Uplink Multicell Processing with Limited Backhaul via Per-Base-Station Successive Interference Cancellation

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***Abstract*—This paper studies an uplink multicell joint processing model in which the base-stations are connected to a centralized processing server via rate-limited digital backhaul links. We propose a simple scheme that performs Wyner-Ziv compress-and-forward relaying on a per-base-station basis followed by successive interference cancellation (SIC) at the central processor. The proposed scheme has a significantly reduced complexity as compared to joint decoding, resulting in an easily computable achievable rate region. Although suboptimal in general, this paper shows that the proposed per-base-station SIC scheme can achieve the sum capacity of a special Wyner cellular model to within a constant gap. This paper also establishes that in order to achieve to within a constant gap to the maximum SIC rate with infinite backhaul, the limited-backhaul system must have backhaul capacities that scale logarithmically with the signal-to-interference-and-noise ratios (SINRs) at the base-stations. Further, this paper studies the optimal backhaul rate allocation problem for the per-base-station SIC model with a total backhaul capacity constraint, and shows that the sum-rate maximizing allocation should also have individual backhaul rates that scale logarithmically with the SINR at each base-station. Finally, the proposed per-base-station SIC scheme is evaluated in a practical multicell network to quantify the performance gain brought by multicell processing.**

***Index Terms*—Coordinated multi-point (CoMP), interference channel, limited backhaul network multiple-input multiple-output (MIMO), relay channel, successive interference cancellation, Wyner-Ziv coding**

I. Introduction

Traditional wireless cellular networks operate on a cell-by-cell basis where out-of-cell interference is treated as part of the noise. However, as cellular base-stations become more densely deployed and heterogeneous networks become commonplace where small cells overlap heavily with macro-cells, modern cellular networks are becoming increasingly interference limited. Because of the intercell interference, the per-cell achievable rates in traditional cellular deployments are typically much smaller than that of a single isolated cell.

Multicell joint processing is a promising technique that has the potential to significantly improve the cellular throughput by taking advantage of its capability for intercell interference mitigation. When base-stations share the transmitted and received signals, the codebooks, and the channel state information (CSI) with each other through high-capacity backhaul, it is theoretically possible to perform joint transmission in the downlink and joint reception in the uplink to eliminate out-of-cell interference completely. This paper deals with the information theoretical capacity analysis of the joint-processing architecture in the uplink. In this architecture, antennas from multiple base-stations essentially become a virtual multiple-input multiple-output (MIMO) array capable of spatial multiplexing multiple user terminals.

One way to implement a multicell joint processing system is to deploy a centralized processing server which is connected to all the base-stations via high-capacity backhaul links. When the capacities of the backhaul links are sufficiently large, the joint processing system implemented across the different cells in the network can be modeled as a multiple-access channel in the uplink and a broadcast channel in the downlink, giving rise to the concept of network MIMO [1], [2].

The practical implementation of a network MIMO system must also consider the effect of finite capacity in the backhaul. While the capacity of the network MIMO system with infinite backhaul is easy to compute, when finite backhaul is considered, the information theoretical analysis of multicell processing becomes significantly more complicated. This paper focuses on an uplink network MIMO model as shown in Fig. 1. From an information theoretical perspective, the overall system can be thought of as a combination of a multiple-access channel (with remote terminals acting as the transmitters and the centralized processor as the receiver) and a relay channel (with the base-stations acting as relays).

A. Related Work

The uplink network MIMO model considered in this paper, where the joint processing takes place in the central server, is related to but is different from the multicell joint processing model with finite-capacity uni- or bi-direction backhaul links deployed between the base-stations. In this latter setting, the uplink multicell model becomes an interference channel with partial receiver cooperation. Although a complete characterization of capacity for such a model is still an open problem,

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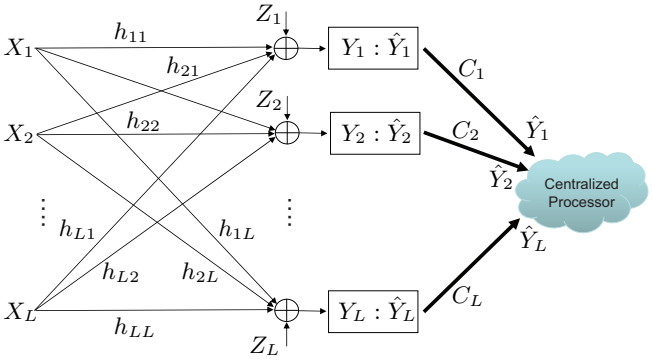


Fig. 1. Uplink multicell joint processing via a centralized processor

information theoretical studies have been carried out for the case of two-user Gaussian interference channel with receiver cooperation [3]–[7]. Likewise, the analysis of uplink cellular network under this setup has been carried out in [8], [9], where strategies such as decode-and-forward and compress-and-forward are proposed, and analytical results under the Wyner model are obtained. We mention briefly here that the downlink counterpart of this model has been studied in [10].

For the uplink multicell joint processing model considered in this paper, where the limited-capacity backhaul links are deployed between the base-stations and a central processor as depicted in Fig. 1, although a complete characterization of its information theoretical capacity region is also an open problem, much work has been done on finding its achievable rate region and the gap to the outer bound [11]–[16]. The early work by Sanderovich et al. [11] considers a joint decoding scheme in which the base-stations quantize the received signals and forward them to the centralized processor, which then performs *joint decoding* of both the source messages and quantized codewords. Instead of joint decoding, [14], [15] describe schemes which *successively* decode the quantization codewords first, then the source messages. A comparison of successive decoding vs. joint decoding is contained in [16]. Alternatively, base-stations can also *decode* part of the messages of users in their own cell, then forward the decoded data along with the remaining part to the centralized processor [11], [12], [17]. The counterpart of these results for the downlink with finite backhaul has been described in [18]. A review of some of these results is available in [2].

The uplink multicell joint processing model is actually a special case of a general multiple multicast relay network studied in [19], [20], for which a generalization of quantize-and-forword can be shown to achieve to within a constant gap to the information theoretical capacity of the overall network. This is proved in [19] using a quantize-map-and-forward technique, and in [20] using noisy network coding, both of which require the joint decoding of the source messages and the quantized codewords. Thus in retrospect, the joint processing scheme of [11] already achieves the approximate capacity of the uplink finite-backhaul multicell model, provided that the quantization noise level is appropriately chosen and joint decoding is done at the central processor.

It should be emphasized that joint decoding is challenging to implement. Not only is the network-wide CSI required at the central processor, the computational complexity of joint decoding typically scales exponentially with the total number of nodes in the network. Moreover, even a mere evaluation of the achievable rate can be computationally prohibitive. The achievable rate region of the joint decoding scheme as presented in [11, Proposition IV.1] involves $2^L - 1$ rate constraints, each requiring a minimization of 2^L terms, where L is the number of users in the uplink multicell model. To obtain analytical results, [11] and likewise [9], [21] assume certain symmetry and obtain rate expressions under a Wyner cellular model. But the rate expression for an arbitrary network remains difficult to evaluate. On top of this, the achievable rate region needs to be optimized over all choices of L quantization noise levels, which is a difficult task, although recent progress has been made in [14], [16], [22].

B. Main Results

To fully overcome the aforementioned difficulty yet still take full advantage of the centralized processing is expected to be challenging. This paper instead aims to design schemes with simple receiver structures that result in computationally feasible achievable rate regions for the uplink multicell joint processing problem. Toward this end, this paper focuses on the uplink multicell model with finite-capacity backhaul to the centralized processor, and proposes suboptimal schemes based on *successive interference cancellation* (SIC). Instead of performing joint decoding of the source messages and quantization codewords [11], or successive decoding of all the quantization codewords first, then the source messages [14], [15], this paper applies the Wyner-Ziv compress-and-forward relaying scheme on a per-base-station basis and performs single-user decoding with SIC at the centralized processor. The Wyner-Ziv coding here also uses the previously decoded messages as side information. The main advantage of this proposed scheme is that it leads to a receiver architecture in which only the decoded transmit signals are shared between the successive stages, so it is more amenable to implementation. Further, the quantization noise levels can now be easily optimized, resulting in an achievable rate region computable with complexity that scales linearly with L.

Although the proposed per-base-station SIC scheme is no longer the best possible for a general uplink joint processing model, this paper shows that for a special Wyner soft-handoff model [23], in which the mobile terminals and the base-stations are placed along a line, the proposed per-base-station SIC scheme achieves a sum rate that is at most a constant gap away from capacity. This paper shows that the constant gap is a linear function of the number of transmitter-receiver pairs in the uplink multicell joint processing model.

Under the proposed per-base-station SIC framework for the finite-backhaul uplink model, we also ask the following question: How much backhaul capacity is needed to approach the theoretical infinite-backhaul successive-decoding rate? The results of this paper show that the backhaul rates need to scale logarithmically with the received signal-to-interference-and-noise ratio (SINR) in order to achieve to within $\frac{1}{2}$ bit of the successive decoding rates attainable with unlimited backhaul capacity.

Further, this paper addresses the question of how to allocate the backhaul rates optimally across the different backhaul links. Under a sum rate constraint on the total backhaul capacity, this paper shows that in order to maximize the sum rate over the entire network, the allocation of individual backhaul link capacity should again scale logarithmically with the SINR under the proposed per-base-station SIC framework.

This paper focuses on capacity analysis with the assumption that the CSI for the entire network is available at the central processor. This is reasonable as the base-stations may estimate the channel in the uplink, then send CSI via the backhaul links to the central processor. For convenience, this paper neglects the additional backhaul capacity needed to support the sharing of CSI. We mention that the capacity of network MIMO under channel estimation error has been investigated in [8]. Study on the use of processing techniques that are robust against channel uncertainty has been reported in [15].

