Utilizing Multiuser Diversity for Multiple Antenna Systems

Wonjong Rhee, Wei Yu and John M. Cioffi

STAR Laboratory
Packard Building, Stanford University, CA 94305-9515
wonjong@dsl.stanford.edu

***Abstract*—Recent research has shown that the capacity of a multiple antenna system grows linearly with increasing number of antennas for rich-scattering environments. However, this is not true for wireless channels with small number of independent paths. To overcome this problem, this paper investigates the possibility of exploiting multiuser dimension with and without channel side information at transmitter. First, the single user capacity per antenna is shown to converge to zero with increasing number of antennas for channels with finite number of independent paths. Then multiuser capacity per antenna at limit is shown to be positive. Simulation results are presented for a single user system and a multiuser uplink system.

I. Introduction

Researchers have long desired to come up with wireless communication systems whose spectral efficiency is as high as in wire-line systems. This is difficult mainly due to the problems of Doppler effect and multipath fading. The problem of Doppler can be reduced for indoor wireless systems where all users move very slowly, and for outdoor fixed wireless systems where antennas are fixed at certain locations. For these systems, it is possible for the receiver to estimate the slowly varying channel and to utilize the channel side information. Channels for these types of applications can usually be assumed to be quasi-static, meaning that the channel is stationary for a block of transmission.

Furthermore, by adopting multiple antennas, the problem of multipath fading can be used as an advantage rather than being thought as an impairment. To utilize multipath fading, multi-element array (MEA) antennas have been adopted in numerous applications. For instance, by using MEA, spectral efficiency as high as 20-40 bps/Hz has been reported for the indoor communication environment [1].

However, the channel characteristics of an outdoor fixed wireless system is known to be quite different from that of an indoor system. The main difference is that the outdoor environment typically lacks scatterers, and hence the MIMO (Multiple Input Multiple Output) channel elements are highly correlated. In certain suburban environments, the number of independent paths per user is known to be around one to three. In such situations, high capacity is difficult to achieve even with a large number of antennas. To resolve this problem, we consider utilizing *Multiuser Diversity* for multiple-user multiple-antenna systems. Multiuser diversity can be defined as the diversity stemming from the independence of channels from different users. In other words, it is a diversity coming from the fact that user channels are independent since each user is located at a random position within a cell.

In this paper, we investigate the possibility of large capacity gain for highly correlated fixed outdoor channels by utilizing multiuser diversity. We use the sum capacity as a measure of capacity.

II. System Models

The channels are assumed to be quasi-static and flat fading. Perfect channel side information is assumed to be available either at both transmitter and receiver or only at receiver. Although these assumptions are not likely to be valid for mobile wireless systems, they are reasonable for indoor or fixed wireless. Note that mobiles are not likely to be able to accommodate large number of antennas anyway. For a wideband system, OFDM (Orthogonal Frequency Division Multiplexing) can be adopted to obtain flat fading channels.

A. Single user systems

Consider a single user system with n_T transmit antennas and n_R receive antennas. For narrowband channels, the system can be modeled as follows :

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} \tag{1}$$

$\mathbf{Y} = [y_1 \dots y_{n_R}]^T$ is the received signal vector, $\mathbf{X} = [x_1 \dots x_{n_T}]^T$ is the transmitted signal vector, $\mathbf{V} = [v_1 \dots v_{n_R}]^T$ the is i.i.d AWGN noise vector at receiver with covariance matrix given by $R_V = E[\mathbf{V}\mathbf{V}^*] = {\sigma_v}^2 \mathbf{I}_{n_R}$, and $\mathbf{H}$ is a $n_R \times n_T$ MIMO channel.

MIMO channel $\mathbf{H}$ can be modeled in various ways depending on environments [2]. In an indoor system [1], elements of $\mathbf{H}$ were modeled as i.i.d Gaussian complex random variables as a large number of independent paths exists between transmitter and receiver.

