Coordinated Beamforming for the Multi-Cell Multi-Antenna Wireless System

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***Abstract*—In a conventional wireless cellular system, signal processing is performed on a per-cell basis; out-of-cell interference is treated as background noise. This paper considers the benefit of coordinating base-stations across multiple cells in a multi-antenna beamforming system, where multiple base-stations may jointly optimize their respective beamformers to improve the overall system performance. This paper focuses on a downlink scenario where each remote user is equipped with a single antenna, but where multiple remote users may be active simultaneously in each cell. The design criterion is the minimization of the total weighted transmitted power across the base-stations subject to signal-to-interference-and-noise-ratio (SINR) constraints at the remote users. The main contribution is a practical algorithm that is capable of finding the joint optimal beamformers for all base-stations globally and efficiently. The proposed algorithm is based on a generalization of uplink-downlink duality to the multi-cell setting using the Lagrangian duality theory. The algorithm also naturally leads to a distributed implementation. Simulation results show that a coordinated beamforming system can significantly outperform a conventional system with per-cell signal processing.**

I. Introduction

Conventional wireless systems are designed with a cellular architecture in which base-stations from different cells communicate with their respective remote terminals independently. Signal processing is performed on a per-cell basis; intercell interference is treated as background noise. Conventional cellular systems are typically designed to be intercell-interference limited. Consequently, the performance of conventional systems can be significantly improved if joint signal processing is enabled across the different base-stations to minimize or even to cancel inter-cell interference.

This paper evaluates the benefit of a particular type of base-station coordination for the multi-cell downlink system. The focus here is a scenario in which the base-stations are equipped with multiple antennas and the remote receivers are equipped with a single antenna each. Within each cell, multiple remote users may be active simultaneously and are separated via spatial multiplexing using beamforming. In a conventional system, the beamforming vectors in each cell are set indepedently. The main point of this paper is that significant performance gain is possible if the beamforming vectors for different base-stations are optimized jointly.

Downlink beamforming for multi-antenna wireless systems has been studied extensively in the past. A concept known as uplink-downlink duality has emerged as a main tool for the beamforming problem. In particular, Rashid-Farrokhi, Liu and Tassiulas [1] proposed an iterative algorithm to design the transmit beamforming vectors and power allocations to satisfy a target SINR for an arbitrary set of transmission links. Their main contribution is a beamformer-power update algorithm based on uplink-downlink duality that converges to a feasible solution to the problem. In the single-cell multi-user downlink case, the optimality of their algorithm was later proved by Visotsky and Madhow [2] and Schubert and Boche [3], [4]. Recently, Wiesel, Eldar and Shamai [5] showed that the single-cell downlink beamforming problem can be formulated as a second-order cone-programming problem. This crucial insight allows a new interpretation of duality via Lagrangian theory in convex optimization [6].

This paper further generalizes the above series of work by rigorously establishing an uplink-downlink duality for the multicell multi-user case. It is shown that the multi-cell downlink problem for minimizing the total weighted transmit power subject to received signal-to-noise-and-interference-ratio (SINR) constraints can be solved via a dual uplink problem. A main contribution of this paper is a novel algorithm, which is capable of efficiently finding the globally optimal downlink beamforming vector across all base-stations. This algorithm is a multi-cell generalization of a similar algorithm proposed in [5] for the single-cell case. A key advantage of the proposed algorithm as compared to previous solutions based on beamformer-power update [1] is that the new algorithm leads naturally to a distributed implementation.

The multi-cell uplink-downlink duality considered in this paper is related to the concept of network duality proposed by Song, Cruz and Rao [7], where a general setting with multiple antennas at both the transmitter and the receivers is considered. However, the approach in [7] does not allow multiple data streams per transmitter. Consequently, the network duality of [7] reduces to a simpler linear programming duality. The problem setting of this work is also related to the fully coordinated multi-cell system considered in [8], [9], [10], [11] in which multiple base-stations are considered as a single antenna array for transmitting multiple data streams to all users. The approach proposed in this paper is a first step toward this vision. As the simulation results of this paper show, significant performance gain can already be obtained with a beamformer-level coordination, which is much more practical to implement than full signal-level coordination.