Finally, under the perfect CSI assumption, this paper evaluates the performance of the proposed per-base-station SIC scheme in an OFDMA network. Numerical simulations show that the proposed per-base-station SIC scheme can almost double the per-cell sum-rate over a baseline scheme where no centralized processor is deployed and each base-station simply treats interference as noise. Further, with optimized backhaul capacity allocation, most of such gains can already be obtained when the backhaul sum capacity is at about 1.5 times the achieved per-cell sum rate.

C. Organization of the Paper

The rest of the paper is organized as follows. Section II presents the multicell joint processing model. Section III introduces the per-base-station SIC scheme and compares it with other achievability schemes such as noisy network coding and joint-base-station processing. The optimality of the per-base-station SIC scheme for the Wyner model is provided in Section IV. The backhaul rate allocation problem with a total backhaul constraint is solved in Section V. Finally, Section VI numerically evaluates the per-base-station SIC scheme for a two-user symmetric setting and for a realistic OFDMA network.

II. Channel Model

Consider the uplink of a multicell network with a central processor. Assuming that there is only one user operating in each time-frequency resource block in each cell, the multicell network can be modelled by L users each sending a message to their corresponding base-stations. Base-stations serve as intermediate relays for the centralized server, which eventually decodes all the transmit messages.

As depicted in Fig. 1, the uplink joint processing model consists of two parts. The left half is an L-user interference channel with X_i as the input signal from the ith user, Y_i as the output signal, Z_i as the additive white Gaussian noise (AWGN), and h_{ij} as the channel gain from user i to base-station j, where $i, j = 1, 2, \cdots, L$. The right half can be seen as a noiseless multiple-access channel with capacities C_i, $i = 1, 2, \cdots, L$, modelling the backhaul links between the base-stations and the central processor. The input signal X_i is power constrained, i.e., $E[|X_i|^2] \leq P_i$; the receiver noises are assumed to be independent and identically distributed with $Z_i \sim \mathcal{N}(0, N_0)$, $i = 1, 2, \cdots, L$. The signal-to-noise ratios (SNRs) and the interference-to-noise ratios (INRs) are defined as follows:

$$\mathsf{SNR}_i = \frac{h_{ii}^2 P_i}{N_0}, \quad \mathsf{INR}_{i,j} = \frac{h_{ij}^2 P_i}{N_0}, \quad i, j = 1, \cdots, L \tag{1}$$

In addition, define the vectors $\mathbf{X} = \{X_1, X_2, \cdots, X_L\}$ and $\mathbf{Y} = \{Y_1, Y_2, \cdots, Y_L\}$.

III. Multicell Processing Schemes

This paper focuses on uplink multicell processing schemes in which the base-stations quantize the received signals, then subsequently transmit a function of the quantized signal to the central processor via noiseless backhaul links. Quantize-and-forward is a natural strategy in the uplink setting. This is because in order to reap the benefit of multicell processing, the user terminals must transmit at rates higher than that decodable at its own base-station alone. This prevents the use of the decode-and-forward strategy. In Fig. 1, the received signal is denoted as Y_i; its quantized version is denoted as $\hat{Y}_i$. This paper also assumes that the base-stations either transmit $\hat{Y}_i$ directly, or performs a binning operation on $\hat{Y}_i$. The task at the central processor is to decode $\{X_1, X_2, \cdots, X_L\}$ based on either $\{\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_L\}$ or their bin indices. In the following, we examine the information theoretical achievable rates for various quantization and decoding strategies.

A. Joint Decoding

Consider a coding scheme in which each base-station quantizes its received signal using a Gaussian codebook at certain distortion or quantization noise level q_i, bins the quantized message, then forwards the bin index to the central processor. In the joint decoding strategy, the central processor, upon receiving all the bin indices, decodes $\{X_1, X_2, \cdots, X_L\}$ and $\{\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_L\}$ jointly based on joint typicality. This strategy is first proposed by Sanderovich et al. [11, Proposition IV.1]. (Note that there is no requirement that $\{\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_L\}$ are decoded correctly; only $\{X_1, X_2, \cdots, X_L\}$ need to.)

The key parameter here is the setting of the quantization noise levels q_i. For example, q_i's can be set to be such that $\{\hat{Y}_1, \hat{Y}_2, \cdots, \hat{Y}_L\}$ can be correctly decoded based on the bin indices. The values of q_i's can then be determined based on the capacities of the backhaul links $(C_1, C_2, \cdots, C_L)$. But, such a setting of q_i may not be optimal. Indeed, as shown in [19] and also later in [20], for a much larger class of general relay networks, setting the quantization noise levels to be at the background noise level, i.e., $\mathsf{q}_i = N_0$, leads to an achievable rate that is within at most a constant gap from the capacity outer bound. This gap scales linearly with the number of nodes in the network, but is independent of the channel gains and the backhaul capacities. Applying this result to the uplink multicell model in Fig. 1, we immediately have the following joint decoding achievable rate. The achievable rate

expression can be derived based on the noisy-network-coding theorem [20], and is equivalent to the expression in [11].

Lemma 1. *For the multicell joint processing model as shown in Fig. 1, the following rate region is achievable*

$$R(\mathcal{S}) \leq \min_{\mathcal{T}\subseteq\mathcal{L}} \left\{ \frac{1}{2}\log\left|I+\Lambda_{\mathcal{T}^c}^{-1}(N_0+\mathsf{q})\mathbf{H}_{\mathcal{S}\mathcal{T}^c}\Lambda_{\mathcal{S}}(P)\mathbf{H}_{\mathcal{S}\mathcal{T}^c}^T\right| + \sum_{i\in\mathcal{T}}\left(C_i-\frac{1}{2}\log(1+\frac{N_0}{\mathsf{q}_i})\right)\right\}, \quad \forall\, \mathcal{S}\subseteq\mathcal{L}\triangleq\{0,1,\cdots,L\} \tag{2}$$

where $R(\mathcal{S})=\sum_{i\in\mathcal{S}}R_i$, q_i *is a positive real number representing the quantization noise level at base-station* i, $\mathbf{H}_{\mathcal{S}\mathcal{T}^c}$ *denotes for the transfer matrix from input* $\mathbf{X}(\mathcal{S})$ *to output* $\mathbf{Y}(\mathcal{T}^c)$, $\Lambda_{\mathcal{T}^c}^{-1}(N_0+\mathsf{q})$ *is a diagonal matrix of* $\frac{1}{N_0+\mathsf{q}_i}$ *with* $i\in\mathcal{T}^c$, *and* $\Lambda_{\mathcal{S}}(P)$ *is a diagonal matrix of* P_i *with* $i\in\mathcal{S}$. *This rate region is within a constant gap to the capacity region of the multicell joint processing model if we set* $\mathsf{q}_i=N_0, i=1,2,\cdots,L$.

The above achievable rate region is valid for any choice of q_i, and is approximately optimal when $\mathsf{q}_i=N_0$. But, the evaluation of such an achievable rate region can be difficult. For the uplink joint processing model, the achievable rate region as derived in Lemma 1 requires a minimization of 2^L terms for each rate constraint, and there are 2^L-1 different rate constraints describing the rate region. Even when the size of the network is in a reasonable range, for example as in a 19-cell topology, it is computationally prohibitive to minimize over 2^{19} terms each involving $2^{19}-1$ different rate constraints. Further, the implementation of the joint decoding scheme itself tends to have exponential complexity, so the achievable rate given in Lemma 1 is not practically implementable for a reasonably-sized network.

In order to derive more tractable performance analysis of the multicell joint processing scheme, [11] resorts to a modified Wyner model (see [23]), where each transmitter-receiver pair interferes only with one other neighboring transmitter-receiver pair, and is subject to interference from only one neighboring transmitter-receiver pair. Further, certain symmetry is introduced so that all the direct channels are identical, and so are all interfering channels. With this symmetric cyclic structure, computation of the sum rate becomes tractable under joint decoding [11].

B. Per-Base-Station SIC

This paper focuses on the general multicell model (instead of the symmetric Wyner model) and proposes suboptimal schemes based on the successive decoding of source messages. Based on the observation that the exponential complexity of the rate expression in Lemma 1 is due to the joint decoding step at the destination, this paper proposes a per-base-station SIC approach (in contrast to joint-base-station SIC to be described in detail later) at the centralized processor that significantly reduces the complexity of computation of the achievable rate region. In addition, the proposed scheme involves the use of compress-and-forward relaying technique [24] at each base-station independently, which also significantly reduces the complexity of its eventual implementation in a practical system.

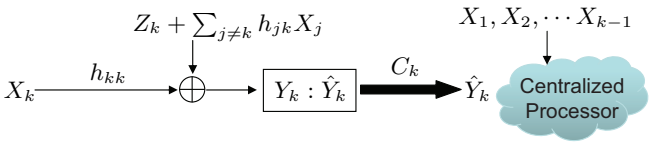


Fig. 2. Equivalent channel of user k in the k^{th} decoding stage

Specifically, we use an SIC approach assuming a *fixed* order of decoding first X_1, then $X_2, X_3, \cdots, X_L$. A central feature of the proposed scheme is that only the decoded signals are used as side information in subsequent stages. This simplifies the sharing of information among the base-stations and facilitates practical implementation.