This uncorrelated channel model is conceptually simple, but it is inadequate for modeling outdoor channels where the number of scatterers is much smaller. As an example, the MIMO channel rank of a system with few far-field reflectors is known to be small even with a large number of transmit and receive antennas. This is true even with many local scatterers around transmitter and receiver since the number of independent paths is limited by the number of far-field reflectors. Therefore, the outdoor MIMO channels are best modeled by correlated channel models.

A correlated MIMO channel $\mathbf{H}$ with L independent paths can be modeled as below :

$$\mathbf{H} = \sum_{l=1}^{L} \mathbf{H}_l = \sum_{l=1}^{L} h_l \mathbf{A}_{R,l} \mathbf{A}_{T,l} \tag{2}$$

h_l is a complex gain, $\mathbf{A}_{R,l}$ is $n_R \times 1$ array response of receiver antennas, and $\mathbf{A}_{T,l}$ is $1 \times n_T$ array response of transmit antennas for path l. Each element of $\mathbf{A}_{R,l}$ and $\mathbf{A}_{T,l}$ is assumed to have unit variance. Depending on the number of local scatterers around receiver (or transmitter), $\mathbf{A}_{R,l}$ (or $\mathbf{A}_{T,l}$) can be modeled in different ways. If a receiver (or a transmitter) is surrounded by a sufficiently large number of local scatterers, elements of $\mathbf{A}_{R,l}$ (or $\mathbf{A}_{T,l}$) can be approximated by i.i.d Gaussian random variables. If a receiver (or a transmitter) is not surrounded by a large number of local scatterers, elements of $\mathbf{A}_{R,l}$ (or $\mathbf{A}_{T,l}$) can be modeled as correlated variables. The correlation is lower when a large number of local scatterers is present. For simulations in this paper, we choose Raleigh's far-field channel model [3] with L independent paths. The results depend mostly on L, not on the choice of models for $\mathbf{A}_{R,l}$ or $\mathbf{A}_{T,l}$.

B. *Multiuser systems*

We consider a multiuser system with a single base station. Signals from other cells are assumed to be included in AWGN noise. The channel between a user and the base station can be modeled as an uncorrelated or correlated MIMO channel as in single user systems.

For uplink with K users, we have the following system model.

$$\mathbf{Y} = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{X}_k + \mathbf{V} \tag{3}$$

Index k is used to represent the signals and channels related to user k.

For downlink with K users, we have the following system model.

$$\mathbf{Y}_k = \mathbf{H}_k \sum_{k=1}^{K} \mathbf{X}_k + \mathbf{V}_k \tag{4}$$

$\mathbf{Y}_k$ is the sum of signals received at user k's receiver, $\mathbf{H}_k$ is a MIMO channel between base station and the k'th user, $\mathbf{X}_k$ is the signal containing user k's information bits, and $\mathbf{V}_k$ is the AWGN noise of user k's receiver.

Also, the correlated MIMO channel model (2) can be used with index k denoting the channel for user k.

$$\mathbf{H}_k = \sum_{l=1}^{L} \mathbf{H}_{k,l} = \sum_{l=1}^{L} h_{k,l} \mathbf{A}_{R_k,l} \mathbf{A}_{T_k,l} \tag{5}$$

III. Asymptotic behavior of capacity : Single user systems

In this section, we deal with the single user case. The treatment here is also applicable to FDMA or TDMA multiuser systems because in such systems only one user is assigned to a particular frequency band at a particular time, so the system can be modeled by (1). For simplicity, we assume the same number of transmit and receive antennas, i.e. $n_T = n_R = n$.