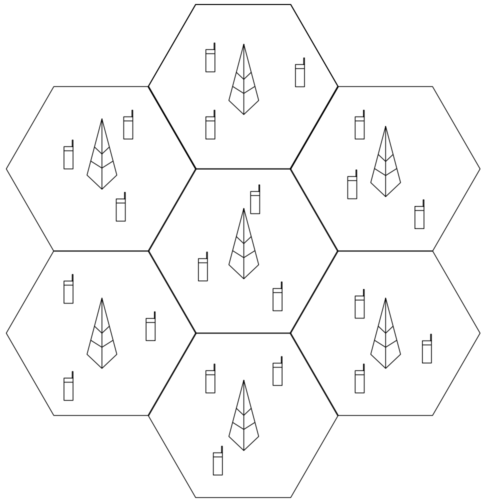

Fig. 1. A wireless network with seven base-stations and three users per cell.

II. System Model and Problem Formulation

A. Channel Model

This paper considers a multi-cell multiuser spatial multiplex system with N cells and K users per cell with N_t antennas at each base-station and a single antenna at each remote user. Multiuser transmit beamforming is employed at each base-station. Let $x_{i,j}$ be a complex scalar denoting the information signal for the jth user in the ith cell, and $w_{i,j} \in \mathcal{C}^{N_t \times 1}$ be its associated beamforming vector. The channel model can be written down as follows:

$$y_{i,j} = \sum_{l} h_{i,i,j}^H w_{i,l} x_{i,l} + \sum_{m \neq i,n} h_{m,i,j}^H w_{m,n} x_{m,n} + z_{i,j} \quad (1)$$

where $y_{i,j} \in \mathcal{C}$ is the received signal at the jth remote user in the ith cell, $h_{l,i,j} \in \mathcal{C}^{N_t \times 1}$ is the vector channel from the base-station of the lth cell to the jth user in the ith cell, and $z_{i,j}$ is the additive white Gaussian complex noise with variance $\sigma^2/2$ on each of its real and imaginary components. Fig. 1 illustrates the system model for a network with seven cells and three users per cell.

B. Problem Formulation

The beamformer design problem in this paper consists of minimizing total transmit power across all base-stations subject to SINR constraints at the remote users. In practice, as each base-station has its own power constraint, it is useful to consider a more general problem of minimizing a weighted total transmit power, with the power at the ith base-station weighted by a factor α_i.

With $w_{i,j}$ as the beamforming vectors, the SINR for the jth user in the ith cell can be expressed as:

$$\Gamma_{i,j} = \frac{|w_{i,j}^H h_{i,i,j}|^2}{\sum_{l \neq j} |w_{i,l}^H h_{i,i,j}|^2 + \sum_{m \neq i,n} |w_{m,n}^H h_{m,i,j}|^2 + \sigma^2} \quad (2)$$

Let $\gamma_{i,j}$ be the SINR target for the jth user in the ith cell. The transmit power minimization problem can then be formulated as

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} \alpha_i w_{i,j}^H w_{i,j} \\ \text{subject to} \quad & \Gamma_{i,j} \geq \gamma_{i,j}, \quad \forall i = 1 \cdots N,\ j = 1 \cdots K \end{aligned} \quad (3)$$

where the minimization is over the $w_{i,j}$'s.

The SINR target constraints in (3) may appear to be nonconvex at a first glance. However, in a study of single-cell downlink beamforming problem, [5] showed that nonconvex constraints of this type can be transformed into a second-order-cone constraint. This crucial observation enables methods for solving (3) via convex optimization.