At the kth stage, the base-station k, upon receiving Y_k, quantizes Y_k into $\hat{Y}_k$ using the compress-and-forward technique and sends bin index of $\hat{Y}_k$ to the destination via the noiseless link of capacity C_k. Note that the quantization process at the base-station k includes interference from all other users. The quantization noise level at each base-station k is chosen so as to fully utilize C_k such that $\hat{Y}_k$ can be immediately recovered at the central processor. This quantization process can be done either with Wyner-Ziv coding that takes advantage of the already-decoded user messages $(X_1,\cdots,X_{k-1})$ as side information, or without Wyner-Ziv coding for ease of implementation.

At the central processor, to decode user k's message X_k the central processor first decodes the quantization message $\hat{Y}_k$ upon receiving its description from the digital link C_k. It then decodes the message of user k using joint typicality decoding between X_k and the quantized message $\hat{Y}_k$. The decoding of X_k also takes advantage of the knowledge of previously decoded messages $(X_1,\cdots,X_{k-1})$ at the centralized processor. In this way, the impact of interference from $X_1,\cdots,X_{k-1}$ eventually disappears and the effective interference is only due to users not yet decoded, i.e., X_j, for $j>k$. After decoding X_k, the central processor moves to the next decoding stage, adding X_k to the set of known side information.

1) Per-Base-Station SIC with Wyner-Ziv: The following theorem gives the achievable rate of the proposed per-base-station SIC scheme with Wyner-Ziv compress-and-forward relaying.

Theorem 1. *For the uplink multicell joint processing channel depicted in Fig. 1, the following rate is achievable using Wyner-Ziv compress-and-forward relaying at the base-stations followed by SIC at the centralized processor with a fixed decoding order:*

$$R_k=\frac{1}{2}\log\frac{1+\overline{\mathsf{SINR}}_k}{1+2^{-2C_k}\overline{\mathsf{SINR}}_k}, \tag{3}$$

where

$$\overline{\mathsf{SINR}}_k=\frac{\mathsf{SNR}_k}{1+\sum_{j>k}\mathsf{INR}_{j,k}}. \tag{4}$$

Proof: In the kth stage of the SIC decoder, $X_1, \cdots, X_{k-1}$ decoded in the previous decoding stages serve as side information for stage k. The equivalent channel of user k is depicted in Fig. 2. This is a three-node relay channel without the direct source-destination link. Specifically, the source signal X_k is sent from the transmitter to the relay, which receives Y_k, quantizes it into $\hat{Y}_k$ and forwards its description to the centralized processor via the noiseless digital link of capacity C_k. At the centralized processor, $X_1, \cdots, X_{k-1}$ serve as side information and facilitate the decoding of $\hat{Y}_k$ and X_k. According to [24, Theorem 6], the achievable rate of user k using Wyner-Ziv compress-and-forward can be written as

$$R_k = I(X_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) \tag{5}$$

subject to the constraint

$$I(Y_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) \le C_k. \tag{6}$$

We constrain ourselves to Gaussian input signals $X_k \sim \mathcal{N}(0, P_k)$, and the Gaussian quantization scheme[1], i.e.,

$$\hat{Y}_k = Y_k + e_k, \tag{7}$$

where $e_k \sim \mathcal{N}(0, \mathsf{q}_k)$ is the Gaussian quantization noise independent of everything else. To fully utilize the digital link, it is natural to set

$$I(Y_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) = C_k. \tag{8}$$

Now, substituting $Y_k = \sum_{j=1}^{L} h_{jk} X_j + Z_k$ and $\hat{Y}_k = Y_k + e_k$ into (8), we have

$$\begin{aligned} C_k &= I(Y_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) \\ &= h(\hat{Y}_k | X_1, \cdots, X_{k-1}) - h(e_k) \\ &= \frac{1}{2} \log \left(1 + \frac{N_0 + \sum_{j \ge k} h_{jk}^2 P_j}{\mathsf{q}_k} \right), \end{aligned} \tag{9}$$

which results in the following quantization noise level that fully utilizes the digital links C_k:

$$\mathsf{q}_k^* = \frac{N_0 + \sum_{j \ge k} h_{jk}^2 P_j}{2^{2C_k} - 1}. \tag{10}$$

With the above q_k^*, the achievable rate of user k can be calculated as

$$\begin{aligned} R_k &= I(X_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) \\ &= h(\hat{Y}_k | X_1, \cdots, X_{k-1}) - h(\hat{Y}_k | X_1, \cdots, X_k) \\ &= \frac{1}{2} \log \frac{\mathsf{q}_k^* + N_0 + \sum_{j \ge k} h_{jk}^2 P_j}{\mathsf{q}_k^* + N_0 + \sum_{j > k} h_{jk}^2 P_j} \\ &= \frac{1}{2} \log \frac{N_0 + \sum_{j > k} h_{jk}^2 P_j + h_{kk}^2 P_k}{N_0 + \sum_{j > k} h_{jk}^2 P_j + 2^{-2C_k} h_{kk}^2 P_k} \\ &= \frac{1}{2} \log \frac{1 + \overline{\mathsf{SINR}}_k}{1 + 2^{-2C_k} \overline{\mathsf{SINR}}_k}. \end{aligned} \tag{11}$$

Finally, note that $\overline{\mathsf{SINR}}_k = \frac{h_{kk}^2 P_k}{N_0 + \sum_{j>k} h_{jk}^2 P_j}$, which is equivalent to (4). ■

[1]Gaussian input and Gaussian quantization are used for convenience and for simplicity only. They are not necessarily optimal; see [12] for an example.

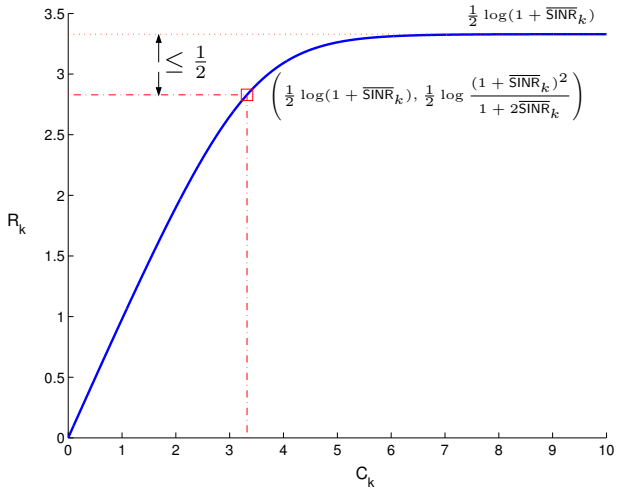


Fig. 3. Achievable rate of user k versus the backhaul capacity C_k

The rate expression (3) shows how the achievable rates are affected by the limited capacities of the digital backhaul links under the proposed per-base-station SIC decoding framework. Fig. 3 plots the achievable rate of R_k as a function of the backhaul link capacity C_k with $\overline{\mathsf{SINR}}_k$ equal to 20dB. When C_k is small, R_k grows almost linearly with C_k, which means that each bit of the backhaul link provides approximately one bit increase in the achievable rate for user k. The digital backhaul is efficiently exploited in this regime. However, as C_k grows larger, each bit of the backhaul link gives increasingly less achievable rate improvement. In the limit of unbounded backhaul capacity, i.e., $C_k = \infty$, R_k saturates and approaches $\frac{1}{2}\log(1+\overline{\mathsf{SINR}}_k) \triangleq \bar{R}_k$, which is the upper limit for the rate of user k when the SIC decoder is employed.

Since backhaul link capacity can be costly in practical implementations, it is natural to ask how large C_k needs to be in order to achieve a rate R_k that is close to the maximum SIC rate with unlimited backhaul. It is easy to see that when $C_k = \frac{1}{2}\log(1+\overline{\mathsf{SINR}}_k)$, $\bar{R}_k - R_k$ is upper bounded by one-half bit, i.e.,

$$\begin{aligned} \bar{R}_k - R_k &= \frac{1}{2}\log(1+\overline{\mathsf{SINR}}_k) \\ &\quad - \frac{1}{2}\log\frac{1+\overline{\mathsf{SINR}}_k}{1+2^{-2C_k}\overline{\mathsf{SINR}}_k}\Big|_{C_k=\frac{1}{2}\log(1+\overline{\mathsf{SINR}}_k)} \\ &= \frac{1}{2}\log\left(1+\frac{\overline{\mathsf{SINR}}_k}{1+\overline{\mathsf{SINR}}_k}\right) \\ &\le \frac{1}{2}. \end{aligned} \tag{12}$$

Therefore, when the backhaul link has capacity $C_k = \frac{1}{2}\log(1+\overline{\mathsf{SINR}}_k)$, the achievable rate is half a bit away from the SIC upper limit. This is also the point under which the utilization of C_k is the most efficient, as shown in Fig. 3.

It is worth noting that the above result, which suggests that C_k should scale as $\log(\overline{\mathsf{SINR}}_k)$, is analogous to the conclusion of [11, Corollary IV.6], which states that the backhaul rate should scale as $\log(P)$ in order to approach the capacity of the infinite backhaul case for the symmetric Wyner model. The

main difference is that this paper uses SIC, so interference not yet cancelled still appears in the SINR expression.