A. *Capacity formulation*

For a single user system with n transmit and n receive antennas, and total power constraint P_t, capacity per Hz with transmitter optimization can be evaluated from the following equation [4].

$$C_{S,Opt} = \max_{\mathbf{R}_X} \log_2 \left(\left| \mathbf{I}_n + \frac{1}{{\sigma_v}^2} \mathbf{H} \mathbf{R}_X \mathbf{H}^* \right| \right) \tag{6}$$

$$\text{subject to } Tr(\mathbf{R}_X) \le P_t$$

$\mathbf{R}_X = E[\mathbf{X}\mathbf{X}^*]$ is the covariance matrix of $\mathbf{X}$, $|\cdot|$ is the determinant operation, and $Tr(\cdot)$ is the trace operation. The optimal $\mathbf{R}_X$ for this problem can be analytically

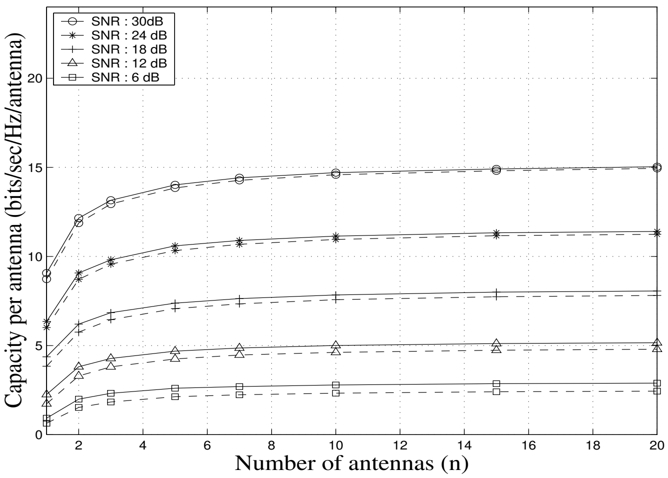

Fig. 1. Single user, uncorrelated channels : 3% outage capacity per antenna for i.i.d Gaussian MIMO channels for various SNR's ($=\frac{P_t}{{\sigma_v}^2}$). Solid lines are outage capacity per antenna with optimal transmitter, and the dashed lines are outage capacity per antenna with equal power at all transmit antennas.

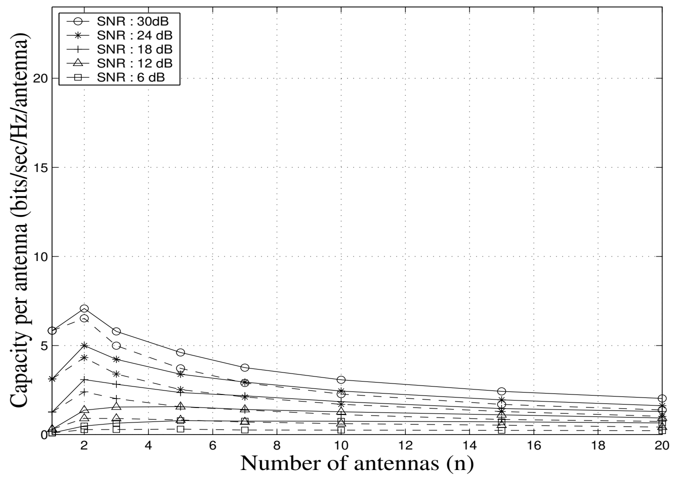

Fig. 2. Single user, correlated channels : 3% outage capacity per antenna for correlated channels and various SNR's ($=\frac{P_t}{{\sigma_v}^2}$). Two independent paths are assumed (L=2). Solid lines are capacity per antenna with optimal transmitter, and the dashed lines are outage capacity per antenna with equal power at all transmit antennas.

found from singular value decomposition of channel $\mathbf{H}$ followed by waterfilling [3].

When channel side information is not available at the transmitter, we adopt the approach in [1] and put equal power in each transmit antenna. In such cases, $\mathbf{R}_X$ equals $\frac{P_t}{n}\mathbf{I}_n$.

$$C_{S,Eq} = \log_2\left(\left|\mathbf{I}_n + \frac{P_t}{n{\sigma_v}^2}\mathbf{H}\mathbf{H}^*\right|\right) \tag{7}$$

This serves as an upper bound for channel capacity as exact bit loading is impossible without channel side information at transmitter side. However, this is known to be a good measure to use when outage capacity is of concern. For quasi-static channel, where channel slowly varies with time, capacity is not easy to define [5], and outage capacity is one of the best figure of merits.

