Optimization of Wireless Access Point Placement in Realistic Urban Heterogeneous Networks

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***Abstract*—The placement of the access points (APs) has a significant impact on the wireless system performance, especially for irregular heterogeneous networks with hierarchical APs such as macro/micro base-stations, pico-stations, and femto-stations. Traditional system modeling and optimization are based on the regular 2D hexagonal cellular topology and a set of predefined large-scale propagation models, which is highly abstract and may be inaccurate. This paper considers the AP placement optimization problem in realistic deployment environments, where radio-wave propagation characteristics are accurately modeled using ray-tracing techniques. Toward this end, this paper proposes a novel concept of area proportional fairness utility for the entire network under a given user geographic distribution, and proposes an iterative method to optimize the placement of the APs for utility improvement while taking into account the mutual interference between the APs. The significant benefit of the placement optimization is shown in a numerical experiment conducted in a realistic Chicago downtown topology, where placement optimization is shown to improve the sum rate by up to 40%.**

I. Introduction

Future wireless networks are characterized by ubiquitous coverage and significantly higher data rates in comparison to the current conventionally deployed cellular networks. To achieve this goal, wireless operators are increasingly augmenting the traditional regular planned hexagonal wireless cellular topology with irregular heterogeneous infrastructure which may include hierarchical access points (APs), such as micro-, pico-, femto-, and relay-stations in addition to the conventional macro base-stations [1], [2]. In doing so, the wireless service providers are now facing new challenges in configuring such a multi-tier topology and in optimizing the placement of these lower-power nodes in realistic deployment scenarios.

Several issues need to be considered when planning for the deployment of the APs. In a dense urban area with various of artificial and natural structures (e.g., buildings, vegetation), the distance-dependent wireless signal attenuation law is no longer accurate because of the reflection and diffraction of the transmitted signals. Second, the average user traffic varies in different area, thus different levels of coverage are required in different parts of the network. In addition, the APs are typically deployed with maximal frequency reuse, so the placement of APs greatly affect the mutual inter-node interference. A good AP placement strategy must consider all the above factors. The optimal placement of APs is expected to have a significant impact on the overall performance of the network.

This paper addresses the AP placement optimization problem for the heterogenous network, and makes two main contributions to the state-of-the-art. First, this paper proposes a network utility metric that takes into account both the user geographical distribution throughout the network and the fair allocation of radio resources among the users. The proposed concept of area proportional fairness utility is easily computable, and can therefore be used as the objective function of an iterative method for AP placement optimization. Second, this paper utilizes realistic channel models obtained via an accurate 3D ray-tracing characterization of electromagnetic wave propagation. These models allow the effectiveness of AP placement optimization to be quantified in an actual deployment, while accounting for the mutual interference between the APs. Using an actual urban propagation model in downtown Chicago, the simulation results of this paper show that significant improvements in both the throughput and network utility can be obtained with intelligent placement of APs in a heterogenous network. To the best of authors' knowledge, this is the first concrete characterization of the benefit of AP placement optimization in a realistic scenario.

The site placement problem has been studied for many different scenarios in the literature, e.g., the distributed antenna systems (DAS) [3], [4], [5], the wireless relay networks [6], [7], [8], and the wireless sensor networks (WSN) [9], [10], [11]. Some of these studies on site placement impose restrictions on the network topology, e.g., linear cell structure [3], even site deployment on a circle [4], identical relative site positions in each cell [5], which, although make the model amendable to algorithmic development, are not realistic in practices. In particular, in [6], [7], [8], [11], relays cannot be placed anywhere but in a set of pre-defined candidate positions due to physical restrictions, which results in integer constraints on the location parameter. The present paper formulates a more general problem where the APs can be placed anywhere within the considered area, (although our algorithm can also be applied to the case where restrictions to candidate positions are imposed).

In the site placement literature for WSN (e.g., [9], [10], [11]), a common assumption is that the relay and base-station nodes are deployed in proper locations for aggregation and fusion of the data, which are captured by sensor nodes with

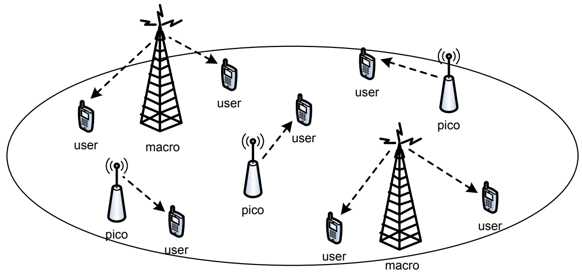


Fig. 1. An example of heterogeneous network

fixed locations throughout the designated area. This is in contrast to the wireless access network model considered in the present paper, where users are not fixed (due to mobility), thus the AP site locations need to be optimized for the given user distribution.