2) Per-Base-Station SIC Without Wyner-Ziv: Implementing the Wyner-Ziv compression may not be easy in practice. The main difficulty lies in the binning operation that is necessary in order to take advantage of the side information. To make the proposed per-base-station SIC scheme more amenable to practical implementation, the Wyner-Ziv quantizer can be replaced by a simple vector quantizer that does not take side information (i.e., previously decoded messages in the SIC framework) into account. In this way, the following rate can be achieved for user k:

$$R_k = I(X_k; \hat{Y}_k | X_1, \cdots, X_{k-1}) \tag{13}$$

subject to the constraint

$$I(Y_k; \hat{Y}_k) \le C_k, \tag{14}$$

for $k = 1, 2, \cdots, L$. Following the same lines of the proof of Theorem 1, it is easy to see that by replacing (9) with $C_k = \frac{1}{2}\log\left(1 + \frac{N_0 + \sum_j h_{jk}^2 P_j}{\mathsf{q}_k}\right)$, which gives $\mathsf{q}_k^* = \frac{N_0 + \sum_j h_{jk}^2 P_j}{2^{2C_k} - 1}$, we obtain

$$\begin{aligned} R_k &= \frac{1}{2}\log\frac{\mathsf{q}_k^* + N_0 + \sum_{j \ge k} h_{jk}^2 P_j}{\mathsf{q}_k^* + N_0 + \sum_{j > k} h_{jk}^2 P_j} \\ &= \frac{1}{2}\log\frac{N_0 + \sum_{j \ge k} h_{jk}^2 P_j + 2^{-2C_k}\sum_{j<k} h_{jk}^2 P_j}{N_0 + \sum_{j > k} h_{jk}^2 P_j + 2^{-2C_k}\sum_{j \ge k} h_{jk}^2 P_j}. \end{aligned} \tag{15}$$

Define

$$\overline{\mathsf{SINR}}'_k = \frac{h_{kk}^2 P_k}{N_0 + \sum_{j>k} h_{jk}^2 P_j + 2^{-2C_k}\sum_{j<k} h_{jk}^2 P_j}. \tag{16}$$

We have just proved that the following rate is achievable using per-base-station SIC without Wyner-Ziv compress-and-forward.

Theorem 2. *For the uplink multicell joint processing channel depicted in Fig. 1, the following rate is achievable using vector quantization at the base-stations followed by SIC at the centralized processor with a fixed decoding order:*

$$R_k = \frac{1}{2}\log\frac{1 + \overline{\mathsf{SINR}}'_k}{1 + 2^{-2C_k}\overline{\mathsf{SINR}}'_k}, \tag{17}$$

where

$$\overline{\mathsf{SINR}}'_k = \frac{\mathsf{SNR}_k}{1 + \sum_{j>k}\mathsf{SIR}_{j,k} + 2^{-2C_k}\sum_{j<k}\mathsf{SIR}_{j,k}}. \tag{18}$$

Comparing Theorem 2 with Theorem 1, it is easy to see that achievable rates in (3) and (17) share the same structure. However, without Wyner-Ziv compression, the effective SINR (18) becomes smaller due to the extra terms $2^{-2C_k}\sum_{j<k}\mathsf{SIR}_{j,k}$. This means that the impact of previously decoded signals $X_1, \cdots, X_{k-1}$ is not fully cancelled at stage k. We also note that as $C_k \to \infty$, we have $\overline{\mathsf{SINR}}'_k \to \overline{\mathsf{SINR}}_k$. Thus, Wyner-Ziv coding is most useful only when the backhaul link capacity is small. For reasonably large C_k, the benefit of Wyner-Ziv coding is expected to be marginal.

C. Joint Base-Station SIC Schemes

The per-base-station SIC scheme takes advantage of multicell processing only in so far as to the use of the decoded messages as side information. It is possible to further improve upon these schemes by joint decoding across the base-stations.

Note that in the per-base-station SIC scheme, the decoding of source messages and quantized messages follow the order $\hat{Y}_1 \to X_1 \to \hat{Y}_2 \to X_2, \to \cdots, \to \hat{Y}_L \to X_L$, where in the kth compression and decoding stage, previously decoded source signals $X_1, \cdots, X_{k-1}$ are used as side information. However, note that in the decoding process the quantization codewords $\hat{Y}_1, \cdots, \hat{Y}_{k-1}$ are also decoded along the way, and thus are also available for possible joint processing at stage k, which can lead to better performance. In particular, we can incorporate $\hat{Y}_1, \cdots, \hat{Y}_{k-1}$ in the expressions for C_k and R_k, leading to the following achievable rate:

$$R_k = I(X_k; \hat{Y}_1, \cdots, \hat{Y}_k | X_1, \cdots, X_{k-1}), \tag{19}$$

subject to

$$C_k \ge I(Y_k : \hat{Y}_k | X_1, \cdots, X_{k-1}, \hat{Y}_1, \cdots, \hat{Y}_{k-1}), \tag{20}$$

for $k = 1, 2, \cdots, L$.

Alternatively, we can also proceed in a two-stage successive process of decoding all of $\{\hat{Y}_k\}_{k=1}^L$ first, then $\{X_k\}_{k=1}^L$ [14], [15]. Further, each of these two stages can be accomplished in an SIC fashion, resulting in rate expressions

$$R_k = I(X_k; \hat{Y}_1, \cdots, \hat{Y}_L | X_1, \cdots, X_{k-1}), \tag{21}$$

subject to

$$I(Y_k; \hat{Y}_k | \hat{Y}_1, \cdots, \hat{Y}_{k-1}) \le C_k, \tag{22}$$

for $k = 1, 2, \cdots, L$. We note here that the above rate expressions (19) and (21) can outperform the rates (3) in Theorem 1 and (17) in Theorem 2, because in the above expressions each X_k is decoded based on the quantized observations of all base-stations (or that of all previous decoded base-stations), rather than just the kth base-station in the per-base-station SIC scheme. However, for this same reason, the implementations of the above joint-base-station SIC schemes are also expected to be somewhat more complicated. For the rest of this paper, we only focus on the per-base-station SIC schemes of Theorem 1 and Theorem 2, and leave the joint-base-station SIC schemes as subject for future studies.

IV. Approximate Optimality of Per-Base-Station SIC in a Wyner Model

The proposed per-base-station SIC scheme is in general suboptimal. However, in certain asymmetric settings, the per-base-station SIC scheme can be shown to be approximately optimal in the sense of achieving the sum capacity to within a constant gap. This section studies a class of such asymmetric channels in which each transmitter-receiver pair interferes only with one neighbor and gets interfered by only one other neighbor as shown in Fig. 4. This model is called the soft-handoff Wyner model in the literature [23]. The Wyner model is an abstraction of the cellular network to a one-dimensional setting. It can, for example, model a communication scenario where base-stations are placed along a highway and mobile terminals

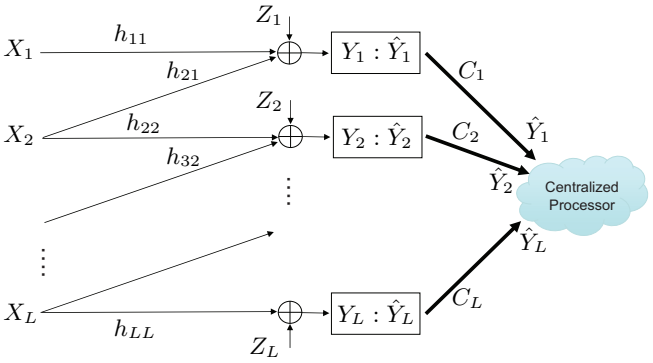


Fig. 4. Wyner model where each transmitter-receiver only interferes with one neighbor.

travel between the base-stations. This section proves that the per-base-station SIC scheme achieves to within a constant gap to capacity for the Wyner model. This result holds either with or without Wyner-Ziv coding.

Consider first the per-base-station SIC with Wyner-Ziv coding. Intuitively, decoding order plays an important role in this asymmetric setting. Since receiver L sees a non-interfered version of the source signal X_L, decoding should take place starting from user L. After X_L is decoded, it can serve as side information, which facilitates the decoding of user $L-1$, then $L-2$, etc. With this decoding order and applying the result of Theorem 1 to the channel setting of Fig. 4, we arrive at the following achievable rate region:

$$\begin{cases} R_i = I(X_i; \hat{Y}_i | X_{i+1}), & i = 1, 2, \cdots, L-1 \\ R_L = I(X_L; \hat{Y}_L) \end{cases} \tag{23}$$

subject to

$$\begin{cases} I(Y_i; \hat{Y}_i | X_{i+1}) \le C_i, & i = 1, 2, \cdots, L-1 \\ I(Y_L; \hat{Y}_L) \le C_L \end{cases} \tag{24}$$

which, when specialized to Gaussian inputs $X_i \sim \mathcal{N}(0, P_i)$ and the Gaussian quantization scheme: $\hat{Y}_i = Y_i + e_i$, where e_i is the quantization noise independent of everything else and follows Gaussian distributions $e_i \sim \mathcal{N}(0, \mathsf{q}_i)$, gives the following achievable rate

$$R_i = \frac{1}{2} \log \left(\frac{1 + \mathsf{SNR}_i}{1 + 2^{-2C_i} \mathsf{SNR}_i} \right), \quad i = 1, 2, \cdots, L. \tag{25}$$

The achievable sum rate for the Wyner model with decoding order from user L to user 1 is then

$$R_{\text{sum}} = \sum_{i=1}^{L} \frac{1}{2} \log \left(\frac{1 + \mathsf{SNR}_i}{1 + 2^{-2C_i} \mathsf{SNR}_i} \right). \tag{26}$$

The above sum rate can be slightly improved by noticing that the Lth base-station can use decode-and-forward instead of compress-and-forward, which leads to

$$R_L = \min \left\{ \frac{1}{2} \log(1 + \mathsf{SNR}_L), C_L \right\}. \tag{27}$$

The main result of this section is that under a mild weak interference condition, the sum rate achieved by the per-base-station SIC scheme is at most a constant number of bits away from the sum capacity for the Wyner model, so it is approximately optimal in the high SNR regime.

