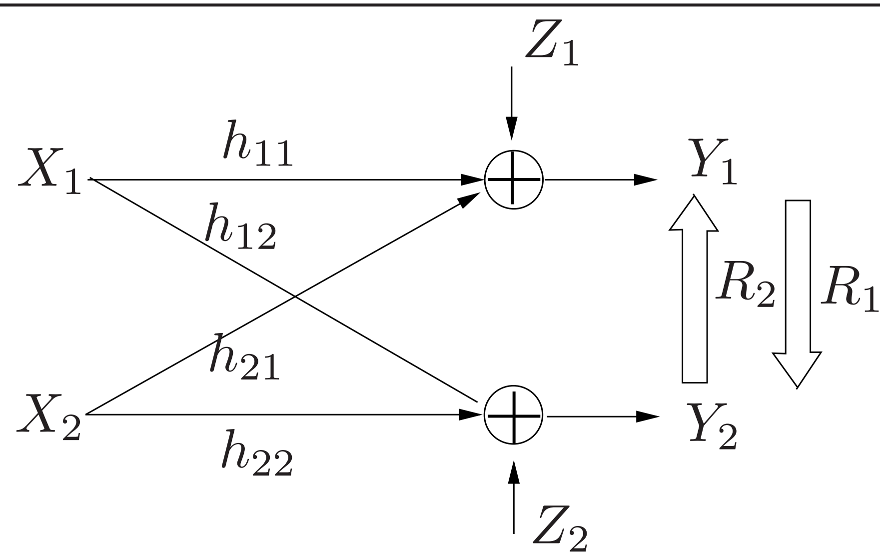
Motivation: Mobile Users at the Cell Edge



- Cellular networks are fundamentally limited by intercell interference.
  - However, receivers are often capable of communicating with each other via independent relay links.
  - Question: Can we use relay links for interference mitigation?

Gaussian Interference Channel with Relay Links



- Fundamental coding strategies:
  - Han-Kobayashi common-private scheme for the interference channel
  - Decode-and-forward or quantize-and-forward for the relay channel
  - Multi-session conversation for the broadcast channel

Two Simplest Models

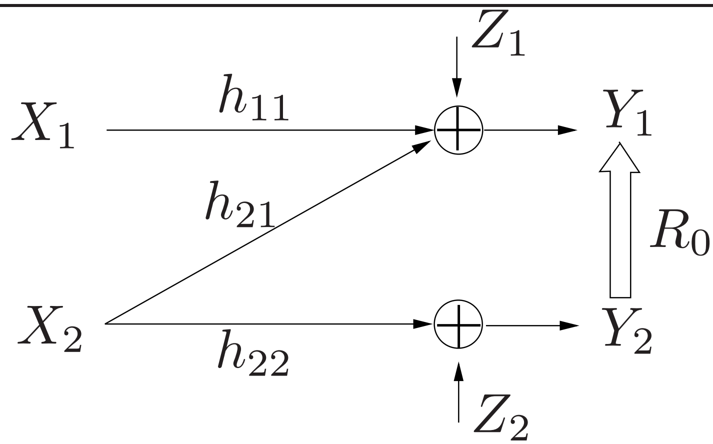


Figure 1: Type I

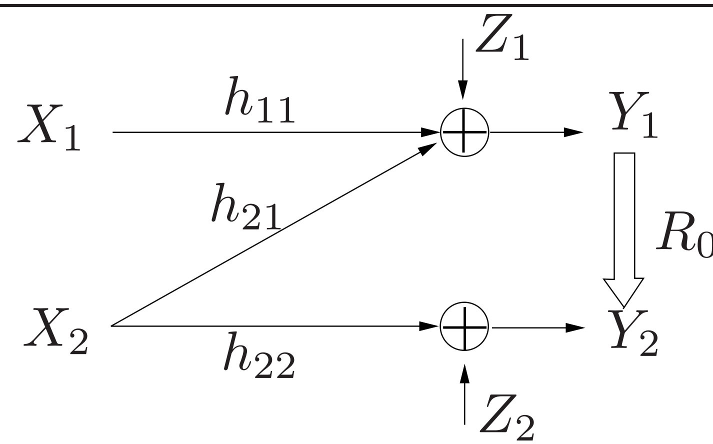
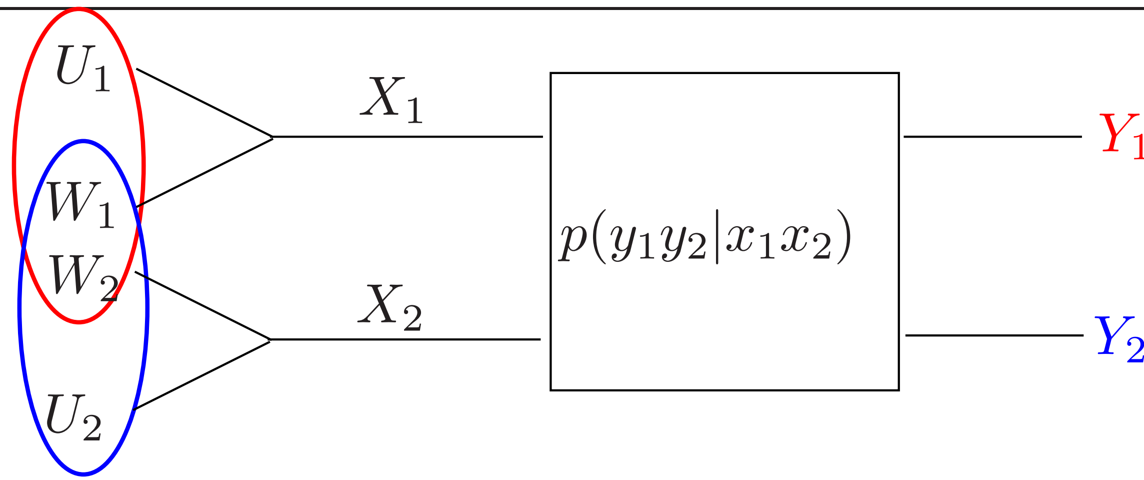