Finally, almost all existing works (with an exception of [5]) ignore inter-node interferences by either assuming a noise-limited environment or frequency reuse. In contrast, this paper assumes maximal reuse of resources among the APs in a dense urban scenario. This is a crucial consideration which significantly affects the interference pattern and therefore the optimal placement of APs in the network.

The AP placement problem can be formulated with various objectives (e.g., capacity enhancement, network lifetime maximization, power minimization, node number minimization). This paper proposes to use an area proportional fairness utility as the optimization objective function for the placement problem, which is defined in accordance with the standard fairness criterion used for resource allocation and scheduling. The area proportional fairness utility is a function of the users distribution, the transmitter spatial location, and the propagation environment. The computation of this utility can be facilitated by constructing a quantized grid in the considered area with sufficient resolution, where the radio-wave propagation from any AP to the test points in the grid can be computed in a deterministic way using ray-tracing prediction software such as the Volcano Lab [12]. We propose to optimize the placement of the APs from a given initial spatial locations to improve the utility in an iterative manner.

This rest of the paper is organized as follows. The general AP placement problem is formulated in Section II, where the concept of area proportional fairness utility is proposed and a quantized grid model is used to approximate and facilitate the computation of such utility in a realistic scenario. In Section III, an iterative algorithm is proposed to optimize the AP spatial locations. Results of numerical experiment are given in Section IV and conclusion is drawn in Section V.

II. System Model and Problem Formulation

Fig. 1 shows a two-tiered wireless heterogeneous network, where multiple macro-stations and pico-stations are deployed within the area to serve their designated users. Both the macro- and pico-stations have dedicated backhauls and signaling channels so that they can handle independent traffic, but may have different level of power and height and thus different coverage areas. From the operators' point of view, macro-stations are more regularly deployed to provide a basic ubiquitous coverage, and pico-stations may be irregularly deployed to extend coverage and further boost the system capacity.

A. Area Proportional Fairness Utility

For a fixed set of AP placement, we would like to quantify the performance of the network using a network-wide utility function that takes into account both the user density and fair allocation of resources. Toward this goal, we define an area proportional fairness utility as follows.

Denote the considered area as $\mathcal{S}$, and the number of APs and mobile users within this area as K and M, respectively. The total available resource blocks (e.g., frequency and time) for AP k is N_k. Each user is associated with one AP according to a maximum received power metric. Thus the covered zone of AP $k \in \{1, ..., K\}$, denoted as $\mathcal{S}_k \subset \mathcal{S}$, has an area of

$$|\mathcal{S}_k| = \iint_{\mathcal{S}} \mathbf{1}\left(P_k^{(x,y)} > P_{l,\forall l \neq k}^{(x,y)}\right) dxdy \qquad (1)$$

where $\mathbf{1}(\cdot)$ is the indicator function, and $P_k^{(x,y)}$ is the measured received power from AP k at the coordinate (x, y). Note that $P_k^{(x,y)}$ corresponds to the narrow-band response containing only the large-scale fading components (pathloss and shadowing). At the coordinate (x, y) which is covered by AP k, the average signal-to-interference ratio (SIR) can be approximated as follows (ignoring the noise):

$$\eta_k^{(x,y)} = \frac{P_k^{(x,y)}}{\sum_{l \neq k} P_l^{(x,y)}} \qquad (2)$$

Let $\delta^{(x,y)}$ be the density of 2D user distribution at coordinate (x, y). The average number of users covered by AP k can be computed as:

$$\begin{aligned} M_k &= \iint_{\mathcal{S}_k} \delta^{(x,y)} dxdy \\ &= \iint_{\mathcal{S}} \mathbf{1}\left(P_k^{(x,y)} > P_{l,\forall l \neq k}^{(x,y)}\right) \delta^{(x,y)} dxdy \qquad (3) \end{aligned}$$

For uniform user distribution, the number of associated users of any AP is proportional to its coverage, i.e., $M_k \propto |\mathcal{S}_k|$.