Theorem 3. *For a multicell processing Wyner model shown in Fig. 4, in the weak-interference regime of* $\mathsf{INR}_{i+1,i} \le \mathsf{SNR}_i, i = 1, 2, \cdots, L-1$, *the per-base-station SIC scheme with Wyner-Ziv coding achieves a sum rate that is within* $L - \frac{1}{2}$ *bits of the sum capacity.*

Proof: To show that the difference between the achievable sum rate in (26) and the sum capacity is bounded by a constant gap, we first write down the cut-set bound [25, Theorem 14.10.1] for the Wyner channel model. Let $\mathcal{L} = \{1, 2, \cdots, L\}$ represent the index set for the base-stations. We partition $\mathcal{L}$ into two sets, $\mathcal{S}$ and $\mathcal{S}^c$, and only consider cuts for which all the X_i's and a selected subset of Y_i's, with $i \in \mathcal{S}$, are on one side, and the rest of the Y_i's, with $i \in \mathcal{S}^c$, and the central processor are on the other side. Denote $\mathbf{X}(\mathcal{L}) = \{X_i \mid i \in \mathcal{L}\}$ and $\mathbf{Y}(\mathcal{S}^c) = \{Y_i \mid i \in \mathcal{S}^c\}$. We have an upper bound to the cut-set bound as follows:

$$\begin{aligned} R_{\text{sum}}^{\text{cut-set}} &\le \max_{p(\mathbf{X}(\mathcal{L}))} \min_{\mathcal{S} \subseteq \mathcal{L}} \left\{ I\left(\mathbf{X}(\mathcal{L}); \mathbf{Y}(\mathcal{S}^c)\right) + \sum_{i \in \mathcal{S}} C_i \right\} \\ &\le \min_{\mathcal{S} \subseteq \mathcal{L}} \left\{ \max_{p(\mathbf{X}(\mathcal{L}))} I\left(\mathbf{X}(\mathcal{L}); \mathbf{Y}(\mathcal{S}^c)\right) + \sum_{i \in \mathcal{S}} C_i \right\}, \end{aligned} \tag{28}$$

where the maximization is taken over all the possible joint distributions of $\mathbf{X}(\mathcal{L})$, and the minimization is taken over 2^L different choices of cuts that separate the L transmitters and the centralized processor.

Evaluate the mutual-information term in (28) as follows:

$$\begin{aligned} & I\left(\mathbf{X}(\mathcal{L}); \mathbf{Y}(\mathcal{S}^c)\right) \\ &= h\left(\mathbf{Y}(\mathcal{S}^c)\right) - \sum_{i \in \mathcal{S}^c} h(Z_i) \\ &\le \sum_{i \in \mathcal{S}^c} \left(h(Y_i) - h(Z_i) \right) \\ &\overset{(a)}{\le} \sum_{i \in \mathcal{S}^c} \frac{1}{2} \log(1 + \mathsf{SNR}_i + \mathsf{INR}_{i+1,i}) \\ &\overset{(b)}{\le} \begin{cases} \sum_{i \in \mathcal{S}^c} \frac{1}{2} \log(1 + \mathsf{SNR}_i) + \frac{|\mathcal{S}^c|}{2}, & \text{if } L \in S \\ \sum_{i \in \mathcal{S}^c} \frac{1}{2} \log(1 + \mathsf{SNR}_i) + \frac{|\mathcal{S}^c| - 1}{2}, & \text{if } L \in S^c \end{cases} \end{aligned} \tag{29}$$

where (a) holds because Gaussian distribution maximizes the entropy $h(Y_i)$ and (b) is due to the fact that $\mathsf{INR}_{i+1,i} \le \mathsf{SNR}_i$ thus $\frac{1}{2}\log(1+\mathsf{SNR}_i+\mathsf{INR}_{i+1,i}) \le \frac{1}{2}\log(1+\mathsf{SNR}_i)+\frac{1}{2}$ for $i = 1, \cdots, L-1$. For $i = L$, $h(Y_L) - h(Z_L) = \frac{1}{2}\log(1+\mathsf{SNR}_L)$.

Now, we are going to choose a particular cut to further upper bound the cut-set bound. We compare $\frac{1}{2}\log(\mathsf{SNR}_i)$ at each base-station with the backhaul capacity C_i, and let the cut go through either the user-base-station link or the backhaul link, whichever corresponds to the smaller of the two quantities above, as shown in Fig. 5. More precisely, define

$$\mathcal{S} = \left\{ i \;\middle|\; \frac{1}{2} \log \mathsf{SNR}_i \ge C_i \right\}. \tag{30}$$

With this particular cut, we can now bound the difference between the cut-set bound and the achievable sum rate. First,

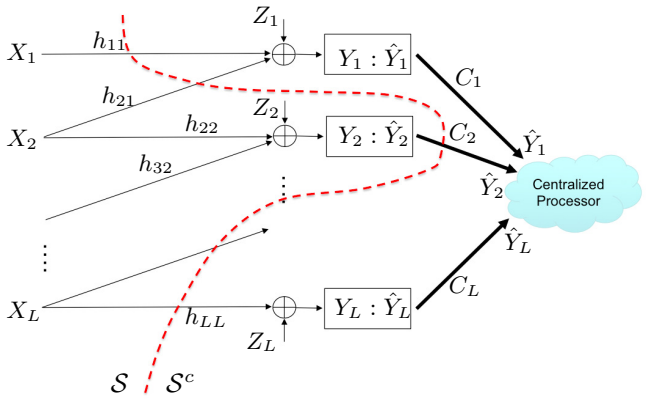


Fig. 5. Choice of cut-set for the Wyner model. The cut goes through the access link or the backhaul link depending on whether $\frac{1}{2}\log(\mathsf{SNR}_i)$ is greater or smaller than C_i.

consider the case that $L \in S^c$, we use the achievable sum rate as given in (26):

$$\begin{aligned} R_{\text{sum}}^{\text{cut-set}} - R_{\text{sum}} \\ \leq & \sum_{i\in\mathcal{S}^c}\left\{\frac{1}{2}\log(1+\mathsf{SNR}_i) - \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_i}{1+2^{-2C_i}\mathsf{SNR}_i}\right)\right\} \\ & + \sum_{i\in\mathcal{S}}\left\{C_i - \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_i}{1+2^{-2C_i}\mathsf{SNR}_i}\right)\right\} + \frac{|\mathcal{S}^c|-1}{2} \\ = & \sum_{i\in\mathcal{S}^c}\frac{1}{2}\log\left(1+2^{-2C_i}\mathsf{SNR}_i\right) \\ & + \sum_{i\in\mathcal{S}}\frac{1}{2}\log\left(\frac{2^{2C_i}+\mathsf{SNR}_i}{1+\mathsf{SNR}_i}\right) + \frac{|\mathcal{S}^c|-1}{2} \\ \overset{(a)}{\leq} & \frac{|\mathcal{S}^c|}{2} + \frac{|\mathcal{S}|}{2} + \frac{|\mathcal{S}^c|-1}{2} \\ \leq & L - \frac{1}{2} \end{aligned} \tag{31}$$

where in (a) we used the definition of the cut (30), i.e., $\mathsf{SNR}_i \leq 2^{2C_i}$ for $i \in \mathcal{S}^c$, and $\mathsf{SNR}_i \geq 2^{2C_i}$ for $i \in \mathcal{S}$, and the last inequality is due to the fact that $|\mathcal{S}^c|+|\mathcal{S}| = L$ and $|\mathcal{S}^c| \leq L$.

Now, consider the case that $L \in \mathcal{S}$. We tighten the sum rate expression (26) by noticing that since by the definition of $\mathcal{S}$ we have $\frac{1}{2}\log \mathsf{SNR}_L \geq C_L$, so by (27) we have $R_L = C_L$. In this case,

$$\begin{aligned} R_{\text{sum}}^{\text{cut-set}} - R_{\text{sum}} \\ \leq & \sum_{i\in\mathcal{S}^c}\left\{\frac{1}{2}\log(1+\mathsf{SNR}_i) - \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_i}{1+2^{-2C_i}\mathsf{SNR}_i}\right)\right\} \\ & + \sum_{i\in\mathcal{S}\setminus\{L\}}\left\{C_i - \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_i}{1+2^{-2C_i}\mathsf{SNR}_i}\right)\right\} + \frac{|\mathcal{S}^c|}{2} \\ \leq & \frac{|\mathcal{S}^c|}{2} + \frac{|\mathcal{S}|-1}{2} + \frac{|\mathcal{S}^c|}{2} \leq L - \frac{1}{2} \end{aligned} \tag{32}$$

This completes the proof. ■

It turns out that even without Wyner-Ziv compress-and-forward, it is still possible to achieve the sum capacity to within a constant, albeit larger, gap under the same weak-interference condition.

Theorem 4. *For a multicell processing Wyner model shown in Fig. 4, in the weak-interference regime of* $\mathsf{INR}_{i+1,i} \leq \mathsf{SNR}_i, i = 1, 2, \cdots, L-1$, *the per-base-station SIC scheme without Wyner-Ziv coding achieves a sum rate that is within* $\frac{1}{2}(1+\log(3))L - \frac{1}{2}$ *bits of the sum capacity.*

Proof: Applying Theorem 2 to the Wyner model of Fig. 4 with the decoding order from user L to user 1, it is easy to see that the achievable rate of user L remains the same while the rates of other users become

$$R_i = \frac{1}{2}\log\frac{1+2^{-2C_i}\mathsf{INR}_{i+1,i}+\mathsf{SNR}_i}{1+2^{-2C_i}\mathsf{INR}_{i+1,i}+2^{-2C_i}\mathsf{SNR}_i}. \tag{33}$$

Under the weak-interference condition $\mathsf{INR}_{i+1,i} \leq \mathsf{SNR}_i$, when $i \in \mathcal{S}^c$, we have $0 \leq 2^{-2C_i}\mathsf{INR}_{i+1,i} \leq 1$, and when when $i \in \mathcal{S}$, we have $0 \leq 2^{-2C_i}\mathsf{INR}_{i+1,i} \leq 2^{-2C_i}\mathsf{SNR}_i$. Replace the R_i expressions in (31) and (32) by (33) while noting the inequalities above, we obtain the $\frac{1}{2}(1+\log(3))L - \frac{1}{2}$ bound. ■

Note that a looser bound can be obtained by noting that under the weak-interference condition $\mathsf{INR}_{i+1,i} \leq \mathsf{SNR}_i$, the achievable rate without Wyner-Ziv coding (33) is already within $\frac{1}{2}$ bits of the rate with Wyner-Ziv coding as in (25). As a result, a looser sum-rate gap to the cut-set upper bound can immediately be obtained as $L - \frac{1}{2} + \frac{L-1}{2} = \frac{3}{2}L - 1$ bits[2].