Figure 2: Type II

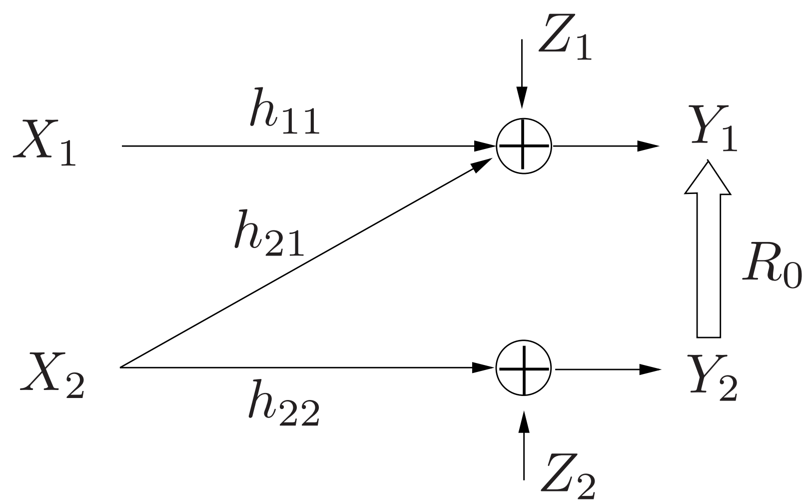
- Related work:
  - Sahin and Erkip ('07): Interference channel with an extra relay node
  - Ng, Jindal, Goldsmith, Mitra ('07): Tx and Rx cooperation
  - Maric, Dabora, Goldsmith ('08): Interference-forwarding strategy
  - Simeone, Somekh, Poor, Shamai ('08): 1-D cellular model with relay

Interference Channel: Han-Kobayashi Scheme ('81)



- Basic idea: Allow interference to be partially subtracted.
  - U_1 and U_2 are private messages, W_1 and W_2 are common messages;
  - Achievable region based on intersection of two multiple access regions
- Etkin, Tse, Wang ('07) shows Han-Kobayashi is within 1-bit to capacity.

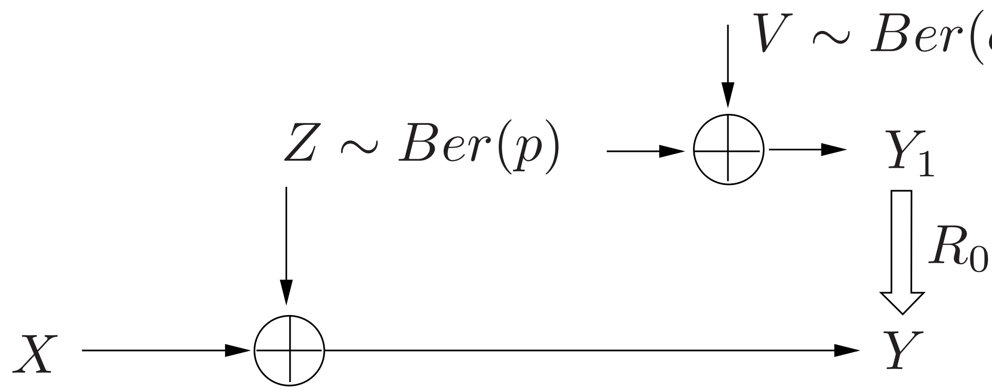
Type I Gaussian Z-relay interference Channel



- How can Y_2 help Y_1?
- Y_2 does not observe X_1, but it observes the noise at Y_1.

Natural idea: forward the interference to allow subtraction.

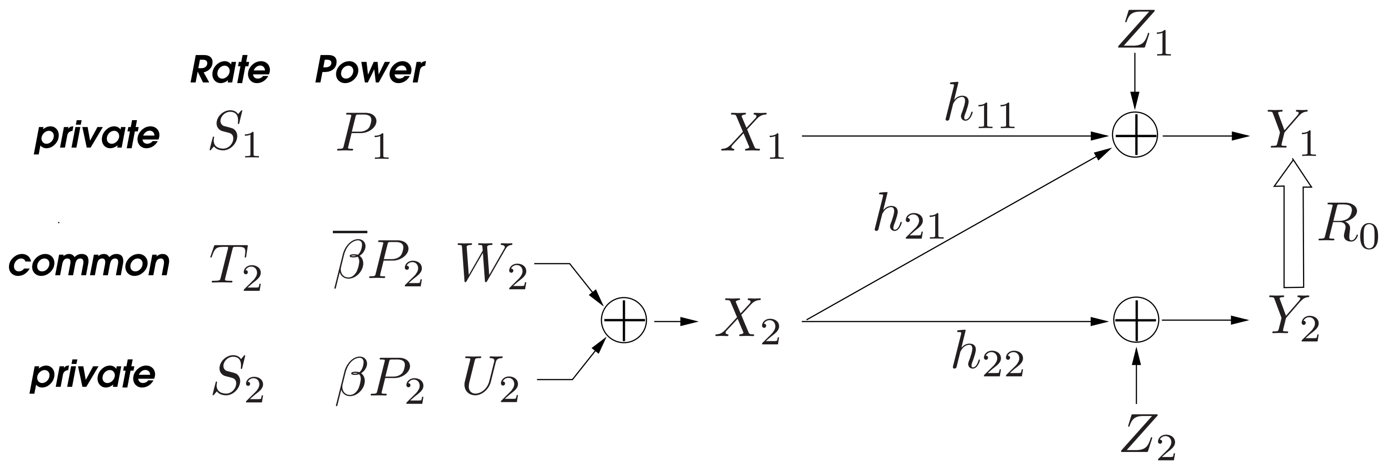
Related Work: A Class of Modulo-Sum Relay Channel



- Y_2 observes a noisy version of the noise Z.
- Quantize-and-forward is capacity achieving (Aleksic, Razaghi, Yu '07)