C. Conventional Systems

In a conventional wireless cellular system, the multiuser beamforming problem is solved on a per-cell basis; out-of-cell interference is regarded as a part of background noise. In particular, for a fixed base-station $\hat{i}$, a conventional system finds the optimal set of $w_{\hat{i},j}$, $j = 1 \cdots K$, assuming that all other $(N-1)K$ beamformers are fixed:

$$\begin{aligned} \text{minimize} \quad & \sum_{j} w_{\hat{i},j}^H w_{\hat{i},j} \\ \text{subject to} \quad & \Gamma_{\hat{i},j} \geq \gamma_{\hat{i},j}, \quad \forall j = 1 \cdots K \end{aligned} \quad (4)$$

where $\Gamma_{\hat{i},j}$ is given by (2). This single-cell downlink problem has a classic solution as given in [1], [2], [3], [5].

Note that in a conventional system, the choice of beamformers at each base-station affects the background noise level at neighboring cells, and hence the setting of beamformers in neighboring base-stations. Thus, the above per-cell optimization is in practice performed iteratively until the system converges to a per-cell optimal solution.

D. Motivating Example for Joint Optimization

One of the main points of this paper is that the per-cell optimization above does not necessarily lead to a joint optimal solution. Significant performance improvement may be obtained if base-stations coordinate in jointly optimizing all of their beamformers at the same time. The following motivating example illustrates this point.

Consider a multi-cell network but with only a single user per cell. The per-cell optimization reduces to the optimal transmit beamforming problem for a multi-input single-output (MISO) system with a background noise level which includes out-of-cell interference. Note that regardless of the level of the background noise, the optimal per-cell transmit beamformer is a vector that matches the channel. Thus, in this example, per-cell optimization across the cells converges in one iteration – every base-station uses a transmit beamformer that matches the MISO channel.

This channel-matching solution is not necessarily the joint optimum. For example, when two users belonging to two different cells are near each other at the cell edge, it may be advantageous to steer the beamforming vectors for the two base-stations away from each other so as to minimize the mutual interference. Such a joint optimal beamforming solution may lead to higher received SINRs at a fixed transmit power, or conversely a lower transmit power at fixed SINRs.

One of the first algorithms for solving the multi-cell joint beamforming optimization problem is given by Rashid-Farrokhi, Liu and Tassiulas [1]. They showed that the optimal downlink beamforming problem under SINR constraints can be solved efficiently by an iterative uplink beamformer and power update algorithm. It is well known that the uplink beamforming problem is much easier to solve [12]. Thus, by transforming the downlink problem into the uplink domain, the downlink problem may be solved efficiently as well.

The global optimality of the beamformer-power iteration algorithm has been shown for the single-cell case in [2], [3], [5]. This paper will first give a rigorous derivation of duality for the multi-cell case, then propose a new algorithm for solving the joint multi-cell downlink beamforming problem.

III. Uplink-Downlink Duality for Multi-Cell Systems

Uplink-downlink duality refers to the fact that the minimum transmit power needed to achieve a certain set of SINR constraints in a downlink channel is the same as the minimum total transmit power needed to achieve the same set of SINR targets in an uplink channel, where the uplink channel is obtained by reversing the input and the output of the downlink. This paper establishes uplink-downlink duality for a multi-cell network. The development here uses a Lagrangian technique, similar to the approach used in [6].

Theorem 1: The optimal transmit beamforming problem (3) for the downlink multiuser multi-cellular network can be solved via a dual uplink channel in which the SINR constraints remain the same and the noise power is scaled by α_i. More precisely, a Lagrangian dual of the optimization problem (3) is the following minimization problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} \lambda_{i,j}\sigma^2 \\ \text{subject to} \quad & \Lambda_{i,j} \geq \gamma_{i,j} \end{aligned} \tag{5}$$

where the minimization is over $\lambda_{i,j}$, and

$$\Lambda_{i,j} \;=\; \max_{\hat{w}_{i,j}} \frac{\lambda_{i,j}|\hat{w}_{i,j}^H h_{i,i,j}|^2}{\sum_{(m,l)\neq(i,j)} \lambda_{m,l}|\hat{w}_{i,j}^H h_{i,m,l}|^2 + \alpha_i||\hat{w}_{i,j}||^2}$$

Further, the optimal $\hat{w}_{i,j}$ has the interpretation of being the receiver beamformer of the dual uplink channel, and is a scaled version of the optimal $w_{i,j}$. The optimal $\lambda_{i,j}$ has the interpretation of being the dual uplink power, and it corresponds to the dual variable associated with the SINR constraint of (3).