V. Optimal Rate Allocation with a Total Backhaul Capacity Constraint

A practical system may have a constraint on the sum capacity of all digital backhaul links, for example, when the backhaul links are implemented in a wireless medium with shared bandwidth. So, it may be of interest to optimize the allocation of backhaul capabilities among the base-stations in order to achieve an overall maximum sum rate under a total backhaul capacity constraint. The structure of the solution of such an optimization can also yield useful insight. The optimization problem can be formulated as the following:

$$\begin{aligned} \text{maximize} \quad & \sum_{k=1}^{L} R_k \\ \text{subject to} \quad & C_k \geq 0, \quad k = 1, 2, \cdots, L \\ & \sum_{k=1}^{L} C_k \leq C \end{aligned} \tag{34}$$

where $R_k, k = 1, 2, \cdots, L$ are functions of C_k as derived in Theorem 3, and $C > 0$ is the total available backhaul capacity. The following theorem provides an optimal solution to the above optimization problem for the per-base-station SIC scheme with Wyner-Ziv coding.

Theorem 5. *For the uplink multicell joint processing model shown in Fig. 1, with the per-base-station SIC at the central processor with Wyner-Ziv coding, the optimal allocation of backhaul link capacities subject to a total backhaul capacity constraint* C *is given by*

$$C_k^* = \max\left\{\frac{1}{2}\log(\overline{\mathsf{SINR}}_k) - \alpha, 0\right\}, \tag{35}$$

[2]The authors wish to thank Yuhan Zhou to point out the tighter bound in Theorem 4.

where α is chosen such that $\sum_{k=1}^{L} C_k^* = C$.

Proof: Substituting the rate expression (19) for R_k into the optimization problem (34), we obtain the following equivalent minimization problem:

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{L} \frac{1}{2} \log\left(1 + 2^{-2C_k} \overline{\mathsf{SINR}}_k\right) \\ \text{subject to} \quad & C_k \geq 0, \quad k = 1, 2, \cdots, L. \\ & \sum_{k=1}^{L} C_k \leq C \end{aligned} \tag{36}$$

It can be easily seen that (36) is a convex optimization problem, as the constraints are linear and the objective function is the sum of convex functions, as can be verified by taking their second derivatives.

Now introducing Lagrange multipliers $\boldsymbol{\nu} \in \mathbb{R}_+^L$ for the positivity constraints $C_k \geq 0, k = 1, 2, \cdots, L$, and $\lambda \in \mathbb{R}_+$ for the backhaul sum-capacity constraint $\sum_{k=1}^{L} C_k \leq C$, we form the Lagrangian

$$\begin{aligned} L(C_k, \boldsymbol{\nu}, \lambda) = & \sum_{k=1}^{L} \frac{1}{2} \log\left(1 + 2^{-2C_k} \overline{\mathsf{SINR}}_k\right) \\ & - \sum_{k=1}^{L} \nu_k C_k + \lambda \left(\sum_{k=1}^{L} C_k - C \right) \end{aligned} \tag{37}$$

Taking the derivative of the above with respect to C_k, we obtain the following Karush-Kuhn-Tucker (KKT) condition

$$-\frac{2^{-2C_k^*} \overline{\mathsf{SINR}}_k}{1 + 2^{-2C_k^*} \overline{\mathsf{SINR}}_k} - \nu_k + \lambda = 0, \tag{38}$$

for the optimal C_k^*, where $k = 1, 2, \cdots, L$. Note that $\nu_k = 0$ whenever $C_k > 0$. Now, the optimal C_k^* must satisfy the backhaul sum-capacity constraint $\sum_{k=1}^{L} C_k^* \leq C$ with equality, because the objective of the minimization R_k monotonically increases with C_k. Solving the condition (38) together with the fact that $\sum_{k=1}^{L} C_k^* = C$ gives the following optimal C_k^*:

$$C_k^* = \max\left\{ \frac{1}{2} \log \frac{\overline{\mathsf{SINR}}_k}{\beta}, 0 \right\}, \tag{39}$$

where β is chosen such that $\sum_{k=1}^{L} C_k^* = C$. This is equivalent to (35). ∎

An interpretation of (39) is that whenever the SINR of user k is above a threshold β, $\frac{1}{2} \log \frac{\overline{\mathsf{SINR}}_k}{\beta}$ bits of the backhaul link should be allocated to user k. Otherwise, this user should not be active as far as maximizing the uplink sum rate is concerned. This optimal rate allocation is quite similar to the classic water-filling solution for the sum-capacity maximization problem for a parallel set of Gaussian channels, in which more power (backhaul capacity in this case) is assigned to users with a better channel.

When written as (35), the optimal backhaul capacity allocation can be interpreted as follows: $C_k = \frac{1}{2} \log(\overline{\mathsf{SINR}}_k)$ can be thought of as the nominal optimal backhaul link capacity. If the total backhaul capacity is above (or below) the nominal $\sum_k \frac{1}{2} \log(\overline{\mathsf{SINR}}_k)$, then the extra capacity must be distributed (or taken away) from each base-station equally. In other words, all base-station should nominally operate at the point 1/2 bits away from the SIC limit (as shown in Fig. 3). If more (or less) backhaul capacity is available than the nominal value, all base-stations should move above (or below) that operating point in the same manner.

Finally, we remark that the decoding order at the centralized processor plays an important role in the optimal rate allocation. Different decoding orders result in different rate expressions in Theorem 1 and thus different rate allocations in Theorem 5, and as a consequence different achievable sum rates. In order to determine the decoding order that results in the largest sum rate (or the maximum network utility), we need to exhaustively search over $K!$ different decoding orders. This is a nontrivial problem that is also encountered in other contexts involving successive decoding such as in V-BLAST [26].

VI. Numerical Examples

A. Two-User Symmetric Scenario

To obtain numerical insight on the per-base-station SIC-based schemes proposed in this paper, the achievable rate regions of Theorem 1 and Theorem 2 are compared with that obtained by two other schemes: 1) single-user decoding without joint processing; 2) joint base-station processing as defined in (21) in Section III-C, for a two-user symmetric scenario where $L = 2$, $P_1 = P_2 = N_0 = 1$, $\mathsf{SNR} = h_{11}^2 = h_{22}^2$, $\mathsf{INR} = h_{12}^2 = h_{21}^2$, and $C = C_1 = C_2$. Under the symmetric setting, both Theorem 1 and Theorem 2 give two symmetric achievable rate pairs depending on the decoding order. Time-sharing of the two achievable rate pairs gives a pentagon shaped achievable rate region.

Single-user decoding without joint processing is considered as a baseline, in which each receiver decodes its own signal while treating the other user's signal as noise. This gives the following achievable rate pair

$$R_1 = R_2 = \min\left\{ \frac{1}{2} \log\left(1 + \frac{\mathsf{SNR}}{1 + \mathsf{INR}}\right), C \right\} \tag{40}$$

which in the symmetric setting results in a square shaped achievable rate region with (R_1, R_2) as the top-right corner.

We also compare with the joint-base-station processing SIC scheme of (21). We restrict ourselves to symmetric quantization noise levels here. The quantization noise level q is chosen such that the resulting average backhaul link capacity is C, i.e.,