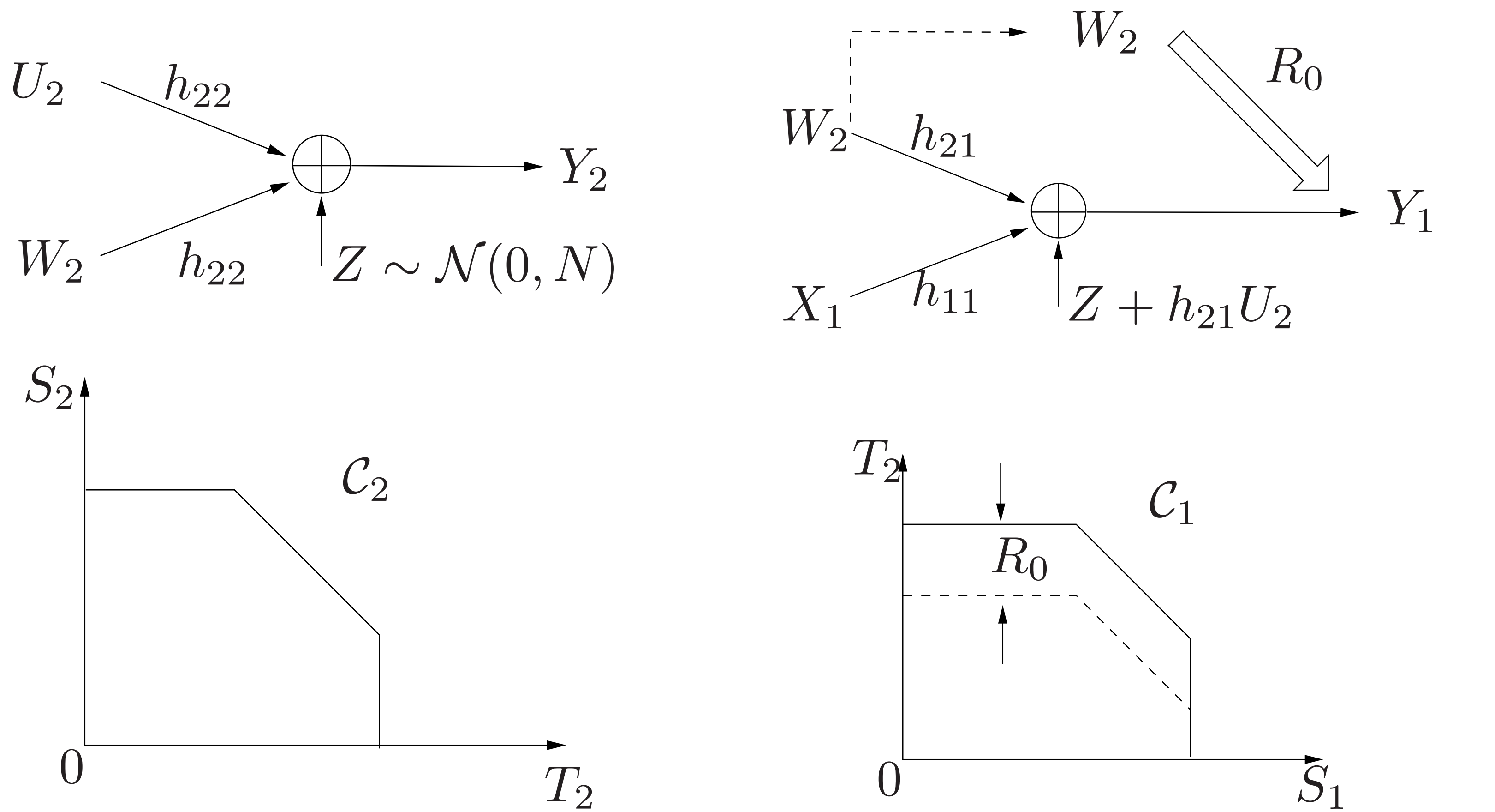
What if the noise comes from a codebook?

Partial Interference Forwarding for Type I Channel

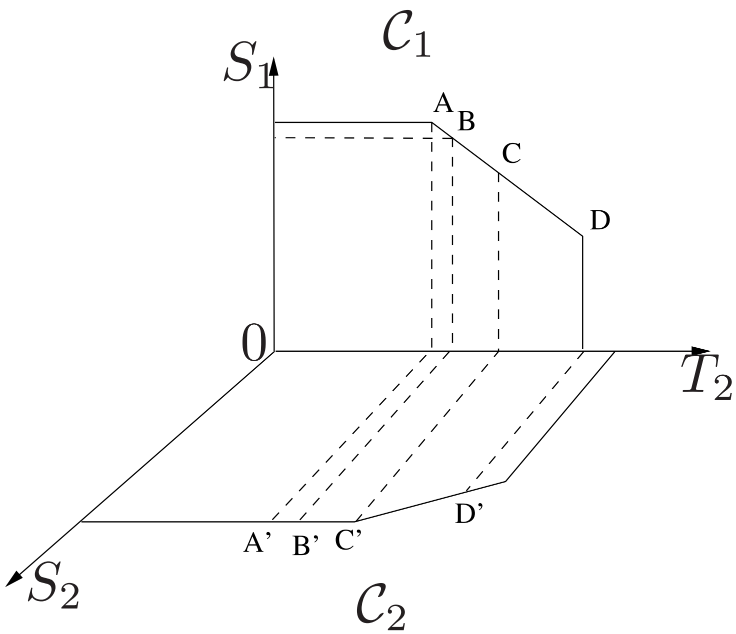


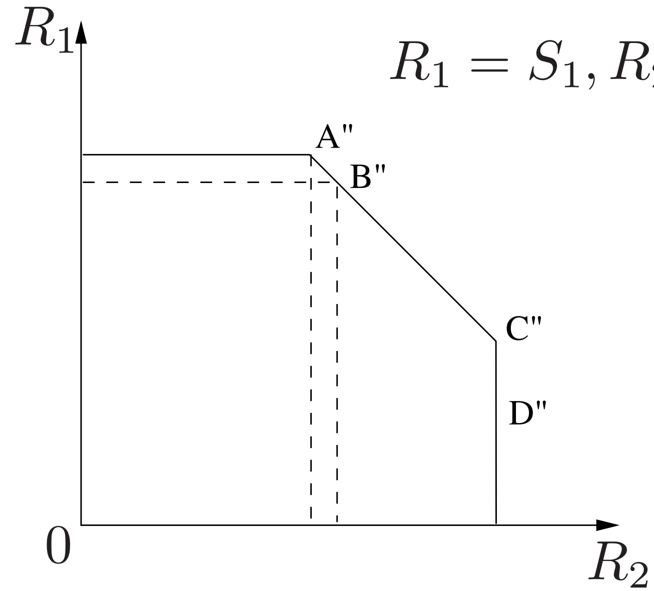
- Based on Han-Kobayashi common-private splitting:
  - β is the portion of the power assigned to the private message U_2.
- Achievable rate region is based on the intersection of two multiple access regions: $\mathcal{C}_1 : (X_1, W_2) \to Y_2$ with R_0, and $\mathcal{C}_2 : (W_2, U_2) \to Y_2$.

Two Multiple Access Channel Components



Intersecting Two Pentagons





$$R_1 = S_1, R_2 = S_2 + T_2$$

Union of Pentagons

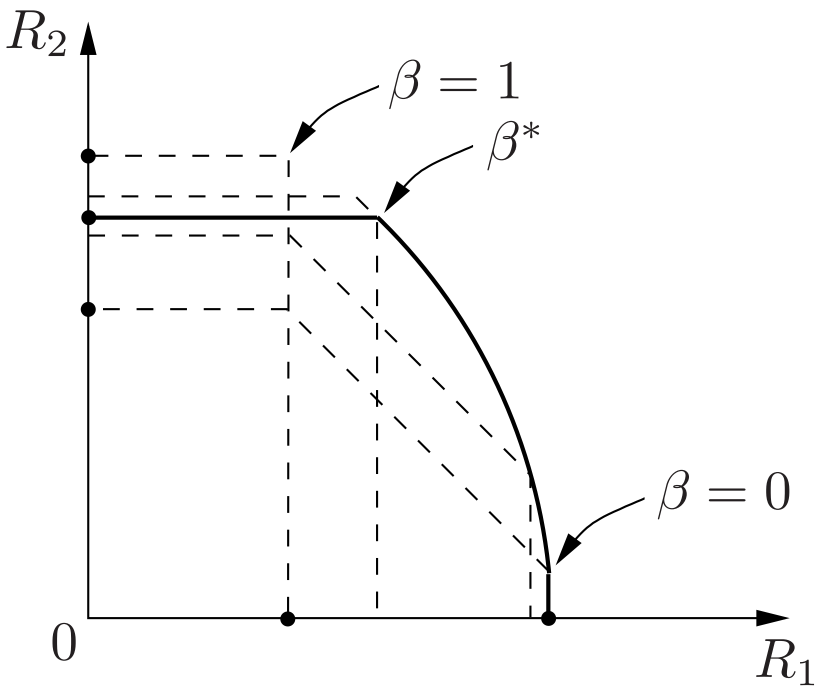


Figure 3: Weak interference regime.