Proof: The proof hinges upon the fact that the SINR constraints can be reformulated as a second-order cone-programming problem as shown in [5]. Therefore, strong duality holds for (3). This allows us to characterize the solution of (3) via its Lagrangian:

$$\begin{aligned} L(w_{i,j}, \lambda_{i,j}) = \sum_{i,j} \alpha_i w_{i,j}^H w_{i,j} - \sum_{i,j} \lambda_{i,j} \Bigg[\frac{|w_{i,j}^H h_{i,i,j}|^2}{\gamma_{i,j}} - \\ \sum_{l\neq j} |w_{i,l}^H h_{i,i,j}|^2 - \sum_{m\neq i,n} |w_{m,n}^H h_{m,i,j}|^2 - \sigma^2 \Bigg] \end{aligned} \tag{6}$$

Rearranging (6), we get:

$$\begin{aligned} L(w_{i,j}, \lambda_{i,j}) = \sum_{i,j} \lambda_{i,j}\sigma^2 + \sum_{i,j} w_{i,j}^H \Bigg[\alpha_i I - \\ \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} h_{i,i,j} h_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \Bigg] w_{i,j} \end{aligned} \tag{7}$$

The dual objective is

$$g(\lambda_{i,j}) = \min_{w_{i,j}} L(w_{i,j}, \lambda_{i,j}) \tag{8}$$

It is easy to see that if $\alpha_i I - \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} h_{i,i,j} h_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H$ is not a positive definite matrix, then there exists a set of $w_{i,j}$ that would make $g(\lambda_{i,j}) = -\infty$. Thus, the Lagrangian dual of (3), which is the maximum of $g(\lambda_{i,j})$, is

$$\begin{aligned} \text{maximize} \quad & \sum_{i,j} \lambda_{i,j}\sigma^2 \\ \text{subject to} \quad & \Sigma_i \succeq \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} h_{i,i,j} h_{i,i,j}^H \end{aligned} \tag{9}$$

where

$$\Sigma_i \triangleq \alpha_i I + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \tag{10}$$

Next, we show that the above dual is equivalent to (5). The problem (5) corresponds to an uplink channel with receive beamformers $\hat{w}_{i,j}$, where the noise power of the dual channel is scaled by α_i. The optimal receive beamformers $\hat{w}_{i,j}$ that maximize the SINR are the minimum-mean-squared-error (MMSE) receivers, which can be expressed as:

$$\hat{w}_{i,j} = \left(\sum_{m,l} \lambda_{m,l}\sigma^2 h_{i,m,l} h_{i,m,l}^H + \alpha_i \sigma^2 I \right)^{-1} h_{i,i,j} \tag{11}$$

Plugging back $\hat{w}_{i,j}$ into the SINR constraint of (5), one can show that the SINR constraint is equivalent to

$$\alpha_i I + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \preceq \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} h_{i,i,j} h_{i,i,j}^H$$

Thus, one can rewrite (5) as follows:

$$\begin{aligned} \text{minimize} \quad & \sum_{i,j} \lambda_{i,j}\sigma^2 \\ \text{subject to} \quad & \Sigma_i \preceq \left(1 + \frac{1}{\gamma_{i,j}}\right) \lambda_{i,j} h_{i,i,j} h_{i,i,j}^H \end{aligned} \tag{12}$$

Note that the problems in (9) and (12) are identical except that the maximization is replaced by minimization and the inequality constraints are reversed. It can be shown that the optimal solutions for both problems are such that the constraints are satisfied with equality. Thus, (9) and (12) give the same solutions.