$$I(Y_1; \hat{Y}_1) + I(Y_2; \hat{Y}_1 | \hat{Y}_1) = 2C. \tag{41}$$

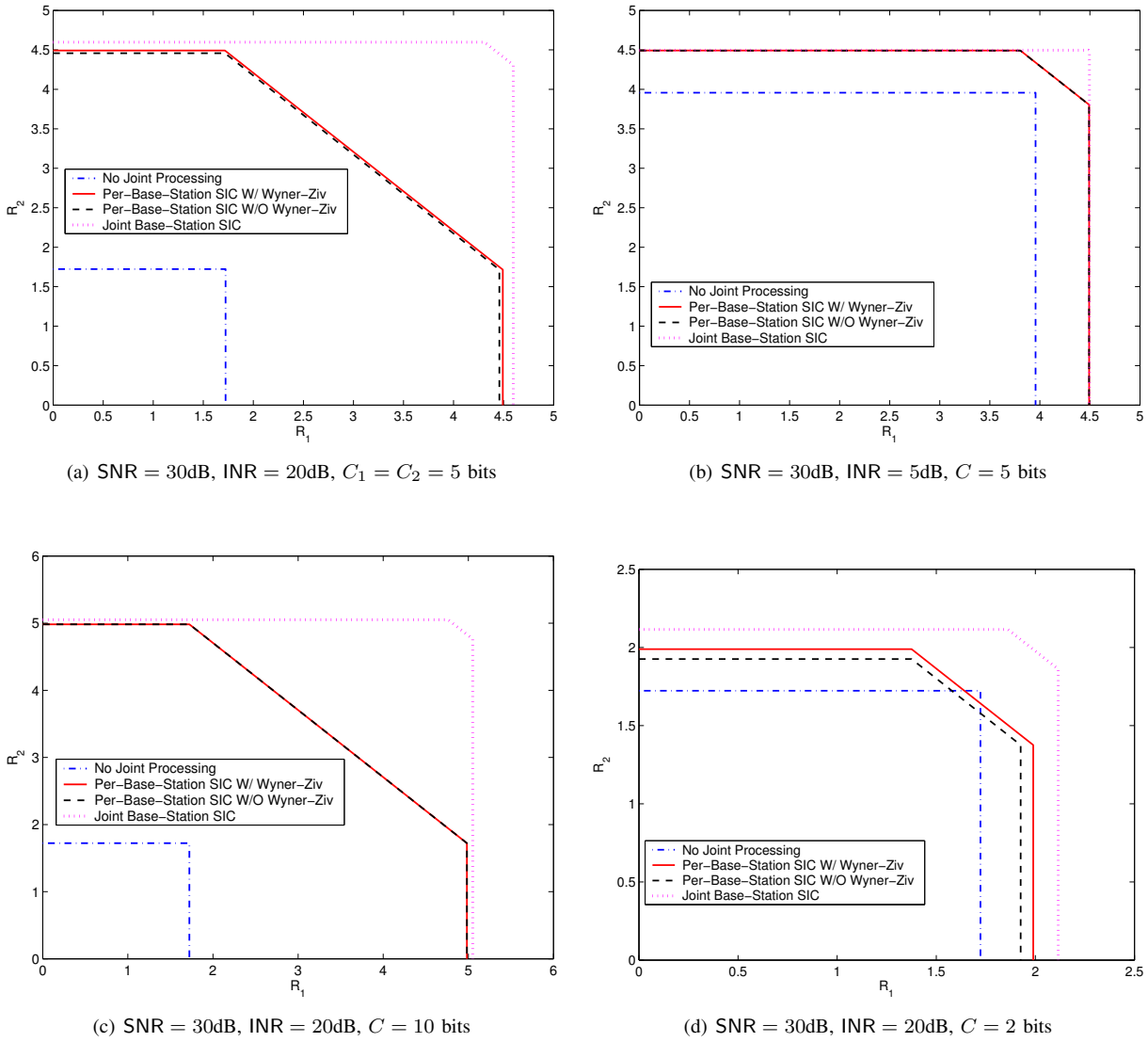
This condition gives the following analytic expression for the quantization noise level:

$$\mathsf{q} = \frac{a + \sqrt{4b + 2^{4C}(a^2 - 4b)}}{2^{4C} - 1}, \tag{42}$$

where $a = 1 + \mathsf{SNR} + \mathsf{INR}$, and $b = \mathsf{SNR} \cdot \mathsf{INR}$. Plots of the joint-base-station SIC are then obtained using (21) with this quantization noise level.

The achievable rate regions obtained above are compared for the following channel settings:

- $\mathsf{SNR} = 30$dB, $\mathsf{INR} = 20$dB, $C = 5$ bits;
- $\mathsf{SNR} = 30$dB, $\mathsf{INR} = 5$dB, $C = 5$ bits;



(a) SNR = 30dB, INR = 20dB, $C_1 = C_2 = 5$ bits

(b) SNR = 30dB, INR = 5dB, $C = 5$ bits

(c) SNR = 30dB, INR = 20dB, $C = 10$ bits

(d) SNR = 30dB, INR = 20dB, $C = 2$ bits

Fig. 6. Comparison of the proposed achievability scheme and another two schemes

- SNR = 30dB, INR = 20dB, $C = 10$ bits;
- SNR = 30dB, INR = 20dB, $C = 2$ bits.

Fig. 6(a) shows that at relatively strong interference level INR = 20dB, the proposed per-base-station SIC schemes (both with and without Wyner-Ziv compression) expand the baseline achievable rate region by about 2.8 bits on either of the individual rates and on the sum rate. The joint base-station SIC regions further outperforms the proposed scheme in sum rate by about 2.5 bits due to the benefits of joint decoding.

However, when the interfering links are weak, as shown in Fig. 6(b) where INR = 5dB, all four achievable rate regions are close to each other. This is the regime where treating interference as noise is close to optimal, so multicell processing does not provide significant benefits.

In the above two examples, the capacities of the backhaul links are already quite abundant, since they are set to be the rate supported by the direct links: $\frac{1}{2}\log(1+\mathsf{SNR}) \approx 5$ bits. In Fig. 6(c), we further increase the backhaul capacity to 10 bits, and show that doing so does not significantly improve the achievable rate region for either proposed SIC-based schemes or the joint base-station processing scheme.

Lastly, we decrease the backhaul capacity from 5 bits to 2 bits. Interestingly, this is a situation in which the baseline scheme can outperform per-base-station SIC as shown in Fig. 6(d). Therefore, the proposed per-base-station schemes can be inefficient in term of sum rate, when the backhaul rates are very limited. Observe that the largest sum rate is still obtained with joint base-station processing.

B. Multicell OFDMA Network

To further understand the performance of the proposed per-base-station SIC scheme in practical systems, in this section, the achievable rates of the two variations of the per-base-station SIC, i.e., with and without Wyner-Ziv coding, are evaluated for a wireless cellular network setup with 19 cells, 3 sectors per cell, and 10 users per sector, where an orthogonal frequency-division multiple-access (OFDMA) scheme with 64 tones over a fixed 10Mbps bandwidth is employed. The cellular topology is shown in Fig. 7. A 19-cell wrap-around layout is used to ensure uniform interference statistics throughout the

TABLE I
WIRELESS CELLULAR MODEL PARAMETERS

Cellular Layout	Hexagonal, 19 cells, 3 sectors/cell
BS-to-BS Distance	600 m
Frequency Reuse	1
Number of User per Sector	10
Channel Bandwidth	10 MHz
MS Transmit PSD	-27 dBm/Hz
Antenna Gain	15 dBi
Background Noise	-169 dBm/Hz
Noise Figure	7 dB
Multipath Time Delay Profile	ITU-R M.1225 PedA
Distance-dependent Path Loss	$128.1 + 37.6 \log_{10}(d)$
Number of Tones	64

network. The assignments of frequency tones to users within each cell are non-overlapping. As a result, users experience only intercell interference and no intracell interference. Both the base-stations and the mobile users are equipped with a single antenna each. Each of the 19 base-stations is connected to the centralized processor via a rate-limited backhaul link. Perfect channel estimation is assumed, and the CSI is made available to all base-stations and to the centralized processor. In the simulation, uniform power allocation of -27dBm/Hz is assumed at all the mobile users. For convenience, a round-robin scheduler is used for user assignment. The base-station-to-base-station distance is set to 600m corresponding to a typical urban deployment. Detailed system parameters are outlined in Table I.

In the first part of the simulation, the capacities of the backhaul links are fixed per base-station and uniformly distributed across the frequency tones, e.g., if the capacity of a backhaul link is 64Mbps, each frequency tone is assumed to have a backhaul of $64\text{Mbps}/64 = 1\text{Mbps}$. Cumulative distribution function (CDF) of the user rates is plotted in order to visualize the performance gain of the proposed schemes over a baseline system, in which base-stations decode the user messages without joint processing at the centralized processor. For the proposed per-base-station SIC scheme, to account for the fairness among users, the decoding order across the cells is chosen to be in decreasing order of the user SINRs (prior to SIC). This decoding order is adopted on each of the OFDM tones independently.

Fig. 8 shows the CDF plots of user rates with a backhaul capacity at each base-station of 180Mbps (i.e., 60Mbps per sector). It is seen that the two per-base-station SIC schemes (with or without Wyner-Ziv coding) both significantly outperform the baseline system. The user rate at 50th-percentile is around 0.8Mbps for the baseline, 2.1Mbps for the per-base-station SIC scheme without Wyner-Ziv coding, and 2.8Mbps for the per-base-station SIC scheme with Wyner-Ziv coding. Compared with the baseline curve, the per-base-station SIC curves also have a better distribution of user rates in terms of fairness. There is a noticeable performance gap between the SIC curve with Wyner-Ziv compression and without Wyner-Ziv-compression. This gap is due to the compression gain brought by side information.

When the capacity of the backhaul per base-station increases to 270Mbps, the proposed per-base-station SIC schemes produce a further performance gain as compared to the 180Mbps case. As can be seen in Fig. 9, the 50-percentile rate becomes 2.6Mbps for the per-base-station SIC scheme without Wyner-Ziv compression, and 3.05Mbps for the per-base-station SIC scheme with Wyner-Ziv compression. However, the gap between the two curves becomes smaller. As the capacity of the backhaul per base-station further increases to 360Mbps, Fig. 10 shows that the per-base-station SIC schemes now only perform marginally better than the 270Mbps case. This is when the benefit of multicell SIC starts to saturate.

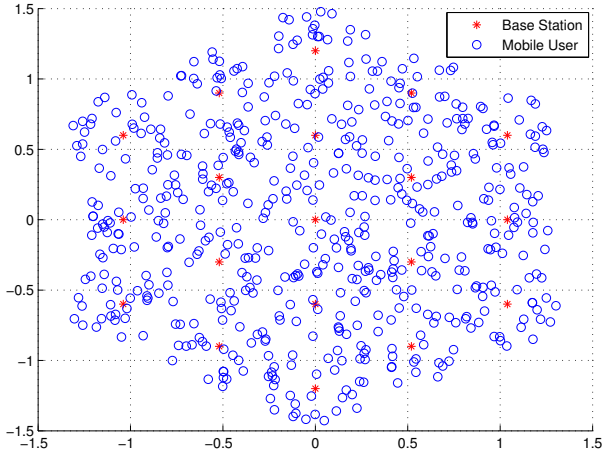


Fig. 7. A cellular topology with 19 cells, 3 sectors per cell, and 10 users per sector placed randomly. The distance between the neighboring base-stations is set to 600m.