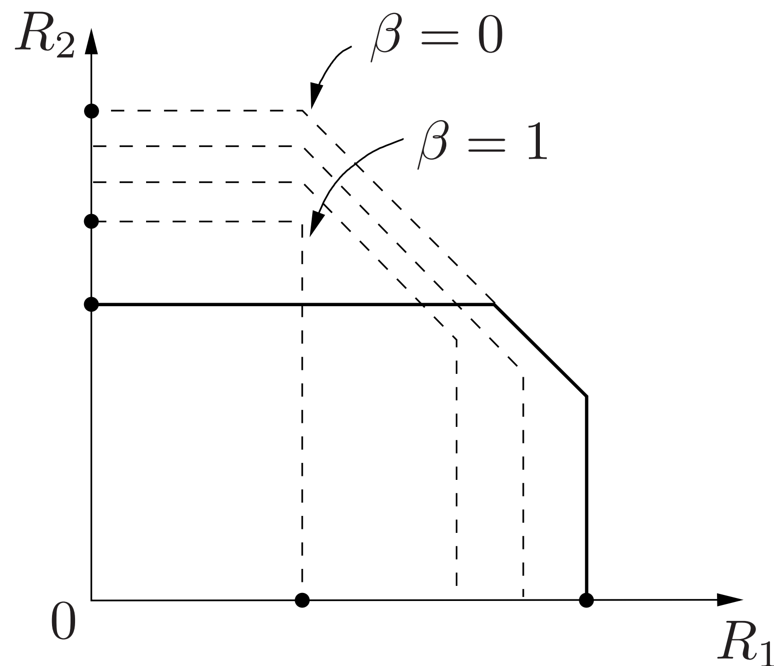


Figure 4: Strong interference regime.

Type I Channel: Achievable Rate Regions

Theorem 1. *For the Gaussian Z-interference channel with a digital link of limited rate R_0 from the clean receiver to the interfered receiver, in the weak interference regime defined by* $\mathsf{INR}_2 \leq \mathsf{SNR}_2$*, the following rate region is achievable:*

$$\bigcup_{0\leq\beta\leq 1} \left\{ (R_1, R_2) \left| \begin{array}{l} R_1 \leq \gamma\left(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}\right) \\ R_2 \leq \min\left\{\gamma(\mathsf{SNR}_2), \gamma(\beta\mathsf{SNR}_2) + \right. \\ \qquad\qquad \left. \gamma\left(\frac{(1-\beta)\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right) + R_0\right\} \end{array} \right. \right\}.$$

where $\mathsf{SNR}_1 = \frac{|h_{11}|^2 P_1}{N}$, $\mathsf{SNR}_2 = \frac{|h_{22}|^2 P_2}{N}$, $\mathsf{INR}_2 = \frac{|h_{21}|^2 P_2}{N}$ *and* $\gamma(x) = \frac{1}{2}\log_2(1+x)$.

In the strong interference regime, defined by

$$\mathsf{SNR}_2 \le \mathsf{INR}_2 \ \le \max\{\mathsf{SNR}_2, \mathsf{INR}_2^*\},$$

where

$$\mathsf{INR}_2^* = (1+\mathsf{SNR}_1)(2^{-2R_0}(1+\mathsf{SNR}_2)-1),$$

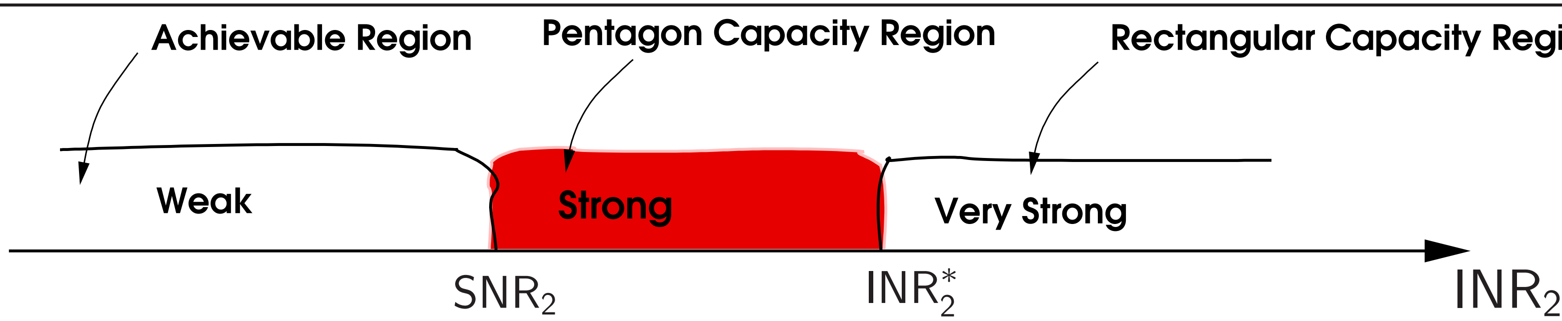
the capacity region is given by:

$$\left\{ (R_1, R_2) \middle| \begin{array}{rcl} R_1 & \le & \gamma(\mathsf{SNR}_1) \\ R_2 & \le & \gamma(\mathsf{SNR}_2) \\ R_1 + R_2 & \le & \gamma(\mathsf{SNR}_1 + \mathsf{INR}_2) + R_0 \end{array} \right\}.$$

In the very strong interference regime defined by $\mathsf{INR}_2 \ge \max\{\mathsf{SNR}_2, \mathsf{INR}_2^*\}$*, the capacity region is given by:*

$$\left\{ (R_1, R_2) \middle| \begin{array}{l} R_1 \le \gamma(\mathsf{SNR}_1) \\ R_2 \le \gamma(\mathsf{SNR}_2) \end{array} \right\}.$$