To ensure that each user has a fair allocation of resources, we now assume all the associated users for a particular AP to have the same channel access probability, i.e., each AP allocates equal resources N_k/M_k to each of its users. This can be achieved for example by round-robin scheduling. This is also valid for proportional fairness scheduling if users have homogeneous small-scale fading statistics [13].

In a fully reused network where $N_k = N, \forall k$, we normalize the per-user resource to be $1/M_k$, and define the density of utility at the coordinate (x, y) covered by the AP k as the logarithm of the normalized average user rate:

$$U_k^{(x,y)} = \delta^{(x,y)} \log\left\{\frac{1}{M_k}\log_2\left(1 + \eta_k^{(x,y)}/\Gamma\right)\right\} \qquad (4)$$

where $\Gamma = -\ln(5\text{BER})/1.5$ is the SNR gap. The total area proportional fairness utility is then:

$$U = \sum_k U_k = \sum_k \iint_{\mathcal{S}_k} U_k^{(x,y)} dxdy$$
$$= \sum_k \iint_{\mathcal{S}} \mathbf{1}\left(P_k^{(x,y)} > P_{l,\forall l \neq k}^{(x,y)}\right) \delta^{(x,y)}$$
$$\times \log\left\{\frac{1}{M_k}\log_2\left(1+\eta_k^{(x,y)}/\Gamma\right)\right\} dxdy. \quad (5)$$

This utility captures the performance of the entire network under standard fairness scheduling criterion. The main idea of this paper is that this utility is a function of the AP spatial locations, can thus be used for the optimization of the AP placement.

B. Realistic Propagation Modeling in Urban Scenario

For a given user density distribution, the network utility (5), which is based on (2) and (3), is only a function of the narrow-band received power $P_k^{(x,y)}$. In traditional wireless propagation models, the received power can be computed from pathloss and shadowing models given the transmitter location and power. The pathloss and shadowing models, however, can be highly inaccurate in actual urban deployments, where reflection and diffraction of the signals have significant impact. This paper adopts a much more realistic ray-tracing-based multipath radio-wave propagation model based on real-world geographic information. Such models are indispensable in the proper optimization of AP placement.

We have conducted an experimental measurement of the realistic received power map in the Chicago downtown environment, as is shown in Fig. 2. The black hexagonal edges are the imaginary cell borders. The blocks model different types of urban structures (e.g., building, bridge), each with distinct physical attenuation properties. Each structure is characterized by a polygon and its associated height information. Macro-stations are deployed outdoors in regular hexagonal grid points, which can be either at the street level or at a rooftop. The 3D transmit radiation pattern is used. The colors represent the maximum received signal power in dBm (from one of the macro-stations) on each pixel of the map at a height of 1.5 meters. It is observed that the greater received signal strength can be found on the streets, while the receivers within the structures experience low power due to the penetration loss.

C. A Quantized Grid Approach

To enable the calculation of the area proportional fairness utility (5), we quantize the 2D receiver area $\mathcal{S}$ into a rectangular grid of test points. Each test point represents a possible receiver location, for which the radio propagation characteristics can be computed. For each AP, we can compute the propagation from its transmitter to this defined area. For each test point in the grid, measurements from all K APs can then be collected. In practice, the measurements from each AP can be stored in a matrix, with entries corresponding to the test points in the grid. For the K APs, a total of K matrices (with the same dimension if all APs have the same receiver area $\mathcal{S}$ and quantization resolution) can be formed. The measurements can be:

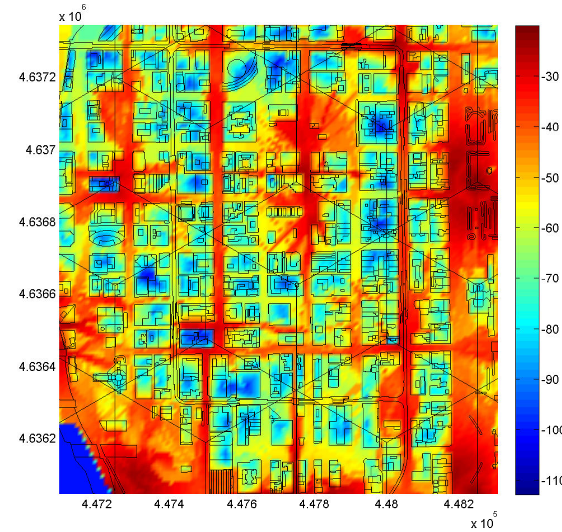


Fig. 2. The received signal power (dBm) in the Chicago downtown, where the x- and y-coordinates are measured in meters.