In addition, it can be shown that $w_{i,j}$ and $\hat{w}_{i,j}$ are scaled versions of each other. Thus, one would also be able to find $w_{i,j}$ by first finding $\hat{w}_{i,j}$, then updating it through scalar multiples named $\delta_{i,j}$ below:

$$w_{i,j} = \sqrt{\delta_{i,j}}\hat{w}_{i,j} \tag{13}$$

It can be shown that these $\delta_{i,j}$ can be found through a matrix inversion. Details derivation of this scaling factor can be found in [6]. ■

IV. Optimal Downlink Beamforming Algorithm

The derivation of uplink-downlink duality via Lagrangian theory forms the basis for numerical algorithms for computing the optimal downlink beamformers for the multi-cell system. Our main algorithm is based on an idea of iterative function evaluation, first proposed for the single-cell case in [5]. This paper generalizes the algorithm to a multi-cell system.

A. Iterative Function Evaluation Algorithm

The main idea is to solve the downlink beamforming problem in the dual uplink domain by first finding the optimal $\lambda_{i,j}$, then the corresponding $\hat{w}_{i,j}$. To find the optimal $\lambda_{i,j}$, we first take the gradient of the Lagrangian (7) with respect to $w_{i,j}$ and set it to zero:

$$\left[\alpha_i I - (1+\frac{1}{\gamma_{i,j}})\lambda_{i,j} h_{i,i,j} h_{i,i,j}^H + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H\right] w_{i,j} = 0. \tag{14}$$

Thus

$$\Sigma_i w_{i,j} = \left(1+\frac{1}{\gamma_{i,j}}\right)\lambda_{i,j} h_{i,i,j} h_{i,i,j}^H w_{i,j} \tag{15}$$

where Σ_i is as defined in (10).

Now, multiply both sides by $h_{i,i,j}^H \Sigma_i^{-1}$, we get:

$$h_{i,i,j}^H w_{i,j} = \left(1+\frac{1}{\gamma_{i,j}}\right)\lambda_{i,j} h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j} h_{i,i,j}^H w_{i,j} \tag{16}$$

Finally, divide both sides of the equation by $h_{i,i,j}^H w_{i,j}$, we obtain a necessary condition for optimal $\lambda_{i,j}$:

$$\lambda_{i,j} = \frac{1}{\left(1+\frac{1}{\gamma_{i,j}}\right) h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j}} \tag{17}$$

which can be used iteratively to obtain the optimal $\lambda_{i,j}$.

The algorithm is summarized as follows:

1) Find the optimal uplink power allocation $\lambda_{i,j}$ using the iterative function evaluation:

$$\lambda_{i,j} = \frac{1}{\left(1+\frac{1}{\gamma_{i,j}}\right) h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j}} \tag{18}$$

where

$$\Sigma_i = \alpha_i I + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \tag{19}$$

2) Find the optimal uplink receive beamformers based on the optimal uplink power allocation $\lambda_{i,j}$:

$$\hat{w}_{i,j} = \left(\sum_{m,l} \lambda_{m,l}\sigma^2 h_{i,m,l} h_{i,m,l}^H + \sigma^2 \alpha_i I\right)^{-1} h_{i,i,j} \tag{20}$$

3) Find the optimal transmit downlink beamformers by scaling $\hat{w}_{i,j}$:

$$w_{i,j} = \sqrt{\delta_{i,j}}\hat{w}_{i,j} \tag{21}$$

The global convergence of this algorithm is guaranteed by both the duality result discussed in the previous section and the convergence of the iterative function evaluation which can be justified by a line of reasoning similar to that in [5]. The proof is based on the property of standard functions [13]. In particular, one can stack the dual variables $\lambda_{i,j}$ into one vector Υ. Then (18) and be rewritten as

$$\lambda_{i,j}^{(t+1)} = f_{i,j}(\Upsilon^{(t)}), \quad i = 1 \cdots N,\ j = 1 \cdots K \tag{22}$$

The function f satisfies the following properties:

1) If $\lambda_{i,j} \geq 0\ \forall i,j$, then $f_{i,j}(\Upsilon) > 0$.
2) If $\lambda_{i,j} \geq \lambda_{i,j}^{'}\ \forall i,j$, then $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon^{'})$
3) For $\rho > 1$, we have $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho\Upsilon)\ \forall i,j$.