B. Uncorrelated channels

In a wireless system with a rich-scattering environment, elements of MIMO channel can be modeled as uncorrelated random variables. For uncorrelated channels where all elements of $\mathbf{H}$ are assumed to have i.i.d Gaussian distributions, linear increase of capacity with increasing number of antennas was reported in [1] under the assumption of perfect channel information only at receiver side. In Figure 1, same results are reproduced. The dashed lines are 3% outage capacity per antenna, $\frac{C_{S,Eq}}{n}$, with equal power distribution at the transmit antennas. The solid lines are 3% outage capacity per antenna, $\frac{C_{S,Opt}}{n}$, with perfect channel side information at both transmitter and receiver. The simulation results are obtained with a large number of random channel realization. From the figure, we observe that the outage capacity per antenna converges to a constant as the number of antenna (n) increases. Also, it is interesting to note that $C_{S,Eq}$ converges to $C_{S,Opt}$ for large SNR, meaning channel side information at transmitter does not result in significant advantage for single user system for uncorrelated channels. This can be easily understood by comparing waterfilling and flat energy distributions at high SNR.

C. Correlated channels

For uncorrelated channels, channel capacity grows linearly with increasing number of antennas, and the capacity per antenna converges to a non-zero constant, which is a function of SNR. However this is not true for highly correlated channel with a fixed number of independent paths, whose channel model is given by (2). We first show that the capacity per antenna goes to zero for fixed rank channels as the number of antennas goes to infinity.

Theorem 1. *For a single user system with n transmit antennas and n receive antennas, if the channel has a fixed rank (=L), the channel capacity per antenna converges to zero as n goes to infinity.*

This is not hard to prove, and we give a simple argument starting from *Lemma 1.*

Lemma 1. *For a MIMO channel with only one path (L=1), the single user capacity per antenna converges to zero as the number of antennas goes to infinity.*
Lemma 1 is true because the channel pass space is one dimensional. In this case, the optimal transmission scheme is to beamform to the pass space ($\frac{\mathbf{A_T}^*}{\sqrt{n_T}}$), and the optimal receiver is a space matched filter in $\frac{\mathbf{A_R}^*}{\sqrt{n_R}}$ direction. Therefore the system can be simplified to a scalar system, $y = h\sqrt{n_T n_R}x + v$ where x is a complex scalar transmitted signal with power P_t, and v is a scalar AWGN noise with variance ${\sigma_v}^2$. Note that we achieved n_T and n_R power gain by beamforming at transmitter and receiver respectively. Thus, for $n_T = n_R = n$,

$$\lim_{n\to\infty} \frac{C_{S,Opt}}{n} = \lim_{n\to\infty} \frac{1}{n} \log_2\left(1 + \frac{n^2|h|^2 P_t}{{\sigma_v}^2}\right) = 0$$

To prove the case for $L > 1$, we upper bound the channel capacity per antenna and show that the upper bound converges to zero. Consider L independent systems with channels $\mathbf{H}_1, \mathbf{H}_2, \cdots \mathbf{H}_L$, each with power constraint P_t. If we choose the same transmit signal for all L systems and sum all L received signals, the sum capacity of these L systems can be shown to be larger than the original single system with channel $\mathbf{H} = \sum_{l=1}^{L} \mathbf{H}_l$. So, the sum capacity of these L systems is an upper bound. Furthermore, each system with channel $\mathbf{H}_l$ has zero capacity per antenna for infinitely large number of antennas according to *Lemma 1.* Consequently, the sum capacity per antenna also converges to zero.

In Figure 2, the simulation results on 3% outage capacity per antenna for channel model (2) are plotted. Number of independent paths are two ($L = 2$), and Raleigh's channel model with rank L was used for random channel generation. We note that the results are similar for other channel models as long as the channel rank is kept at L (cf. channel rank is n if $n < L$).