- Narrow-band received power which contains all the large-scale fading effect, i.e., pathloss and shadowing, but combines all the signals coming from different paths;
- Wide-band response which contains the frequency-selective fading. This is computed by the ray-tracing techniques considering the reflection and diffraction of the signal in a realistic urban environment.

In this paper, the area proportional fairness utility is computed as a function of the narrow-band received power. Subsequently, wide-band response is used for throughput evaluation.

If the coordinate (x, y) is quantized with index (i, j), the received power from AP k at this coordinate is efficiently computed as $P_k^{(i,j)}$, and similarly for the SIR $\eta_k^{(i,j)}$, the utility density $U_k^{(i,j)}$, and the user density $\delta^{(i,j)} = \delta^{(x,y)} s^2$, where s is the width of the grid pixels. The area proportional fairness utility (5) can now be rewritten as:

$$U = \sum_k \sum_{(i,j)\in\mathcal{S}_k} \delta^{(i,j)} \log\left\{\frac{\log_2\left(1+\eta_k^{(i,j)}/\Gamma\right)}{\sum_{(i,j)\in\mathcal{S}_k}\delta^{(i,j)}}\right\} \quad (6)$$

where S_k is the quantized coverage of the AP k:

$$\mathcal{S}_k = \left\{(i,j)|P_k^{(i,j)} > P_{l,\forall l\neq k}^{(i,j)}\right\} \quad (7)$$

In the next section, we develop an algorithm to deploy the APs at places to optimize the above utility.

III. Utility Based AP Placement

With a fixed radiation pattern, power, and geographical information, the area proportional fairness utility is determined by the spatial locations of the APs. The spatial location $L_k = (x_k, y_k, z_k)$ contains both the 2D coordinates and the height information z_k of the transmitter. In this paper, we consider outdoor transmitters which can be either tower-based or rooftop-based. We set the reference height $H_{\text{ref}}^{(x,y)}$ for any coordinate (x, y) to be either 0 if it is not within any structure, or the height of the highest structure covering that coordinate. The height of the AP k at coordinate (x_k, y_k) is set as:

$$z_k = \max\left(H_{\text{tower}},\ H_{\text{ref}}^{(x_k,y_k)} + H_{\text{rooftop}}\right) \tag{8}$$

where H_{tower} is the absolute height of the AP transmitter if it is tower-based, and H_{rooftop} is the relative height of the transmitter above the rooftop of the structure it overlaps. Equation (8) guarantees that the transmitter is placed at least H_{rooftop} meters higher than the rooftop, and is at least higher than the absolute tower height H_{tower}. Note that the APs can be placed at arbitrarily coordinates and are not limited to the quantized test points.

We propose to iteratively adjust the spatial location of the APs, ensuring that each step of the algorithm improves the area proportional fairness utility. The algorithm starts at some initial AP placement which are planned by the operator, and proceeds sequentially and repeatedly for all the APs within the considered area, where the AP is updated to a neighboring candidate location each time. The iteration terminates until no further utility improvement can be found.

To compute the area proportional fairness utility (6), we need to first compute all the received power matrices over the considered area for all the APs given a set of input parameters. *Algorithm 1* summarizes the procedure to obtain such a received power matrix along with the spatial location of a AP k.

Algorithm 1: Compute the spatial location L_k and the received power matrix $\mathbf{P}_k$ for the AP k with coordinate (x_k, y_k).

1) Find the reference height $H_{\text{ref}}^{(x_k,y_k)}$ of the coordinate (x_k, y_k) by first determining whether it is within any of the structures defined by polygons. This is a 2D point-in-polygon (PIP) problem. Given the vertex coordinates of a polygon, the PIP problem can be efficiently solved by the ray-intersection algorithm [14] with a time complexity in the order of the number of edges of that polygon.
2) Set the height z_k of the AP k as in (8).
3) Use ray-tracing techniques to compute the received power matrix $\mathbf{P}_k = \left[P_k^{(i,j)}\right]_{i,j}$ for the AP k over the test points $(i, j) \in \mathcal{S}$ with the spatial location $L_k = (x_k, y_k, z_k)$.