The proof for these three properties is included in the Appendix. These properties guarantee that f is a standard function as defined in [13]. Thus, starting with some initial $\Upsilon^{(0)}$, the iterative function evaluation algorithm converges to a unique fixed point, which must be the optimal downlink power.

B. Comparison with Beamformer-Power Iteration Algorithm

The iterative function evaluation algorithm is based on finding the optimal $\lambda_{i,j}$ independent of the beamformers. In [1], [12], Rashid-Farrokhi, Tassiulas and Liu proposed the following beamformers-power iteration algorithm for the uplink, which, by our previous duality result, must also converge to the global optimal solution for the downlink:

1) Initialize $\hat{w}_{i,j}$;
2) Find the $\lambda_{i,j}$ to satisfy the SINR constraints of (5) with equality;
3) Find the optimal uplink receive beamformers based on the optimal uplink power allocation $\lambda_{i,j}$:

$$\hat{w}_{i,j} = \left(\sum_{m,l} \lambda_{m,l}\sigma^2 h_{i,m,l} h_{i,m,l}^H + \sigma^2 \alpha_i I\right)^{-1} h_{i,i,j} \tag{23}$$

4) Go to step 2 until convergence;
5) Update the transmit downlink beamformers

$$w_{i,j} = \sqrt{\delta_{i,j}}\hat{w}_{i,j} \tag{24}$$

Note that the convergence of the iterations involving steps 2 and 3 was shown in [12].

Both the iterative function evaluation algorithm and the beamformer-power iteration algorithm provide the optimal solution for the multi-cell downlink beamforming problem. However, the iterative function evaluation algorithm has a key advantage – it can be implemented in a distributed fashion.

Consider the dual uplink channel. The function iteration (18) for uplink power $\lambda_{i,j}$ involves channel vectors $h_{i,i,j}$ within each cell, which the base-station typically has the knowledge of, and the matrix Σ_i. Observe that for the uplink channel, Σ_i is precisely the covariance matrix of the received signal at the base-station i, which includes the intended signal, the interference, and the background noise. This covariance matrix may be estimated locally at each base-station. Thus, the iterative function evaluation for $\lambda_{i,j}$ can be performed locally, assuming that all other $\lambda_{i,j}$'s are fixed. Base-station coordination is achieved via power control (i.e. the update of $\lambda_{i,j}$'s, which affect all other Σ_i's.)

In fact, these uplink per-cell updates can even be implemented asynchronously with each base-station using possibly outdated power information. The convergence of such asynchronous update is still guaranteed by the standard function argument as shown in Theorem 4 of [13].

The interpretation that uplink per-cell updates are exactly the global optimum is particularly useful for time-division-duplex (TDD) systems, where uplink and downlink transmissions are reciprocals of each other. In such a system, beamformer and power updates (18) can in fact be done directly in the uplink on a per-cell basis. These uplink per-cell iterations always converge. They converge to the global optimum of the uplink system, which by duality, is also the global optimum for downlink.

Interestingly, uplink-downlink duality holds for the coordinated multi-cell beamforming problem, but it does not hold for the per-cell algorithms. The uplink per-cell algorithm provides the multi-cell optimum; the downlink per-cell algorithm does not.

V. SIMULATIONS

This section presents the simulation results for the beamforming design problem for a 7-cell network with 3 users per cell as shown in Fig. 1. Each base-station is equipped with 4 antennas. Standard WiMax parameters are used in simulation: the noise power spectral density is set to -162 dBm/Hz; the channel vectors are chosen according to the distance-dependent path loss $L = 128.1 + 37.6 \log_{10}(d)$, where d is the distance in kilometers, with 8dB log-normal shadowing, and a Rayleigh component. The distance between neighboring base-stations is set to be 2.8km and the locations of remote users are chosen at random within each cell. An antenna gain of 15dBi is assumed. For illustration purposes, the weighting factors α_i corresponding to base-station power constraints are set to be: $\alpha_1 = \alpha_2 = \cdots = \alpha_7 = 1$.