Different Interference Regimes: Type I

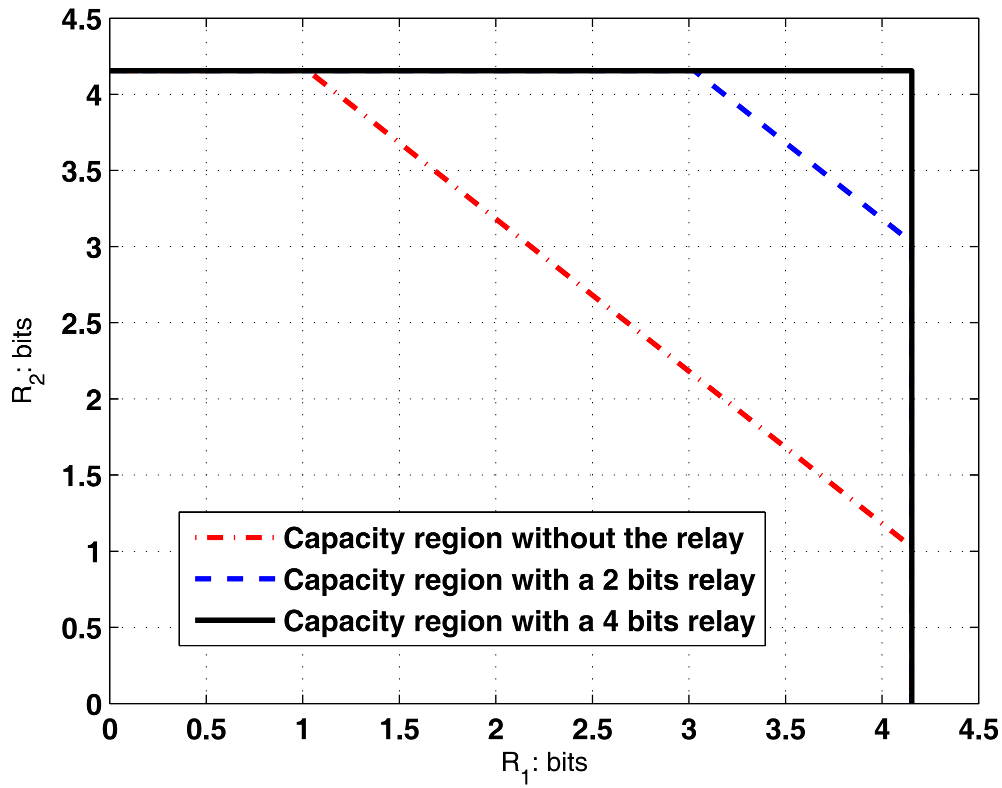


- Both common message and private message are needed in the weak interference regime.

- In the strong and very strong interference regimes, capacity regions are achieved by an all-common-message scheme ($\beta = 0$).

- A strong-interference channel can be converted to the very strong regimes by a relay link. But weak regime can never be converted to strong regime.

Strong & Very Strong Interference Regimes

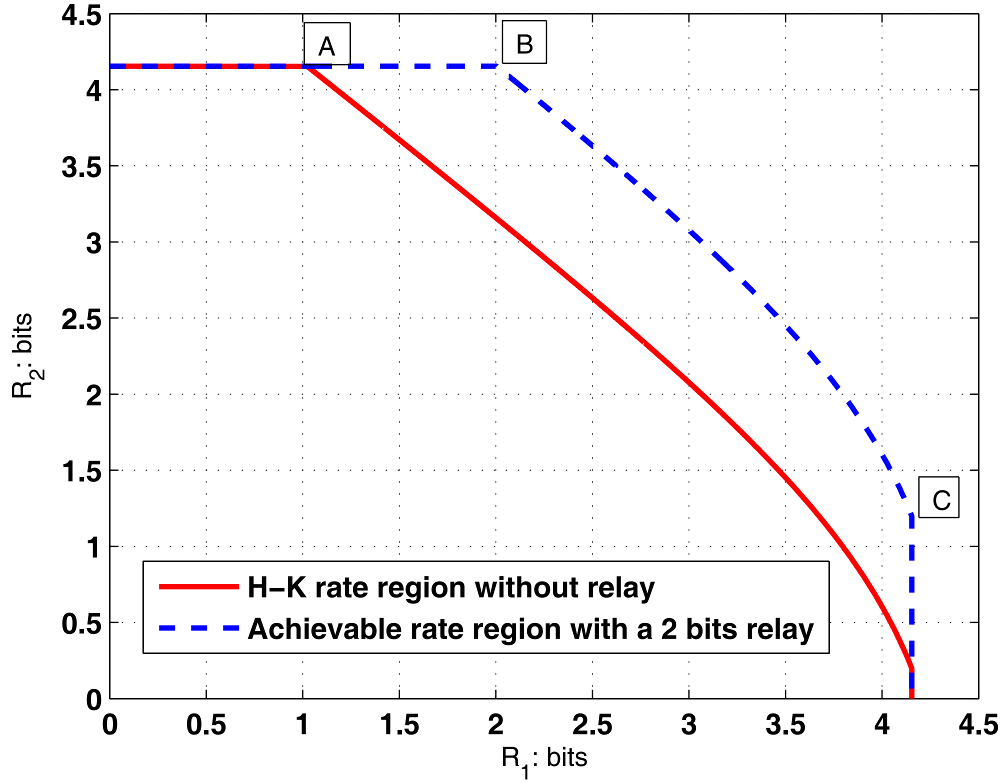


$\text{SNR}_1 = 25\text{dB}$

$\text{SNR}_2 = 25\text{dB}$

$\text{INR}_2 = 30\text{dB}$

Weak Interference Regime

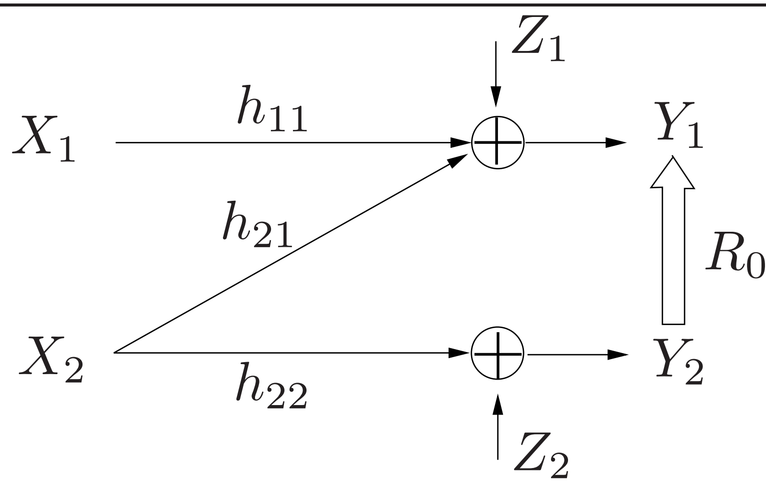


$\mathsf{SNR}_1 = 25\text{dB}$

$\mathsf{SNR}_2 = 25\text{dB}$

$\mathsf{INR}_2 = 20\text{dB}$

Weak Interference Regime: Discussion



- For fixed R_2, the relay increases R_1 by less than R_0 bits.
  - Asymptotically at high SNR/INR, the increase in R_1 approaches R_0
  - Reminiscent of deterministic relay channel result (Kim and Cover '06)

- For fixed R_1, the relay increases R_2 by exactly R_0 bits!