Further, as shown in Fig. 10, the CDF curves for the SIC schemes with and without Wyner-Ziv compression are very close to each other for the 360Mbps backhaul case. Thus, the benefit of performing Wyner-Ziv compress-and-forward relaying at the base-stations becomes negligible when the capacities of backhaul links are high, thus confirming our earlier theoretical analysis.

In order to quantitatively evaluate the performance gain brought by the centralized processor, Table II shows the average per-cell sum rate obtained by different schemes. The baseline scheme gives an average per-cell sum rate of 55.5Mbps. By utilizing a high-capacity backhaul with Wyner-Ziv compression, up to 94% sum rate improvement can be obtained. Note that although the rate improvement without Wyner-Ziv is lower than that with Wyner-Ziv, considerable performance gains in the range of 37% to 84% can still be obtained without Wyner-Ziv coding. As noted before, the gain due to the backhaul saturates at around 270Mbps.

The above simulation is performed assuming that the backhaul capacity is the same for each base-station and is uniformly allocated across the frequencies. It is possible to further optimize the backhaul capacity allocation using (35) of Theorem 5. In the next set of simulations, we choose α in (35) to satisfy an average backhaul constraint across the cells, and present the resulting performance with optimized backhaul allocation in Table II and Fig. 11. It can be seen that $120-150$Mbps optimized backhaul already achieves about the same performance as that of 180Mbps uniform backhaul. Likewise, 180Mbps optimized backhaul already achieves about

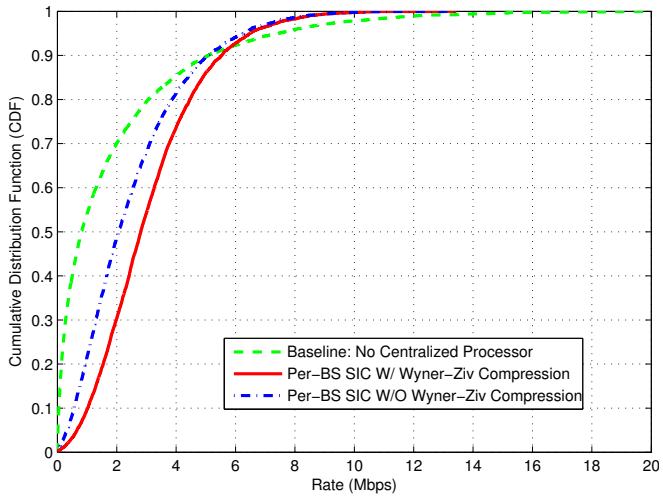


Fig. 8. CDF of user rates with 180Mbps backhaul per base-station uniformly allocated across the frequencies

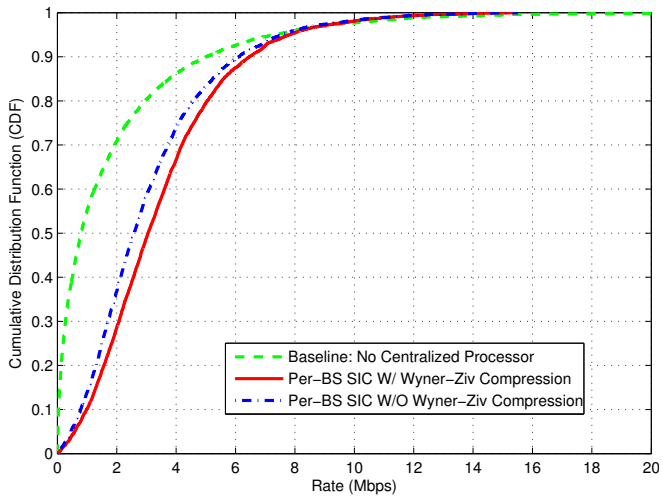


Fig. 9. CDF of user rates with 270Mbps backhaul per base-station uniformly allocated across the frequencies

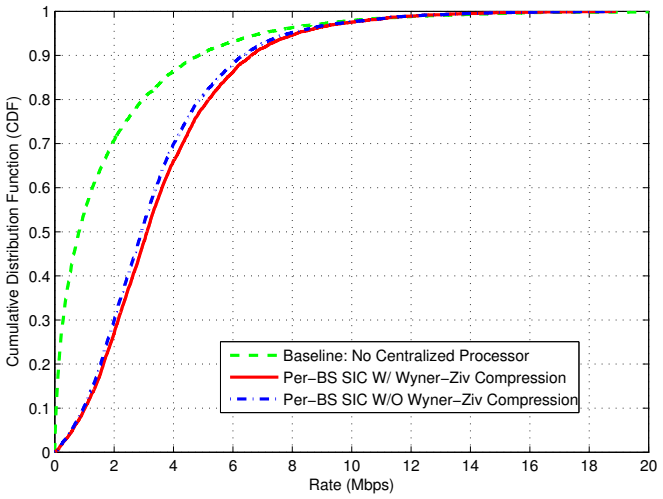


Fig. 10. CDF of user rates with 360Mbps backhaul per base-station uniformly allocated across the frequencies

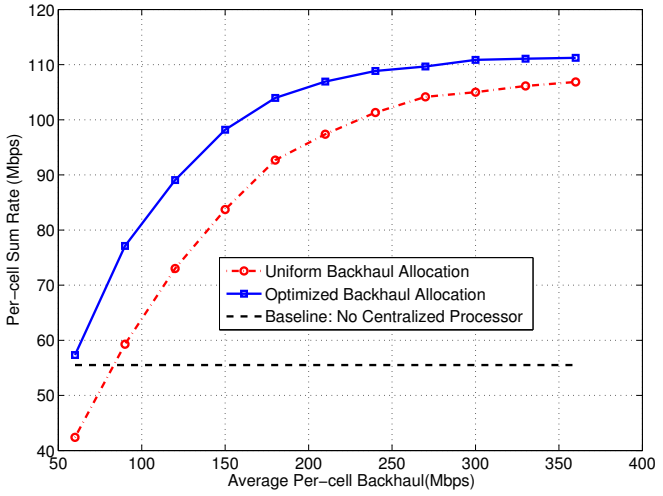


Fig. 11. Per-cell sum rate vs. average per-cell backhaul capacity using per-base-station SIC with uniform and optimized backhaul capacity allocation

the same performance as that of 270Mbps uniform backhaul. Thus, the optimization of the backhaul is quite beneficial.

Further, it can be seen from Fig. 11 that under infinite backhaul, the achieved per-cell sum rate is about 110Mbps for this cellular setting. But when optimized, a finite backhaul capacity at about 1.5 times of the user sum rate (i.e., at about 150Mbps) is already sufficient to achieve about 100Mbps user sum rate, which is 90% of the full benefit of uplink network MIMO. Note that the gain in per-cell sum rate due to the optimization of the backhaul becomes smaller as the backhaul capacity increases, due to the fact that increasing the backhaul capacity eventually offers diminishing return.

Finally, we mention that the performance gain presented here is idealistic because the achievable rates are computed using information theoretical expressions assuming ideal coding, modulation, and perfect CSI. In addition, all users in the 19-cell cluster are assumed to participate in cooperative multicell processing, and no out-of-cluster interference is accounted for. The results in this paper nevertheless serve as upper bound to what is achievable in an uplink network MIMO system.

VII. Conclusion

This paper presents an information-theoretical study of a novel uplink multicell processing scheme employing the compress-and-forward technique with the per-base-station SIC receiver structure for the uplink of a network MIMO system, in which the base-stations are connected to a centralized processor with finite capacity backhaul links. The main advantage of the proposed schemes is that it achieves significant performance gain over the conventional scenario where no centralized processor is deployed, while having an achievable rate region which is easily computable, and that it leads to an architecture that is more amendable to practical implementations than the joint decoding scheme. Furthermore, theoretical analysis shows that the proposed per-base-station SIC scheme is within a constant gap to the sum capacity for a class of Wyner channel models.

The results of this paper also show that when employing the proposed per-base-station SIC scheme, the capacities of the

TABLE II
IMPROVEMENT IN PER-CELL SUM RATE OVER 19 CELLS, 3 SECTORS PER CELL, 10 USERS PER SECTOR. CELL DIAMETER IS 600M. BASELINE PER-CELL SUM RATE IS 55.5MBPS

Per-cell Average Backhaul (Mbps)	Backhaul Capacity Allocation	Per-base-station SIC without WZ (Mbps)	Improvement over Baseline (%)	Per-base-station SIC with WZ (Mbps)	Improvement over Baseline (%)
180	uniform	75.3	37%	92.7	67%
270	uniform	93.2	68%	104.2	88%
360	uniform	102.3	84%	107.6	94%
120	optimized	75.1	35%	89.1	60%
180	optimized	83.3	50%	102.9	85%

backhaul links should scale with the logarithm of the SINR at each base-station, both from a point of view of approaching the theoretical maximum SIC rate with unlimited backhaul, as well as for maximizing the overall sum rate subject to a total backhaul rate constraint. Numerical simulations reveal that significant sum-rate gain can be obtained by the proposed SIC-based schemes with modest backhaul capacity requirement.

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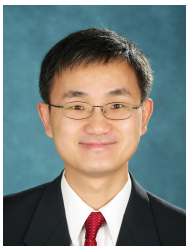

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