IV. Asymptotic behavior of capacity : Multiuser systems

In this section, we show that the sum capacity per antenna does not converge to zero if multiuser diversity is utilized. Total power used by all K users is fixed at P_t for a fair comparison with single user systems. Also, the same amount of power($P_{mu} \doteq \frac{P_t}{K}$) is assumed for each user for the fairness among users. For simplicity, we assume the same number of transmit and receive antennas ($n_T = n_R = n$), and assume a fixed ratio between the number of users and the number of antennas. ($\frac{n}{K}$ = constant).

For multiuser systems, multiple access channel (uplink) and broadcast channel (downlink) need to be considered [4]. From these two situations, multiple access channel is easier to analyze as it has an explicit formulation and an efficient algorithm for sum capacity [6]. Therefore, we concentrate on multiple access channel for simulations.

A. Capacity formulation of multiple access channels

The mutual information between $\{\mathbf{X}_k\}$ and $\mathbf{Y}$ can be formulated as follows when $\{\mathbf{X}_k\}$ has Gaussian distribution.

$$\mathbf{I}(\mathbf{X}_1, \mathbf{X}_2 \cdots \mathbf{X}_K; \mathbf{Y}) = \log_2\left(\left|\mathbf{I}_n + \frac{1}{{\sigma_v}^2}\sum_{k=1}^{K} \mathbf{H}_k \mathbf{R}_k \mathbf{H_k}^*\right|\right)$$

$\mathbf{R}_k = E[\mathbf{X}_k \mathbf{X_k}^*]$ is the covariance matrix of $\mathbf{X}_k$, and $|\cdot|$ is the determinant operation. The power used by user k can be expressed as $Tr(\mathbf{R}_k)$. Hence, finding the sum capacity for the multiple access channel can be formulated as an optimization problem over $\mathbf{R}_1, \mathbf{R}_2 \cdots \mathbf{R}_K$ with power constraints.

$$C_{mu,Opt} = \max_{\mathbf{R}_1 \cdots \mathbf{R}_K} \mathbf{I}(\mathbf{X}_1, \mathbf{X}_2 \cdots \mathbf{X}_K; \mathbf{Y}) \quad (8)$$

subject to $Tr(\mathbf{R}_k) \le P_{mu}$ for all $k = 1, 2, \cdots K$

It can be shown that this problem is convex [7], and therefore efficient numerical optimization algorithms exist. Analysis on this maximization problem can be found from [6].

If channel side information is not available at transmitters, then $\mathbf{R}_k = \frac{P_{mu}}{n}\mathbf{I}_n$ is chosen for transmit signals, and the following can serve as the upper bound of sum capacity.

$$C_{mu,Eq} = \log_2\left(\left|\mathbf{I}_n + \frac{P_{mu}}{n{\sigma_v}^2}\sum_{k=1}^{K} \mathbf{H}_k \mathbf{H_k}^*\right|\right) \quad (9)$$

B. Uncorrelated channels

For uncorrelated channels, full diversity (n_R) gain is already achieved with single user systems. Therefore the only additional gain attainable by utilizing multiuser diversity is power gain. The simulation results are shown in Figure 3. The ratio of capacity gain over a single user system is not negligible for large number of antennas or low SNR, but the gain is much less for small number of antennas or high SNR.

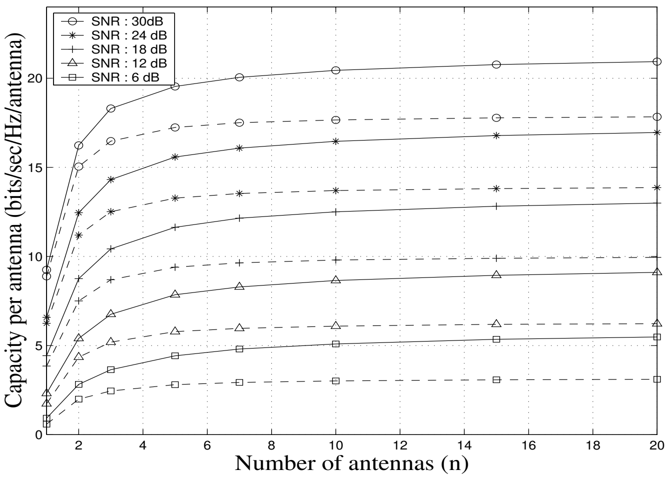

Fig. 3. Multiuser, uncorrelated channels : Sum of 3% outage capacity per antenna of uplink i.i.d Gaussian MIMO channels for various SNR's ($=\frac{P_t}{{\sigma_v}^2}$). Solid lines are sum of outage capacity per antenna with optimal transmitter, and the dashed lines are sum of outage capacity per antenna with equal power at all transmit antennas.