Asymptotic Sum Capacity in Weak Interference

Theorem 2. *For the Type I Gaussian Z-interference channel with a relay link of capacity* R_0, *when* $\mathsf{INR}_2 \leq \mathsf{SNR}_2$ *and*

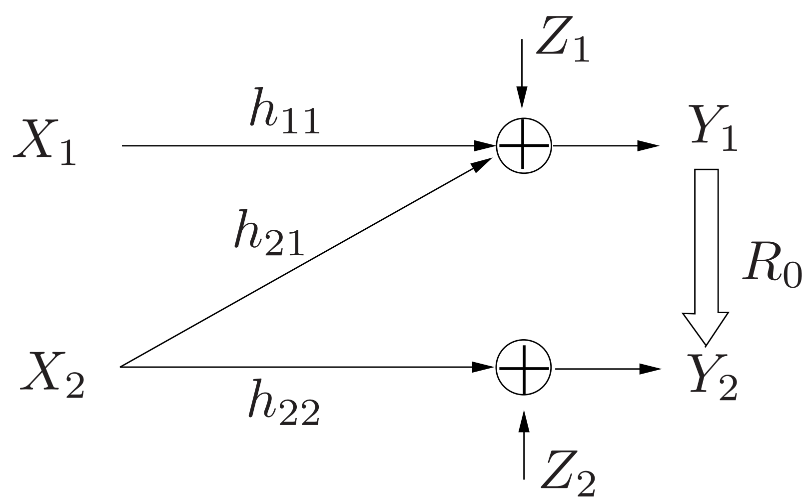
$$\min\{\mathsf{SNR}_1, \mathsf{SNR}_2, \mathsf{INR}_2\} \gg 1. \tag{1}$$

the asymptotic sum capacity is given by:

$$C_{sum}(R_0) \approx C_{sum}(0) + R_0 \tag{2}$$

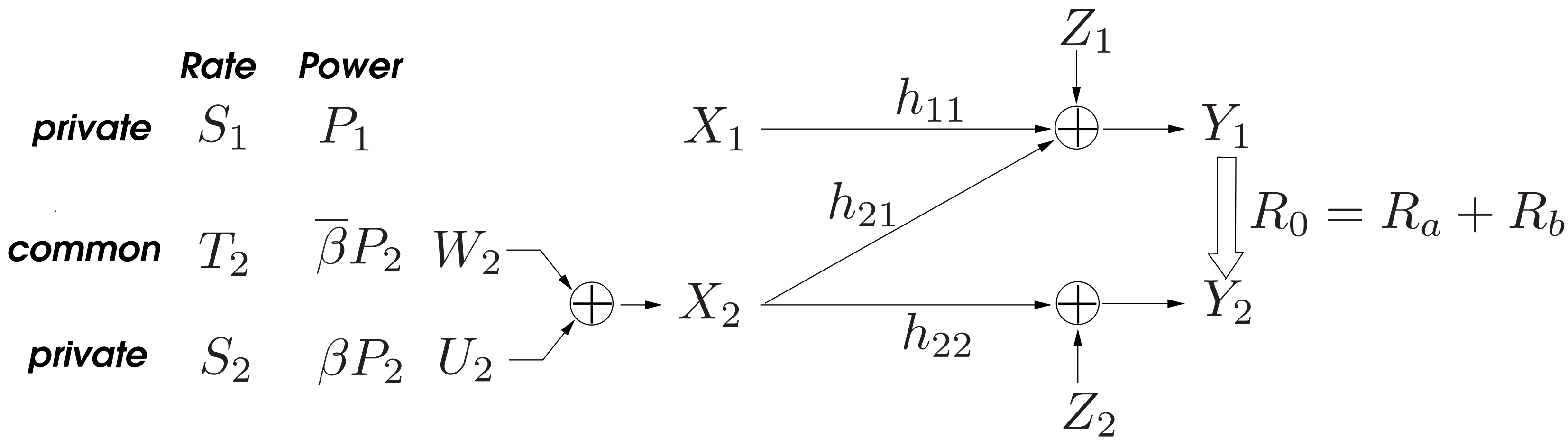
where the notation $f(x) \approx g(x)$ *is used to denote* $\lim_{N \to 0} f(x) - g(x) = 0$.

Type II Gaussian Z-relay Interference Channel



- How can Y_1 help Y_2?
  - Strong interference: Y_1 can decode-and-forward common information
  - Weak interference: Y_1 can quantize-and-forward private information

Relay Strategy for the Type II Channel



- Decode-and-forward W_2 using R_b.
- Quantization-and-forward U_2 using R_a.
  - Strategy #1: Quantize $h_{21}U_2 + Z_1$, with W_2 subtracted.
  - Strategy #2: Quantize $h_{21}(U_2 + W_2) + \alpha W_2 + Z_1$. Optimize α!

Type II Channel: Achievable Rate Regions

Theorem 3. *For the Type II Gaussian Z-relay interference channel with relay rate R_0, in the weak interference regime defined by $\mathsf{INR}_2 \leq \mathsf{SNR}_2$, the following rate region is achievable*

$$\bigcup_{0 \leq \beta \leq 1} \left\{ (R_1, R_2) \middle| R_1 \leq \gamma\left(\frac{\mathsf{SNR}_1}{1+\beta \mathsf{INR}_2}\right), \right.$$

$$\left. R_2 \leq \gamma(\beta \mathsf{SNR}_2) + \gamma\left(\frac{\overline{\beta}\mathsf{INR}_2}{1+\mathsf{SNR}_1+\beta\mathsf{INR}_2}\right) + \delta(\beta, R_0) \right\}, \quad (1)$$

where

$$\delta(\beta, R_0) = \gamma\left(\frac{\beta(2^{2R_0}-1)\mathsf{INR}_2}{2^{2R_0}(1+\beta\mathsf{SNR}_2)+\beta\mathsf{INR}_2}\right). \quad (2)$$