Using the received power matrices from *Algorithm 1*, the initial utility can now be computed as in (6) and expressed as a function of the AP spatial locations, $U(L_1...L_k...L_K)$. The iterative placement optimization algorithm can now be carried out as in *Algorithm 2* below.

Algorithm 2: Iterative placement optimization of the APs with initial spatial location $L_1...L_K$.

1) Fixing the spatial locations of all other APs $l \neq k$ at the current stage and compute their power matrix $\mathbf{P}_l$, do the following for the AP k:
   a) Search for V candidate neighboring coordinates $\left(x_k^{(v)}, y_k^{(v)}\right)$, $1 \le v \le V$, around the current coordinate (x_k, y_k) within a given radius.
   b) Follow *Algorithm 1* to get the spatial location $L_k^{(v)}$ and the power matrix $\mathbf{P}_k^{(v)}$ for each candidate coordinate.
   c) For each candidate v, compute the area proportional fairness utility $U\left(L_1...L_k^{(v)}...L_K\right)$ as in (6).
   d) Update the AP k to the candidate spatial location $L_k = L_k^{(\hat{v})}$ with maximal positive utility increment $\hat{v} = \arg\max_v \left\{U\left(L_1...L_k^{(v)}...L_K\right) - U\left(L_1...L_k...L_K\right)\right\}$.
2) Turn to the AP $k+1$ for placement optimization by repeating the above steps. Return to AP 1 after AP K. Terminate the algorithm when no further positive utility improvement can be found for all APs.

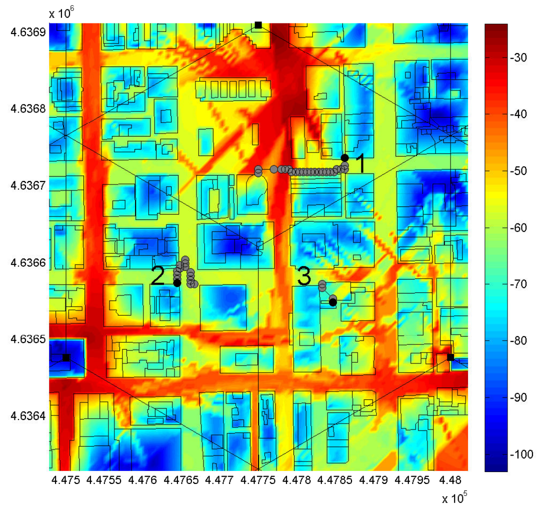
The above procedure is essentially greedy, with complexity scaled as $\mathcal{O}(KV)$ in each iteration. Physical constraints should be imposed such that the APs are not put at any infeasible places, e.g., inside a river. No lower bound on the inter-node distance [11] is needed since the enlarged interference would decrease the utility and thus repel the nodes from being too close to each other. In practice, due to the limited 3D geographical information and the computational burden, the total coverage of a wireless service provider should be partitioned into computable clusters. The algorithm is off-line and needs only to be implemented once for a given topology and user distribution.

IV. Numerical Experiment

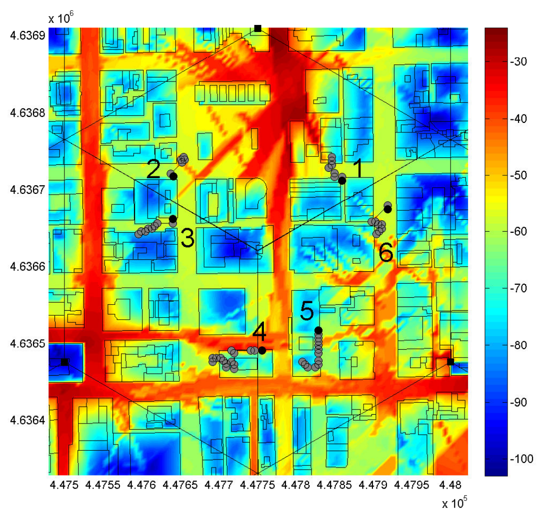
In this section, we optimize the AP placement using the proposed algorithm in a Chicago downtown deployment as in Fig. 2 with an inter-site distance about 500m, corresponding to the typical dense urban scenario with about 21 hexagonal sectors. The carrier frequency is 2GHz. The receiver area $\mathcal{S}$ measured for all APs is rectangular with dimension 600m by 550m, containing the central three sectors in Fig. 2. The APs outside this area are not considered for placement optimization, but nevertheless generate an interference pattern. The grid has a 5m quantization resolution, i.e., $s = 5$. The receiver height at each test point is set to be 1.5 meters, which is the height of typical pedestrian's phone. We assume that regular macro-stations have already been deployed with directional 3D antenna and 46dBm transmit power, whereas the omni pico-stations with 30dBm power are to be placed to further enhance the system performance. Initially, the pico-stations are placed evenly on a circle which is 2/3 radius away from the macro-station in each cell. The height of macro and pico-stations are set to:

$$z_k = \begin{cases} \max\left(32,\ H_{\text{ref}}^{(x_k,y_k)} + 2\right) & \text{for macro} \quad (9a) \\ \max\left(5,\ H_{\text{ref}}^{(x_k,y_k)} + 1\right) & \text{for pico} \quad (9b) \end{cases}$$

For simplicity, the user distribution density is assumed to be a unit constant in our evaluation, i.e., $\delta^{(i,j)} = 1$.



(a)



(b)

Fig. 3. The trajectory of the pico-stations during the optimization process, where the solid black dots are the final placement of the pico-stations. (a) 1 pico-station per sector. (b) 2 pico-stations per sector.

A. The Trajectory of the Access Points

We adopt the following approach in our experiment for step 1a) of *Algorithm 2*. Each AP searches on a circle with radius $t = 5m$ from its current coordinate with 8 evenly distributed directions to find $V = 8$ candidate neighboring coordinates. Follow steps 1b) to 1d) to find a candidate spatial location to update. If no positive utility increment can be found, increase the search radius t by 5m each time up to 30m, and repeat step 1a) to 1d) again until positive utility can be found.

Fig. 3 shows the trajectory of the pico-stations within the placement optimization process. The solid black dots are the final placement of pico-stations. The color bar represents the received power from macro-stations (in dBm). It is interesting to observe that the pico-stations are trying to avoid places with strong interference from the macro-stations (e.g., pico-station 1). It is also interesting to observe that in Fig. 3(b):

- Pico-station 2 and 6 move along a street;
- Pico-station 5 moves on a rooftop;
- Pico-station 1 and 3 move from a rooftop to a street;
- Pico-station 4 moves from a street to a rooftop.

Thus, AP placement optimization process can be quite nontrivial—many different kinds of possibilities exist.

Fig. 4 shows the increase in area proportional fairness utility in the iterative process. As expected as pico-stations move toward optimized placement, the utility monotonically increase until convergence.

B. Throughput Evaluation

To evaluate throughput, we consider 10MHz bandwidth with 50 orthogonal subchannels. Using ray-tracing techniques, it is possible to compute the wide-band frequency-correlated fading of the subchannels in a complex urban deployment. The background noise density is set to -174dBm/Hz. An average of 30 users are uniformly distributed in each sector, but the user locations are quantized to the nearest test points on the grid. A proportional fairness scheduler is used for scheduling.

The cumulative distribution function (CDF) of SIR and per-user rate before and after the placement optimization is shown in Fig. 5. It is interesting to note that without optimizing the placement, the deployment of pico-stations brings almost no improvement in SIR as compared to the no-pico case, while optimized placement can bring about 1dB SIR improvement