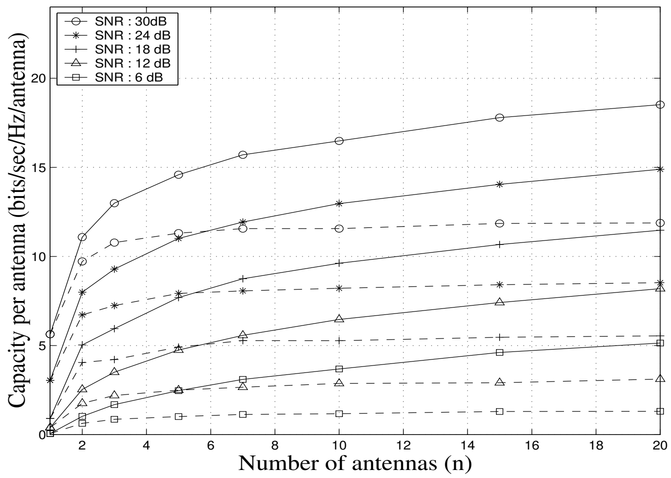

Fig. 4. Multiuser, correlated channels : Sum of 3% outage capacity per antenna of uplink correlated channels for various SNR's ($=\frac{P_t}{{\sigma_v}^2}$). Two independent paths are assumed (L=2). Solid lines are sum of outage capacity per antenna with optimal transmitter, and the dashed lines are sum of outage capacity per antenna with equal power at all transmit antennas.

C. Correlated channels

For a single user system with a correlated channel, capacity per antenna was shown to converge to zero. We show that this problem can be relieved if many users simultaneously share a common frequency band.

Theorem 2. *For a multiuser multiple antenna system with K users each with n antennas, and a base station with n antennas, let each user have a power constraint $P_{mu} = \frac{P_t}{K}$ (so the total power is P_t). Let the MIMO channel between each user and the base station be of fixed rank L and be independent of each other. As both K and n go to infinity while keeping their ratio fixed, the multiuser sum capacity divided by n is bounded away from zero.*

Theorem 2 holds because n multipaths with independent $\{A_{R_k,l}\}$ (arriving vectors for multiple access channels and departing vectors for broadcast channels) can be decoupled with n antennas, and because signal power per path can be kept at a constant with multiple antenna power gain. To prove *Theorem 2*, we lower bound the sum capacity per antenna and show that the lower bound does not converge to zero. For multiple access channel, let $\min(K, \lfloor \frac{n}{L} \rfloor)$ users transmit $\mathbf{R}_k = P_{mu}\mathbf{I}_n = \frac{P_t}{K}\mathbf{I}_n$. Then the number of incoming paths to the base station with non-zero power is guaranteed to be not larger than n, and also guaranteed to increase linearly with n as K grows linearly with n. For each path, there exists a n-dimensional linear filter which nulls all signals except for the desired signal related to the path since there are maximum n independent incoming paths. Hence, signal related to each path can be decoded independently of other signals, and each path has capacity larger than a positive constant for arbitrary large n. The constant depends on the statistics of $A_{R_k,l}$ (i.i.d Gaussian case was shown in [1]). Since the number of paths grows linearly with n, the total sum capacity divided by n is larger than a positive constant.

For the broadcast channel with channel side information at the transmitter, by nulling at the transmitter, it is possible to show that the sum capacity per antenna does not converge to zero. However the result is not clear when channel side information is not available at transmitter.