Fig. 2 shows a plot of the minimum total transmit power (in dBm) over all base-stations versus the SINR target at the remote users. It is observed that while the joint optimization algorithm has the same performance as the conventional per-cell update in low SINRs, it offers significantly better performance at high SINRs. This is due to the fact that at high SINRs, the multi-cell network becomes predominantly interference limited. This is the regime in which the joint optimization approach shows a clear advantage.

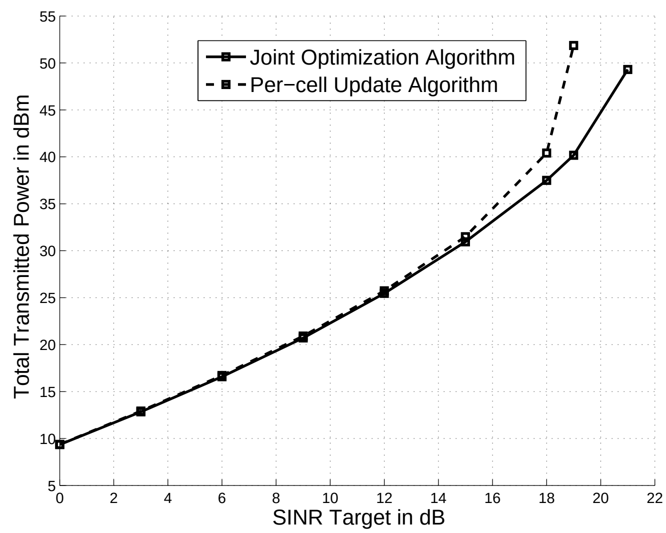

Fig. 2. Plot of the total transmitted power versus the SINR targets for both the joint optimization of beamformers and the per-cell udate algorithm for a wireless network with seven cells and three users per cell.

To illustrate the convergence behavior of the algorithms, Fig. 3 plots the norm residue of the uplink transmitted power (in mW) versus the number of iterations. The norm residue is defined as:

$$R^{(n)} = \sigma^2 ||\Upsilon^{(n)} - \Upsilon^*||_2 \tag{25}$$

where Υ^* represents the optimal power vector.

It is observed that while the beamformer-power update algorithm converges more rapidly than the iterative function evaluation algorithm at the beginning, the iterative function evaluation algorithm eventually provides faster convergence.

VI. CONCLUSION

This paper provides a solution for the optimal downlink beamforming design problem for a multi-cell network with multiple users per cell. Both the uplink and downlink problems are solved by generalizing uplink-downlink duality to the multi-cell case using the Lagrangian theory. An iterative function evaluation algorithm which is capable of finding the global optimum solution is presented. The algorithm is efficient, and it can be implemented in a distributed fashion. The distributed solution outperforms conventional wireless systems with per-cell signal processing.

APPENDIX

This appendix presents the proof of standard function properties satisfied by f. The proof is similar to the one presented in [5], and is included here for completeness.

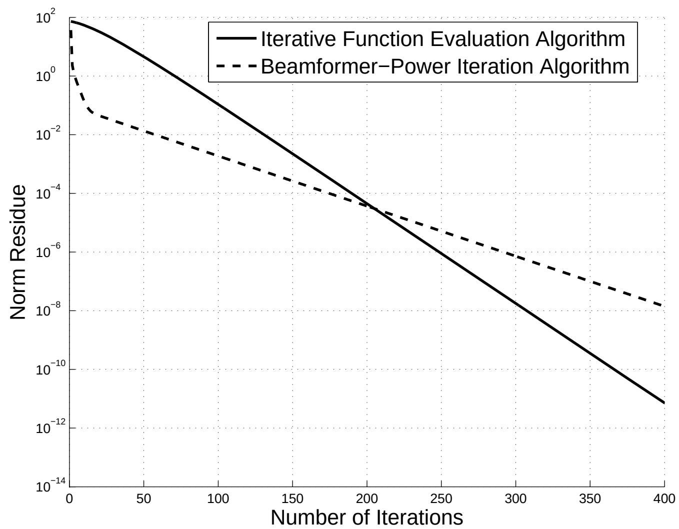

Fig. 3. Plot of the norm residue versus the number of iterations for the two algorithms.