In the moderately strong interference regime, defined by

$$\mathsf{SNR}_2 \le \mathsf{INR}_2 \le 2^{2R_0}(1+\mathsf{SNR}_2) - 1 \triangleq \mathsf{INR}_2^{\dagger}, \tag{3}$$

the convex hull of the following region

$$\left\{ (R_1, R_2) \,\middle|\, \begin{array}{l} R_1 \le \gamma(\frac{\mathsf{SNR}_1}{1+\beta\mathsf{INR}_2}) \\ R_2 \le \min\{\gamma(\mathsf{SNR}_2) + R_b + \eta(\alpha,\beta,R_a), \\ \qquad\qquad \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\bar{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\ \qquad\qquad +\zeta(\alpha,\beta,R_a)\} \\ R_1 + R_2 \le \gamma(\beta\mathsf{SNR}_2) + \gamma\left(\frac{\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}_2}{1+\beta\mathsf{INR}_2}\right) \\ \qquad\qquad +\zeta(\alpha,\beta,R_a) \end{array} \right\}. \tag{4}$$

over all $0 \le \beta \le 1$, $R_a + R_b \le R_0$ *is achievable.*

In the strong interference regime defined by

$$\mathsf{INR}_2^{\dagger} \le \mathsf{INR}_2 \le (1 + \mathsf{SNR}_1)\mathsf{INR}_2^{\dagger} \triangleq \mathsf{INR}_2^{\ddagger}, \tag{5}$$

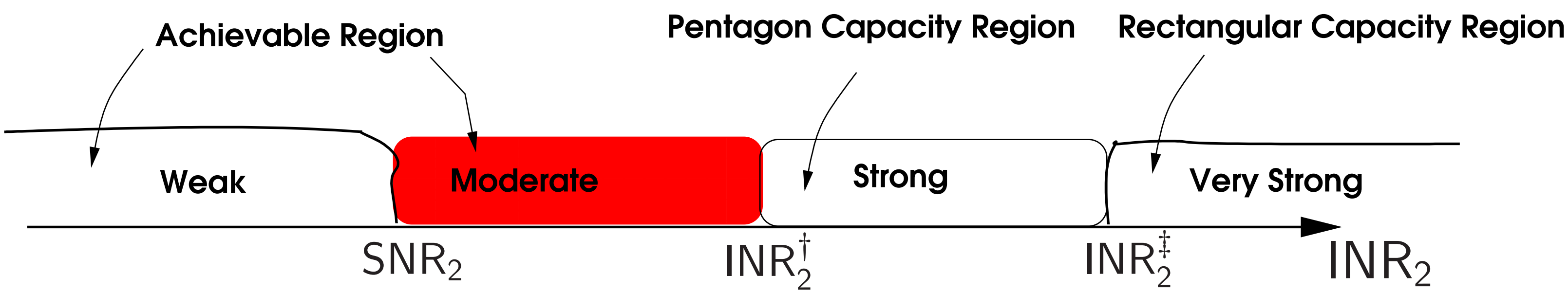
the capacity region is given by

$$\left\{ (R_1, R_2) \middle| \begin{array}{rcl} R_1 & \le & \gamma(\mathsf{SNR}_1) \\ R_2 & \le & \gamma(\mathsf{SNR}_2) + R_0 \\ R_1 + R_2 & \le & \gamma(\mathsf{SNR}_1 + \mathsf{INR}_2) \end{array} \right\}. \tag{6}$$

In the very strong interference regime defined by $\mathsf{INR}_2 \ge \mathsf{INR}_2^{\ddagger}$*, the capacity region is given by*

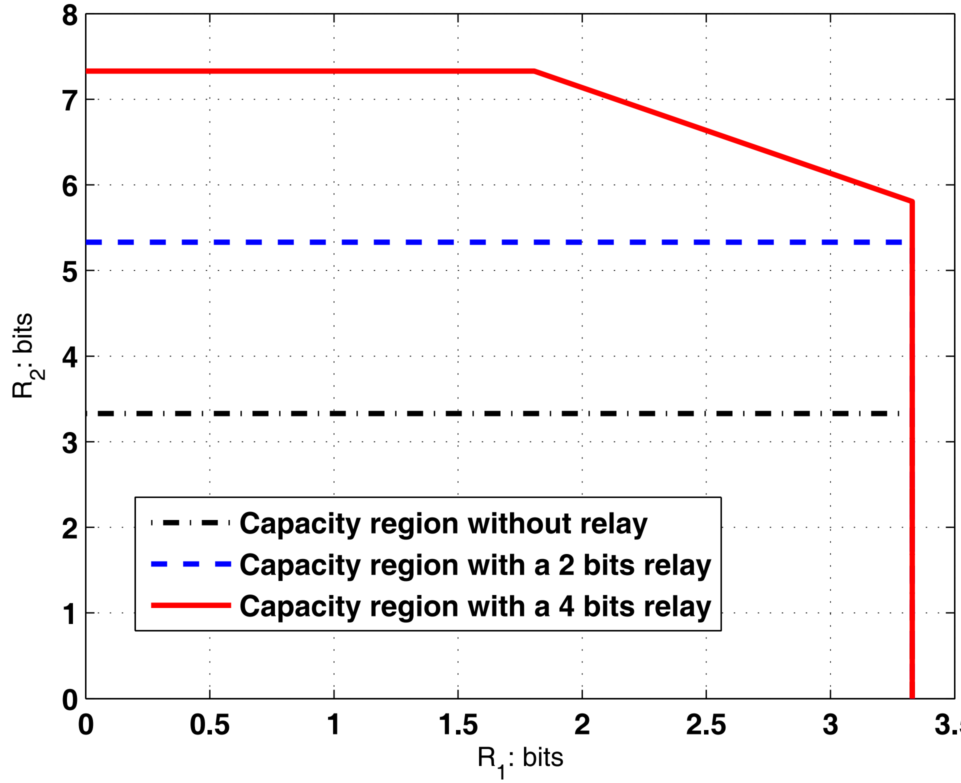
$$\left\{ (R_1, R_2) \middle| \begin{array}{l} R_1 \le \gamma(\mathsf{SNR}_1) \\ R_2 \le \gamma(\mathsf{SNR}_2) + R_0 \end{array} \right\}. \tag{7}$$

Different Interference Regimes: Type II



- Weak interference regime: Both common message and private message are needed. Quantize-and-forward private message only.
- Strong and very strong interference regimes: Decode-and-forward the common message only.
- Moderately strong interference regime: Mixture of decode and quantize.