Figure 4 shows the simulation results for a multiple access channel with $L = 2$ and $\frac{n}{K} = 1$. The dashed lines are 3% outage capacity per antenna, $\frac{C_{mu,Eq}}{n}$, with equal power at all transmit antennas. The solid lines are 3% outage capacity per antenna, $\frac{C_{mu,Opt}}{n}$, with perfect channel side information at both transmitter and receiver. The simulation results are obtained with a large number of random channel realization. If the number of independent paths per user (L) is significantly smaller than the number of antennas (n), the capacity gap between a single user system and a multiuser system is large. Even though we have stated the result only for large n, this effect is readily observed for moderate number of antennas whenever $\frac{L}{n}$ is small enough.

V. Discussion : dimension and power

Two kinds of gain are possible with multiple antenna systems. One of them is *dimension gain* which is similar to diversity gain in [8]. The other one is *power gain* which is similar to coding gain in [8]. For uncorrelated channels, a single user channel already provides n_R dimension, and multiuser diversity can be used for obtaining power gain only. The increase in capacity due to power gain is small for high SNR range as the capacity is a logarithmic function of power. Since high SNR is already achieved by multiple receive antennas, multiuser diversity provides only moderate range of capacity gain for uncorrelated channels.

For correlated channels, each user's channel is rank-deficient, and part of receiver dimensions cannot be exploited for single user systems. These unused dimensions can be used by other users, and full dimensional gain is achieved by allowing a sufficiently large number of users to simultaneously share a common frequency band. Then, the multiuser sum capacity gain comes from both dimension and power, and a large capacity gain is possible at the expense of higher system complexity.

Transmitter optimization can be understood as a tool to achieve power gain. Even when channel information is not available at transmitter, maximum dimension gain can be achieved by transmitting equal amount of power to all dimensions. However, transmitter side power gain cannot be achieved in such case. Hence, as far as capacity is concerned, feedback of channel information to the transmitter is important only when power gain is important, as in the case of very low SNR where logarithmic function can be approximated by a linear function.

VI. Conclusions

In this paper, the multiuser diversity gain was found for both uncorrelated and correlated channels. The uncorrelated channel model is suitable for rich-scattering environments, and the multiuser diversity gain in this case was found to be moderate. This is because with an uncorrelated channel model multiuser diversity brings only power gain, but not dimension gain. The correlated channel model is suitable for outdoor environments where usually only a fixed number of far-field reflectors is present. The multiuser diversity gain in this case was found to be significant. This is because dimension gain as well as power gain is possible for correlated channels.

References

[1] Gerard J. Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment When Using Multi-Element Antennas," Bell Labs Technical Journal, , Autumn 1996

[2] R. B. Ertel, P. Cardieri, K. W. Sowerby and T. S. Rappaport "Overview of spatial channel models for antenna array communication systems," IEEE Personal Communications pp. 10-22, Feb 1998

[3] G.G Raleigh and J.M Cioffi, "Spatio-Temporal Coding for Wireless Communication," IEEE Transactions on Communications, vol 46 3 , pp. 357-366, March 1998

[4] T.M. Cover and J.A. Thomas, "Elements of Information Theory," Wiley, New York, NY 1991

[5] E. Biglieri, J. Proakis and S. Shamai, "Fading Channels: Information-Theoretic and Communications Aspects," IEEE Transactions on Information Theory, vol 44 6, pp. 2619-2692, October 1998

[6] W. Yu, W. Rhee and J.M. Cioffi, "Transmitter Optimization for Vector Multiple Access Channels," to be submitted

[7] S. Wu and S. Boyd, "Design and implementation of a parser/solver for SDPs with matrix structure,"Proceedings of the 1996 IEEE International Symposium on Computer-Aided Control System Design, pp 240-245, 1996

[8] V. Tarokh, N. Seshadri and A.R. Calderbank, "Space-Time Codes for High Data Rate Wireless Communication: Performance and Code Construction," IEEE Transactions on Information Theory, vol. 44, no 2, pp. 744-765, March 1998