1) If $\lambda_{i,j} \geq 0 \; \forall i,j$, then $f_{i,j}(\Upsilon) > 0$. This is true because if $\lambda_{i,j} \geq 0$ then $\Sigma_i \succ 0$ and consequently $\Sigma_i^{-1} \succ 0$. Thus $h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j} > 0$ and consequently $f_{i,j}(\Upsilon) > 0$.
2) If $\lambda_{i,j} \geq \lambda_{i,j}^{'} \; \forall i,j$, then $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon^{'})$

Proof: Assume $\lambda_{i,j} \geq \lambda_{i,j}^{'}$. Then,

$$f_{i,j}(\Upsilon) = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j}} \tag{26}$$

where

$$\begin{aligned} \Sigma_i &= \alpha_i I + \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \\ &= \alpha_i I + \sum_{m,n} \lambda_{m,n}^{'} h_{i,m,n} h_{i,m,n}^H \\ &\quad + \sum_{m,n} (\lambda_{m,n} - \lambda_{m,n}^{'}) h_{i,m,n} h_{i,m,n}^H \end{aligned} \tag{27}$$

Now, since $\lambda_{i,j} \geq \lambda_{i,j}^{'}$, we have $\sum_{m,n}(\lambda_{m,n} - \lambda_{m,n}^{'}) h_{i,m,n} h_{i,m,n}^H \succeq 0$. But as shown in [5], for positive semi-definite matrices C and D and vector x in the range of C:

$$\frac{1}{x^T(C+D)^{-1}x} \geq \frac{1}{x^T C^{-1} x} \tag{28}$$

with equality if and only if $D(C+D)^{-1}x = 0$. Thus

$$\frac{1}{h_{i,i,j}^H \Sigma_i^{-1} h_{i,i,j}} \geq \frac{1}{h_{i,i,j}^H \Sigma_i^{'-1} h_{i,i,j}} \tag{29}$$

where

$$\Sigma_i^{'} = \left(\sum_{m,n} \lambda_{m,n}^{'} h_{i,m,n} h_{i,m,n}^H + \alpha_i I \right) \tag{30}$$

Hence, $f_{i,j}(\Upsilon) \geq f_{i,j}(\Upsilon^{'})$. ■

3) For $\rho > 1$, $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho \Upsilon) \; \forall i,j$.

Proof: Let $\rho > 1$,

$$\rho f_{i,j}(\Upsilon) = \frac{1}{\left(1 + \frac{1}{\gamma_{i,j}}\right) h_{i,i,j}^H (\rho \Sigma_i)^{-1} h_{i,i,j}} \tag{31}$$

where

$$\begin{aligned} \rho \Sigma_i &= \rho \alpha_i I + \rho \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \\ &= (\rho - 1)\alpha_i I + \alpha_i I + \rho \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H \end{aligned}$$

Since $\rho > 1$, we have $(\rho - 1)\alpha_i I \succeq 0$. Based on (28), we get

$$\frac{1}{h_{i,i,j}^H \left(\rho \Sigma_i\right)^{-1} h_{i,i,j}} \geq \frac{1}{h_{i,i,j}^H \left(\alpha_i I + \rho \sum_{m,n} \lambda_{m,n} h_{i,m,n} h_{i,m,n}^H\right)^{-1} h_{i,i,j}} \tag{32}$$

Thus, $\rho f_{i,j}(\Upsilon) \geq f_{i,j}(\rho \Upsilon)$. Finally, it is easy to check that the equality condition is not satisfied. Thus, $\rho f_{i,j}(\Upsilon) > f_{i,j}(\rho \Upsilon)$ strictly. ■

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