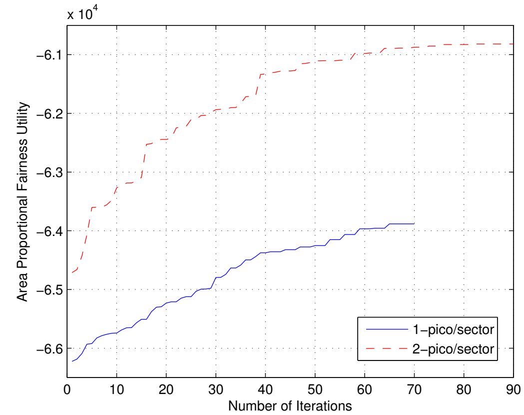
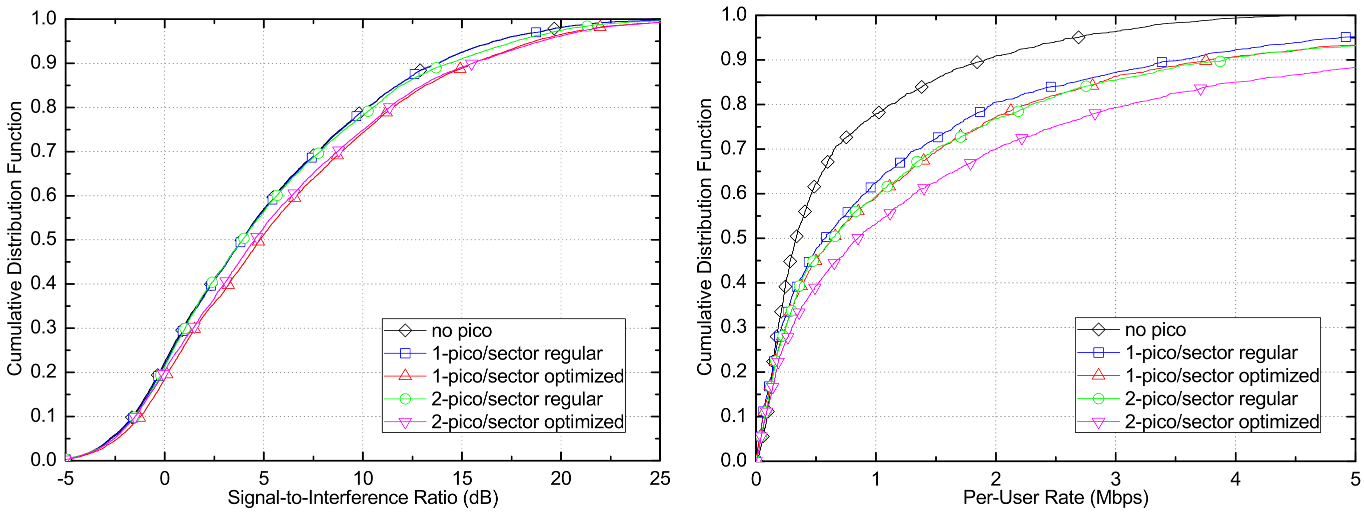


Fig. 4. The area proportional fairness utility versus the number of iterations

TABLE I
AGGREGATE PER-SECTOR PERFORMANCE

	Sum rate (Mbps)	5% rate (Kbps)	Utility[a]
No pico	20.74	50.85	-30.70
1-pico/sec. regular	38.46	28.00	-21.62
1-pico/sec. optimized	49.30	30.40	-18.07
2-pico/sec. regular	45.55	38.03	-17.03
2-pico/sec. optimized	64.68	38.55	-9.71

[a] The utility is computed as a summation of the logarithm of the scheduled users' rate (in Mbps).



(a) (b)

Fig. 5. The cumulative distribution function (CDF) before and after the AP placement optimization. (a) Signal-to-interference ratio (SIR). (b) Per-user rate.

at the 50% range. Thus, without placement optimization, the benefit of pico-cell deployment comes solely from the fact that pico-stations offload users from the macro-station, thus each user gets proportionally more resources. Note that after the placement optimization, the 1-pico/sector case actually has better SIR than 2-pico/sector case. This is because more interference is present in a network with more pico-stations deployed. Differing from the SIR case, the deployment of 1 or more pico-sites per sector has a positive impact on the rates. Fig. 5(b) shows that significant improvement in terms of the user-rate CDF can be obtained from placement optimization.

The aggregate per-sector performance in listed in Table I further confirms the importance of the AP placement. The utility here is the conventional proportional fairness metric, computed with the average transmitted user rates. With optimized placement, the sum rate can be improved by about 28% and 40% for 1-pico/sector and 2-pico/sector case, respectively. Note that the cell edge 5% rate actually decreases when pico-stations are added to the network. This is due to the rise in interference when pico-stations are deployed. The proposed algorithm alleviates the cell edge deterioration.

V. Conclusions

This paper considers the AP placement optimization problem for irregular heterogeneous wireless networks. We show that geographical information and 3D ray-tracing techniques are necessary for realistic evaluation of the heterogeneous network performance, and that placement optimization can bring significant benefit to the overall network performance, improving the sum rate by 40% in a realistic urban environment. The key ingredients of the proposed approach are an accurate radio propagation model integrating geographical structures and 3D spatial locations for the APs, a novel concept of area proportional fairness utility that accounts for both user distribution and fair resource allocation, and an iterative algorithm for optimizing the spatial locations of the AP based on the proposed utility function while accounting for mutual interference.

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