Strong & Very Strong Interference Regimes

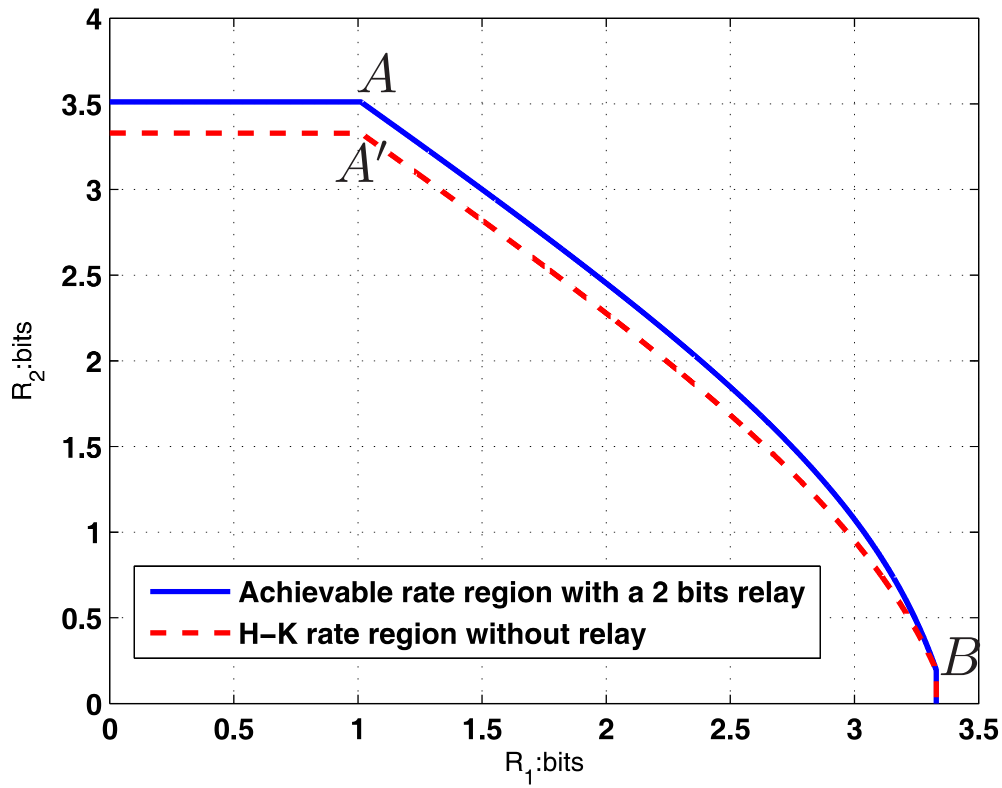


$\text{SNR}_1 = 20\text{dB}$

$\text{SNR}_2 = 20\text{dB}$

$\text{INR}_2 = 55\text{dB}$

Weak Interference Regime

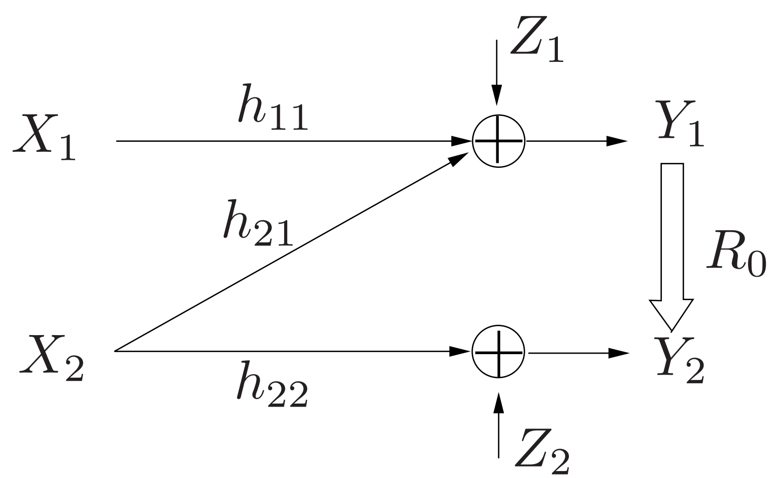


$\mathrm{SNR}_1 = 20\mathrm{dB}$

$\mathrm{SNR}_2 = 20\mathrm{dB}$

$\mathrm{INR}_2 = 15\mathrm{dB}$

Sum Capacity Gain in Weak Interference



- Why is capacity gain limited even as $R_0 \to \infty$?
  - Common info rate is limited by interference link – relay is not useful.
  - Relay helps private info rate by giving Y_2 another look of X_2.
  - SNR gain is at most 3dB. Sum capacity gain is at most $1/2$ bit.

Sum Capacity Bound in Weak Interference

Theorem 4. *For the Type II Gaussian Z-interference channel with a relay link of capacity* R_0 *with* $\mathsf{INR}_2 \le \mathsf{SNR}_2$*, let* $\mathcal{C}(R_0)$ *be its capacity region, and let*

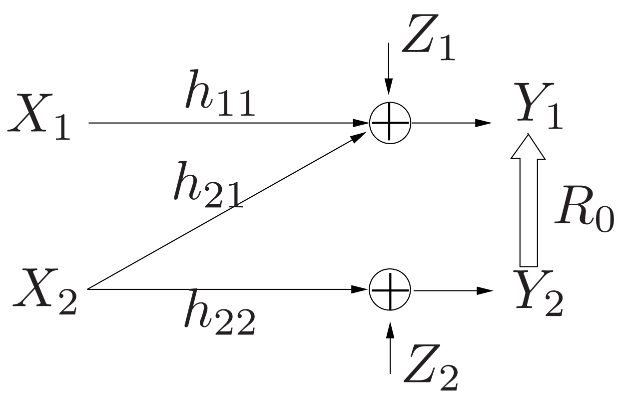
$$C_{sum}^{\theta}(R_0) = \max_{(C_1,C_2)\in\mathcal{C}(R_0)} \theta C_1 + (1-\theta)C_2. \quad (1)$$

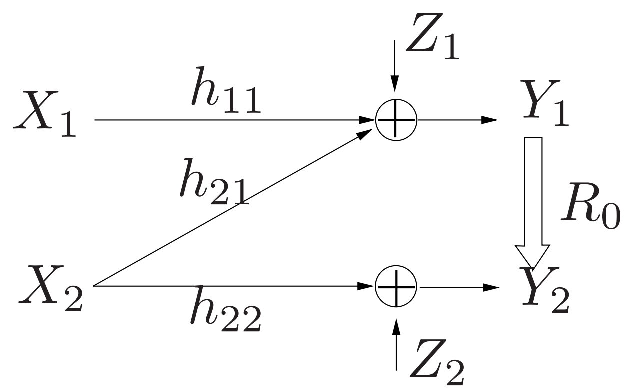
For any $R_0 \ge 0$*,*

$$C_{sum}^{\theta}(R_0) \le C_{sum}^{\theta}(0) + (1-\theta)\gamma\left(\frac{\mathsf{INR}_2}{1+\mathsf{SNR}_2}\right). \quad (2)$$

In particular, the sum capacity of the Type II Gaussian Z-interference channel with a relay link is bounded by the sum capacity without the relay link plus 1/2 bit.

Concluding Remarks





- For a Gaussian Z-interference channels with a relay link, a relay link from clean receiver to interfered receiver is much more efficient:
  - Type I relay asymptotically achieves the cut-set bound.
  - Type II relay increases the sum capacity by at most 1/2 bit.
- Relay link always benefits the clean receiver more!